

# THE PHYSICAL REVIEW.

## IONIZATION AND EXCITATION OF RADIATION BY ELECTRON IMPACT IN NITROGEN.

BY BERGEN DAVIS AND F. S. GOUCHER.

THE purpose of the present paper is to present some of the experimental results obtained in further experiments of the writers on ionization and excitation of radiation by electron impact in various gases.

The method of experimentation employed was in principle that used in connection with our experiments on mercury vapor and hydrogen and the reader is referred to our paper for a detailed description of the same.<sup>1</sup>

This method is simply a modification of that applied by Frank and Hertz and others for the determination of the ionizing potentials of various gases, with the end in view of differentiating between the effects due to radiation from the bombarded atoms and those due to ionization.

This modified method may be summarized by reference to the schematic diagram Fig. 1 as follows. Electrons from a source *A* are accel-

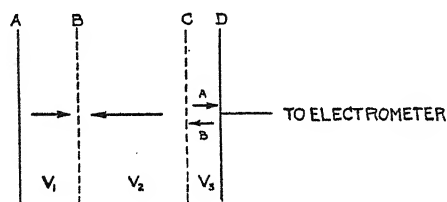


Fig. 1.

erated by an applied potential  $V_1$  through a gauze *B* into a region *BCD*, *C* being a second gauze and *D* a collecting plate connected to an electrometer.  $V_2$  is a retarding field applied between *B* and *C*, and  $V_2$  is maintained at a constant amount greater than  $V_1$  to prevent the original electrons from *A* reaching the plane of the gauze *C*.  $V_3$  is small as compared with  $V_1$  and  $V_2$  and determines the direction of the field in the

<sup>1</sup> PHYSICAL REVIEW, August, 1917.

neighborhood of *C* and *D*. The original electrons from *A* will be capable of making impacts with atoms or molecules in the space between *A* and *C*; and, depending on the energy imparted to them by  $V_1$ , may result in either the production of radiation from some of the impacted atoms or ionization or both. If the gauze *C* were not present and *D* in the place of *C* either of these results might cause a positive charging up of *D* due to the direction of  $V_2$ ; if the cause were radiation alone it might cause a photoelectric emission of electrons from *D*, and if it were ionization alone the positive ions formed would travel to *D*. But with the small controlling field  $V_3$  present the effects arising from these two causes can be differentiated, for if the cause be radiation alone it would produce a photoelectric emission of electrons from either *C* or *D* dependent on the direction of  $V_3$  and a consequent charging up of *D*—negatively if  $V_3$  is in direction (*a*), and positively if in direction (*b*), if however the cause be ionization alone the positive ions formed in the region *BC* will be capable of reaching *D* in either case— $V_3$  being small as compared with  $V_2$ . As however both of these causes may occur simultaneously we may have a combination of these effects. Consequently the method consists in varying the applied potential  $V_1$  maintaining  $V_2 - V_1$  at a constant small value, and obtaining two current curves as against  $V_1$ . One for  $V_3$  being in direction (*a*); and the other for  $V_3$  being in the direction (*b*). The shape of these curves and position of the intercept on the  $V_1$  axis indicates the energy required to produce the different effects for any given gas.

The apparatus and experimental details were essentially the same as those used for mercury vapor and hydrogen. Certain improvements in design of the tube in which the measurements were made are worthy of note however. The same platinum thimble equipotential surface electron source was used, but the amount of metal in the tube was reduced considerably by using spiral wire grids wound on pyrex glass supports in place of the gauzes, with their heavy metal end supports. The grid *B* was of fine platinum wire about no. 30 and about 10 turns to the inch. The grid *C* was of nickel no. 20 and about 5 turns to the inch. The collecting electrode *D* was also of nickel and supported on a glass tube insulated by two grounding rings. All the seals were of glass, thus eliminating the use of cemented joints. This design made it possible to get rid of a much larger portion of the residual gases in the tube both on account of the reduction of the amount of metal and because the tube could be baked out as a whole and the grids glowed by means of current furnished from the outside.

The nitrogen used was prepared by the usual chemical method and



was purified by washing with  $\text{H}_2\text{SO}_4$  and  $\text{NaOH}$  and then passing over hot copper. It was collected over distilled water and before being introduced into the measuring system was dried by passing through a phosphorus pentoxide tube.

The streaming method was employed; the pumps were kept running and the gas was allowed to pass through a porous plug inserted in the bore of a stopcock. By exposing more or less of the surface of the plug and regulating the speed of the pump desirable pressures could be maintained at a constant value as measured by a McLeod gauge.

*Experimental Results.*—From a large number of curves obtained under varying conditions of pressure and intensity of electron emission from the source, the curves shown in Figs. 2-5 were selected as being typical.

Fig. 2 shows that by reversing  $V_3$  which was of the order of 2 volts in

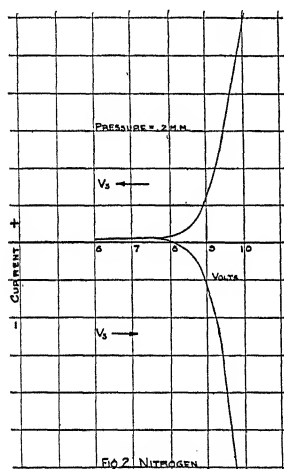


Fig. 2.

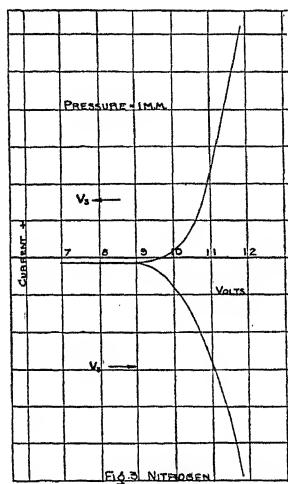


Fig. 3.

all these experiments the current curve could be made to reverse, indicating that the effect is produced largely if not entirely by radiation from the bombarded atoms. It is to be noticed that the curves have a horizontal portion in common. This is due to a very slight leak to the electrometer possibly from the glass. This could not seem to be avoided within small limits and was sometimes positive and sometimes negative. But the essential thing is that the two curves break away from this natural leak line at the same point which is about 7.5 volts, a result consistent with the results obtained by one of us and also as found by other experimenters.

There seems to be quite a marked increase in this radiation at about

9 volts; the curves being very steep after 9 volts. This is better shown in Fig. 3 where the electron emission was much less intense, here the curves do not separate until 9 volts had been reached. The current curve, Fig. 4, was obtained with a very small emission from the source and at low pressure a condition which is most favorable for ionization effects with this type of apparatus. This shows the effect to be radiation largely up to a value of about 17.5 to 18 volts. At this value the curve rises rapidly in spite of the opposing field  $V_3$  showing that ionization has set in.

This then shows that the ionization potential of nitrogen is about 18 volts a value considerably higher than that heretofore supposed. This

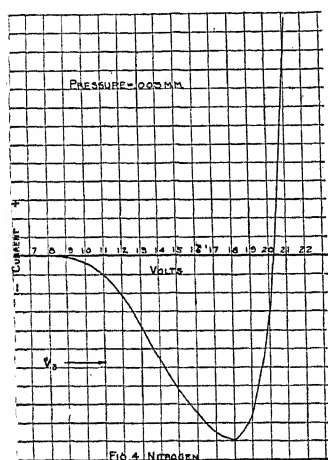


Fig. 4.

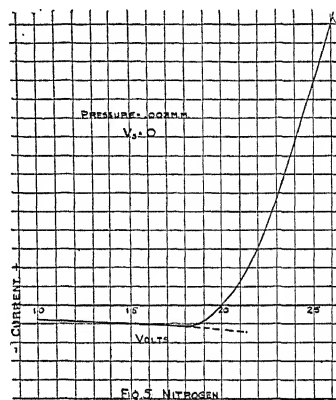


Fig. 5.

result however is more in accord with the work of Johnson on the total ionization by slow electrons. His straight line relation between total ionization and volts points more nearly to 20 than to 10 volts, though this method could not be expected to give an accurate value for the minimum ionization potential.

The intense radiation produced at about 9 volts is of interest in connection with the work of Meyer<sup>1</sup> who found that the wave-length about  $1300 \text{ \AA}^\circ$  stimulated the fluorescent band  $3064 \text{ \AA}^\circ$  in nitrogen. The value  $1300 \text{ \AA}^\circ$  from the quantum relation  $Ve = hn$  would correspond to a voltage of the order of 9 applied to an electron.

Fig. 5 shows the current curve obtained when  $V_3 = 0$ . There is evidently not a complete neutralization of the radiation effect but the ionization effect is shown to set in strongly at about 18 volts.

<sup>1</sup> PHYSICAL REVIEW, November, 1917.

SUMMARY.

These experiments show.

(a) Radiation can be stimulated in nitrogen molecules by electron bombardment without any appreciable ionization of the same.

(b) The value 7.5 volts heretofore accepted as the value of the ionizing potential really corresponds to the least energy an electron must have in order that it may excite radiation by the bombarded molecule.

(c) There is an indication of a more intense type of radiation setting in at about 9 volts.

(d) That ionization sets in at about 18 volts.

These experiments were completed more than a year ago, but stress of circumstance has delayed their publication to the present time.

PHOENIX PHYSICAL LABORATORY,  
September, 1918.

## ON A KINETIC THEORY OF MAGNETISM IN GENERAL.

BY KÔTARÔ HONDA AND JUNZÔ ÔKUBO.

1. In the last fifteen years, our knowledge of the nature of magnetism has made a great advance. The electron theory was first proposed by J. J. Thomson<sup>1</sup> and W. Voigt<sup>2</sup>; but their results were not conclusive. P. Langevin<sup>3</sup> developed a kinetic theory of paramagnetism as well as diamagnetism, and arrived at results which agree in many points with the observed facts; but there are many other facts which cannot be explained by his theory. Prof. P. Weiss founded his well-known theory of magnetons<sup>4</sup> on the basis of Langevin's theory of paramagnetic gas; but subsequent investigations<sup>5</sup> showed that the existence of the magneton—the fundamental unit of magnetism—is very doubtful. One of the present writers<sup>6</sup> modified Langevin's theory of the paramagnetism of gases so as to include the magnetization of liquid and solid states, and made clear the distinction between paramagnetic and diamagnetic substances. This theory explains the fact of the dependency of diamagnetic susceptibility on the temperature, and also its change with the change of state; while according to Langevin's theory, such changes of diamagnetism are improbable. Professor Weiss, by introducing a molecular field of enormously large magnitude (some  $10^7$  gauss) into Langevin's theory of paramagnetic gas, tried to explain the ferromagnetism; but although he succeeded in explaining some of the thermomagnetic properties of ferromagnetic substances, it is very difficult to understand from his theory, why magnetization is so easily induced by a small external field in the presence of the large molecular field. On several occasions, one of the present writers<sup>7</sup> has shown that the existence of the molecular field is very improbable and that the thermomagnetic properties accounted for by means of the molecular field can be equally well explained by other theories.

<sup>1</sup> Phil. Mag., 6, 1903, 673.

<sup>2</sup> Ann. der Phys., 9, 1902, 115.

<sup>3</sup> Ann. de chem. et phy., 5, 1905, 70.

<sup>4</sup> Arch. des Sci., 31, 1911, 401.

<sup>5</sup> K. Honda & T. Ishiwara, Sci. Rep., 4, 1915, 215. K. Honda & H. Takagi, Sci. Rep., 4, 1915, 261; Sci. Rep., 1, 1912, 229.

<sup>6</sup> K. Honda, Sci. Rep., 3, 1914, 171.

<sup>7</sup> K. Honda, Sci. Rep., 4, 1915, 208. K. Honda & J. Okubo, Sci. Rep., 5, 1916, 207. K. Honda, Sci. Rep., 7, 1918, 53.

Various theories of ferromagnetism have been advanced by Poisson, Weber, Maxwell and Ewing. Ewing's theory of molecular magnetism<sup>1</sup> explains the curve of magnetization and the phenomenon of hysteresis quite satisfactorily; but the qualitative nature of the theory is its great defect. R. Gans<sup>2</sup> tried to treat Ewing's model of molecular magnets mathematically; but his theory differs essentially from that of Ewing in assuming the distribution of the molecular magnets in the substance to be quite arbitrary, and in fact the conclusions from his theory are a rough approximation to the observed facts. In the last two years, we have published three papers,<sup>3</sup> in which Ewing's theory of molecular magnetism is treated quantitatively and extended so as to include the theory of hysteresis-loss, as well as the effect of temperature on magnetization. We have obtained the curve of magnetization possessing all the characteristics belonging to iron. Not only has the well-known hysteresis-loop been obtained, but the loss is expressed in terms of the magnetic field and the two constants characteristic of a substance. The effect of temperature on the magnetization, that is, the complicated course of magnetization-temperature curves, is also explained on the basis of the same theory.

2. The theory of magnetism in general may thus be considered as established in the main; but there still remains a great advance to be made. As to Langevin's theories of paramagnetic and diamagnetic substances, some modifications are necessary in order to account for the observed facts. For example, take the case of diamagnetic substances: According to Langevin, the diamagnetic susceptibility  $k$  per unit of volume of a substance is given by

$$k = -\frac{\rho}{12} \left( \frac{e}{m} \right)^2 r^2,$$

where  $\rho$  is the density of electrons,  $e/m$  the ratio of the charge to its mass, and  $r$  the radius of the orbit of electrons. The susceptibility  $x$  per one gram atom or molecule is given by

$$x = -\frac{W\rho}{12D} \left( \frac{e}{m} \right)^2 r^2,$$

where  $W$  is the atomic or molecular weight of the substance and  $D$  its density. Now, for the density of electrons, we have

$$\rho = \frac{DN}{1800W},$$

<sup>1</sup> Phil. Mag., (5), 30, 1891, 205; Magnetic Induction in Iron and other Metals.

<sup>2</sup> Gött. Nachr., 1910, 197; 1911, 118. R. Gans & P. Hertz, Zeitsch. für Math. u. Phys. 61, 1913, 13.

<sup>3</sup> Sci. Rep., 5, 1916, 153; 5, 1916, 325; 6, 1917, 183.

provided each atom or molecule contains  $N$  electrons. Hence for different elements, we have

$$\chi = - \frac{N}{12 \times 1800} \left( \frac{e}{m} \right)^2 \Sigma r^2,$$

where  $\Sigma$  is to be extended for all electrons. Here we may take  $N$  as the atomic number of the elements. Thus, according to Langevin, since  $\Sigma r^2$  is to be considered as increasing with atomic weight, the diamagnetic susceptibility of the elements must increase with their atomic number. But, actually this is not the case; in fact, the susceptibility varies periodically with the atomic weight of the elements. This discrepancy is, however, partially reconciled, if we consider the diamagnetism actually observable to be the differential effect of the paramagnetic and diamagnetic phenomena;<sup>1</sup> that is, denoting the susceptibility observed by  $\chi$ , and Langevin's paramagnetic and diamagnetic susceptibilities by  $\chi_p$  and  $\chi_d$  respectively, we obtain

$$\chi = \chi_p + \chi_d;$$

hence though  $\chi_d$  increases proportionally with the atomic number,  $\chi$  may change periodically, provided  $\chi_p$  makes a similar change.

3. Next, consider the paramagnetic theory of Langevin. According to the kinetic theory of gases, the molecules of a gas are continuously making translational and rotational motions. We suppose each molecule to be a magnet composed of a set of revolving electrons. At a given instant, we may suppose the directions of the axes of these magnetic molecules to be uniformly distributed in all directions, if acted on by no external field. At the next instant, all the magnets change the directions of their axes; but as a whole, these axes are again equally distributed in all directions, and so on. In the case, when an external field acts on them, Langevin considers that the axes of these molecules tend to direct themselves toward the direction of the field; but the thermal impacts prevent this tendency. These two opposing effects make the distribution of the axes of the molecular magnets a little denser in the direction of the field than in the opposite direction. The directions of the magnetic axes are of course changing from instant to instant, but as a whole, the above distribution may be assumed to remain unchanged. By following the analogy of the distribution of the density of a gas under the action of gravity, he assumed the distribution of the axes of the molecular magnets to be given by Maxwell-Boltzmann's law. Resolving their magnetic moments in the direction of the field, and summing up these components, he obtained the well-known expression for the paramagnetic susceptibility.

<sup>1</sup> K. Honda, Sci. Rep., 3, 1914, 171; J. Kunz, Phys. Rev., 6, 1915, 113.

Now, since the molecules are continuously rotating, things are not so simple as Langevin considers. Generally the axis of rotation of a molecule does not coincide with its magnetic axis; hence we consider the magnetic moment to be decomposed into two components parallel and perpendicular to the axis of rotation. The former component is here termed the axial component of the moment and the latter the transverse component. When a magnetic field is suddenly applied to the molecule, in virtue of the axial component of the magnetic moment, its axis of rotation makes a small gyrostatic motion, that is, it makes precession and nutation round the direction of the field. This motion is very little affected by the presence of the transverse component of the moment. The amplitude of the nutational motion measured from the direction of the field extends from an initial angle  $\alpha$  to an angle  $\theta$  towards the field. This motion must therefore make a certain contribution to the increase of the magnetic moment in the direction of the field, while the precessional motion itself does not cause any change in the magnetic moment. If we suppose that by continuous thermal impacts with other molecules, the nutational motion, whose frequency is much larger than that of the precession, rapidly decreases, while the precessional velocity does not considerably alter, a simple precessional motion results with an angle  $\theta$ , which is greater than the initial angle  $\alpha$  and less than the angle  $\theta$ , and causes also an increase of the magnetic moment in the direction of the field. If the field, instead of being applied suddenly, be gradually applied and increased extremely slowly to its final value, the molecules will make such slow nutational and precessional motions that the first precessional motion at the lower limit may be considered as statical. This case is analogous to that of a pendulum acted on extremely slowly by the force of gravity, in which case, the pendulum may be assumed to remain at rest at its lowest point. If, under the action of a magnetic field, the molecules make a simple precessional motion or take definite orientations, as explained above, it is evident that his case is almost the same as that treated by Langevin. Taking the sum of the magnetic moments in the direction of the field due to all molecules, we obtain usually a very small moment, that is, the paramagnetic moment. By increasing the field, the angular distance  $\alpha-\theta$  increases for all molecules, and therefore the paramagnetism of the gas increases with the field. As will be shown later, this result almost coincides with that of Langevin.

4. Consider next the transverse component of the magnetic moment. This component of magnetism is continuously rotating with the molecule in its equatorial plane. Let us at first suppose that this plane coincides with the plane of the magnetic field. Initially its angular velocity is

uniform for all directions during a complete revolution; but if a magnetic field acts on the molecule, the revolution is now not uniform. Toward the direction of the field, the motion is always accelerated; but in the opposite direction, it is always retarded. Hence, if we wish to find its magnetic effect, it is necessary to take the time-mean of the magnetic moment during a complete period of revolution. Since during a revolution, the plus pole of the molecule is a shorter time in the direction of the field than in the opposite direction, the time-mean of the magnetic moment becomes negative, that is, the transverse component of the magnetic moment of rotating a molecule produces a diamagnetic effect. If the equatorial plane of the molecule makes some angle with the plane of the field, it is only necessary to resolve the above effect in the plane of the field. Hence, if there is a number of molecules, the directions of the equatorial planes of which are uniformly distributed between the angles 0 and  $\pi$ , we must take the component moments for these molecules in the direction of the field and integrate them; the result is a diamagnetism. Langevin did not in the least consider the effect corresponding to this kind of diamagnetism. The total magnetic effect of rotating molecules is then the differential effect due to axial and transverse components of magnetic moment, and therefore it may be paramagnetic or diamagnetic, according as the effect due to the axial component is greater or less than that due to the transverse component.

If the above view be correct, Langevin's theory of paramagnetic gas is not valid; and therefore the theory of magnetons, as well as that of molecular field put forward by Prof. Weiss, which are based on the Langevin theory, have no foundation. For this reason, the above view of the magnetization of a gas is very important.

5. We shall next consider the results qualitatively explained above, more in detail. Let us suppose that the magnetic field is acting on a

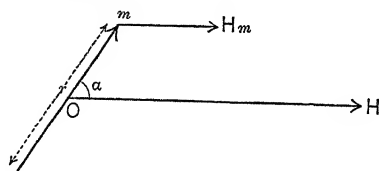


Fig. 1.

molecule of a gas rotating about its magnetic axis; then it will make precessional and nutational motions about the direction of the field. Let  $o$  be the center of the magnetic molecule, its pole-strength and pole-distance being  $m$  and  $r$  respectively. Let  $\alpha$  be the initial angle of a molecular magnet, which is acted on by an external field  $H$ . Then the



magnetic moment  $M$  is

$$M = mr.$$

The following notations are here used:

$K$  = moment of inertia about the magnetic axis,

$\omega$  = angular velocity about the magnetic axis,

$\dot{\psi}$  = precessional velocity,

$\dot{\theta}$  = nutational velocity.

From the theory of the gyroscope, we know that

(i) the angular momentum about  $oH$  remains constant,

(ii) the sum of the kinetic and potential energies of the gyro-molecule is constant;

that is,

$$K\omega \cos \theta + K\dot{\psi} \sin^2 \theta = K\omega \cos \alpha,$$

$$\frac{1}{2}K\omega^2 + \frac{1}{2}K\dot{\psi}^2 \sin^2 \theta + \frac{1}{2}K\dot{\theta}^2 - MH \cos \theta = \frac{1}{2}K\omega^2 - MH \cos \alpha;$$

Hence

$$\dot{\psi} = \frac{K\omega(\cos \alpha - \cos \theta)}{K \sin^2 \theta}$$

and

$$K\omega^2(\cos \theta - \cos \alpha)^2 + K^2\dot{\theta}^2 \sin^2 \theta = 2MHK (\cos \theta - \cos \alpha).$$

The limits of the nutational angle  $\theta$  and  $\theta_1$  are given by

$$\cos \theta = \cos \alpha,$$

$$\cos \theta_1 = -\frac{1}{a} \pm \sqrt{1 + \frac{2}{a} \cos \alpha + \frac{1}{a^2}},$$

where

$$a = \frac{MH}{\frac{1}{2}K\omega^2}.$$

In the last equation,  $\cos \theta$  must always be less than 1; hence the plus sign must be taken for the double sign. We have then

$$\theta = \alpha,$$

$$\cos \theta_1 = -\frac{1}{a} + \sqrt{1 + \frac{2}{a} \cos \alpha + \frac{1}{a^2}}.$$

Hence the gyro-molecule makes a nutational motion between  $\alpha$  and  $\theta_1$ .

Suppose at first that the magnetic field is applied infinitely slowly, or that the field is suddenly applied, but the nutational motion is assumed to subside very rapidly, the angle  $\theta$  for the precessional motion being the same as in the above case. This last supposition may not be strictly true, but as we shall see later, if  $a$  be very small, the result does not materially differ from that of a case free from the above supposition. In a case, where  $a$  is small, the above supposition gives the result, which

exactly coincides with that of Langevin. The magnetic moment in the direction of the field is then  $M \cos \theta_1$ . The total magnetic moment per one gram molecule is

$$\begin{aligned}\sigma &= \frac{Mn}{2} \int_0^\pi \cos \theta_1 \sin \alpha d\alpha \\ &= \frac{Mn}{2} \int_0^\pi \left\{ -\frac{1}{a} + \sqrt{1 + \frac{2}{a} \cos \alpha + \frac{1}{a^2}} \right\} \sin \alpha d\alpha\end{aligned}$$

where  $n$  is the number of molecules per one gram molecule of the gas. Writing  $Mn = \sigma_0$ , we have

$$\begin{aligned}\frac{\sigma}{\sigma_0} &= \frac{1}{2} \int_0^{\pi_0} \left\{ \sqrt{1 + \frac{2}{a} \cos \alpha + \frac{1}{a^2}} - \frac{1}{a} \right\} \sin \alpha d\alpha \\ &= \frac{1}{2} \int_{-1}^{+1} \sqrt{1 + \frac{1}{a^2} + \frac{2}{a} x} dx - \frac{1}{a} = \frac{a}{6} \left| \sqrt{1 + \frac{2}{a} x + \frac{1}{a^2}} \right|_{-1}^{+1} - \frac{1}{a} \\ &= \frac{a}{6} \left| \left( 1 + \frac{1}{a} \right)^2 \right|^{\frac{3}{2}} - \left| \left( 1 - \frac{1}{a} \right)^2 \right|^{\frac{3}{2}} \right| - \frac{1}{a}.\end{aligned}$$

Hence if  $a < 1$ , we have, taking the minus sign for the square root of  $\left( 1 - \frac{1}{a} \right)^2$ ,

$$\frac{\sigma}{\sigma_0} = \frac{a}{3}. \quad (1)$$

but if  $a > 1$ , we get, taking the plus sign for the square root,

$$\frac{\sigma}{\sigma_0} = 1 - \frac{1}{a} + \frac{1}{2a^2}. \quad (1')$$

Since  $\frac{1}{2}K\omega^2$  is the rotational kinetic energy of the molecule,  $a$  has the same meaning as that in the Langevin theory, that is,

$$a = \frac{MH}{rT},$$

where  $r$  is the gas constant referred to one molecule.

According to Langevin,

$$\frac{\sigma}{\sigma_0} = \text{Coth } a - \frac{1}{a}; \quad (2)$$

if  $a$  be small, we have, by expansion,

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a - \frac{2}{90}a^3 + \dots \quad (3)$$

which almost coincides with equation (1). In the following table, the two sets of values for  $\sigma/\sigma_0$ , as calculated from equations (1) and (2), are given side by side:

$a$	$\sigma/\sigma_0$ (Honda & Ôkubo).	$\sigma/\sigma_0$ (Langevin).	$a$	$\sigma/\sigma_0$ (Honda & Ôkubo).	$\sigma/\sigma_0$ (Langevin).
0.0	0.0000	0.0000	5.0	0.8133	0.8000
0.5	0.1667	0.1640	6.0	0.8426	0.8334
1.0	0.3333	0.3130	7.0	0.8638	0.8571
2.0	0.5833	0.5373	8.0	0.8802	0.8750
3.0	0.7037	0.6717	9.0	0.8930	0.8889
4.0	0.7708	0.7507	10.0	0.9033	0.9000

From the above table, we see that for small values of  $a$ , our formula exactly coincides with that of Langevin; but for a moderate value of  $a$ , these two slightly deviate from each other. In Fig. 2, these results (curves 1) are graphically given.

6. Next, we shall consider the case, when the magnetic field is applied

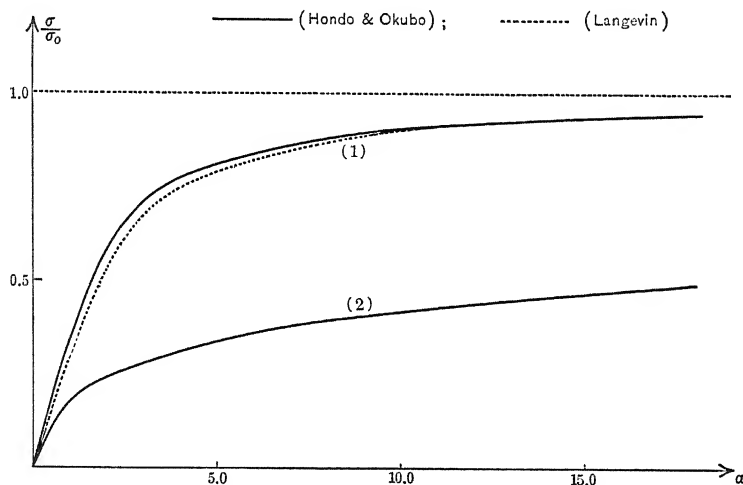


Fig. 2.

impulsively and the consequent nutational motion continues for a long time.

Now, the nutational velocity is given by

$$K \sin^2 \theta \left( \frac{d\theta}{dt} \right)^2 = MH (\cos \theta - \cos \alpha) (\cos \theta - \cos \theta_1) (\cos \theta_2 - \cos \theta).$$

Putting

$$\cos \theta = \cos \theta_1 \cos^2 \varphi + \cos \alpha \sin^2 \varphi$$

and changing the variable from  $\theta$  to  $\varphi$ , we get

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{MH}{4K} (\cos \theta_1 + \cos \theta_2) (1 - \kappa^2 \sin^2 \varphi),$$

where

$$\kappa^2 = \frac{\cos \theta_1 - \cos \alpha}{\cos \theta_1 + \cos \theta_2}.$$

Hence

$$\frac{dt}{p} = \frac{d\varphi}{\sqrt{1 - \kappa^2 \sin^2 \varphi}},$$

where

$$p = \sqrt{\frac{MH}{4K} (\cos \theta_1 + \cos \theta_2)}.$$

Let us suppose that

$$\text{at } t = T, \quad \theta = \alpha, \quad \varphi = \frac{\pi}{2},$$

and

$$\text{at } t = 0, \quad \theta = \theta_1, \quad \varphi = 0,$$

then

$$\frac{T}{p} = \int_0^{\pi/2} \frac{d\varphi}{\sqrt{1 - \kappa^2 \sin^2 \varphi}} = K(\kappa),$$

where  $K(\kappa)$  is the complete elliptic integral of the first kind.

$\theta$  or  $\varphi$  at any instant is given by

$$\frac{t}{p} = \int_0^\varphi \frac{d\varphi}{\sqrt{1 - \kappa^2 \sin^2 \varphi}} = F(\varphi, \kappa).$$

$F(\varphi, \kappa)$  is the Legendre elliptic integral of the first kind, that is,

$$\varphi = am \frac{t}{p}$$

and

$$\cos \theta = \cos \theta_1 cn^2 \frac{t}{p} + \cos \alpha sn^2 \frac{t}{p}.$$

Hence the time-mean of the magnetic moment is

$$\begin{aligned} M_m &= \frac{1}{T} \int_0^T M \cos \theta dt \\ &= \frac{M}{T} \left\{ \cos \theta_1 \int_0^T cn^2 \frac{t}{p} dt + \cos \alpha \int_0^T sn^2 \frac{t}{p} dt \right\} \\ &= \frac{Mp}{T} \left\{ \cos \theta_1 \left\{ \frac{1}{\kappa^2} \left[ E \left( am \frac{t}{p}, \kappa \right) - \kappa'^2 \frac{t}{p} \right] \right\} \right. \\ &\quad \left. + \cos \alpha \left\{ \frac{1}{\kappa^2} \left[ \frac{t}{p} - E \left( am \frac{t}{p}, \kappa \right) \right] \right\} \right\} \Bigg|_0^T. \end{aligned}$$

But since

$$\begin{aligned}\frac{p}{T} &= K(\kappa) \text{ and } amK = \frac{\pi}{2}, \\ \therefore \frac{M_m}{M} &= \frac{\cos \theta_1 - \cos \alpha}{\kappa^2} \frac{E(\kappa)}{K(\kappa)} + \frac{1}{\kappa^2} (\cos \alpha - \kappa'^2 \cos \theta_1) \\ &= (\cos \theta_1 + \cos \theta_2) \frac{E(\kappa)}{K(\kappa)} - \cos \theta_2.\end{aligned}$$

If the axis of the molecular magnets are initially distributed uniformly in all directions, the resultant magnetic moment in the direction of the field is

$$\frac{\sigma}{\sigma_0} = \frac{1}{2} \int_0^\pi \left\{ (\cos \theta_1 + \cos \theta_2) \frac{E(\kappa)}{K(\kappa)} - \cos \theta_2 \right\} \sin \alpha d\alpha. \quad (4)$$

Since  $\cos \theta_1$  and  $\cos \theta_2$  are all functions of  $\alpha$ , the above integral cannot be evaluated in a rational form. If  $a$  be small,  $\cos \theta_1$ ,  $\kappa$ ,  $E(\kappa)$ ,  $K(\kappa)$  can be expanded in the ascending power series of  $a$  and then integrated, we have

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a - 0.0334a^3 + \dots \quad (5)$$

In a case when  $a$  is very large, these quantities can be expanded in an ascending power series of  $\lambda = 1/a$ ; then we get

$$\frac{\sigma}{\sigma_0} = 0.4818 - 0.6875\lambda + 0.6476\lambda^2 + \dots \quad (6)$$

This equation shows that even in an infinitely large field, the magnetic moment does not attain its saturation value; because for  $H = \infty$ ,  $\lambda = 0$ . Hence

$$\frac{\sigma}{\sigma_0} = 0.4818.$$

The reason for this is evident, since according to our first supposition, all the molecules are making continuously nutational oscillations like a pendulum about the direction of the field.

By using equations (5) and (6) for small and large values of  $a$ , and also by mechanical integration of equation (4) for moderate values of  $a$ , we calculated the values of  $\sigma/\sigma_0$  and the result is given in Fig. 2, curve 2. In ordinary cases actually realizable, in which  $a$  is a very small quantity, equations (3) and (5) coincide with each other; but as the values of  $a$  becomes large, the deviation between them becomes always greater. To decide which one of these equations for a large value of  $a$  actually corresponds to the observed facts, requires a further experimental investigation.

7. Next, we shall consider the case of a molecule rotating about an axis perpendicular to the magnetic axis. If the plane of rotation coincides with that of the field, the time-mean of the magnetic moment  $M_m$  in the direction of the field<sup>1</sup> is given by

$$\frac{M_m}{M} = \frac{2}{\kappa^2} \frac{E(\kappa)}{K(\kappa)} - \left( 2 \frac{\kappa'^2}{\kappa} + 1 \right),$$

where

$$\kappa^2 = \frac{4MH}{K\omega^2} = 2a.$$

If the plane of rotation makes an angle  $\theta$  with the direction of the field, its effective moment is  $M_m \cos \theta$ . Hence if the distribution of the planes of rotation for  $n$  molecules be uniform, the resultant moment in the direction of the field will be

$$\sigma = \frac{M_m n}{2} \int_0^\pi \cos \theta \sin \theta d\theta = \frac{M_m n}{2},$$

or

$$\frac{\sigma}{\sigma_0} = \frac{1}{2} \left\{ \frac{2}{\kappa^2} \frac{E(\kappa)}{K(\kappa)} - \left( 2 \frac{\kappa'^2}{\kappa} + 1 \right) \right\}. \quad (7)$$

If  $a$  or  $\kappa$  be small, we have by expansion

$$\frac{\sigma}{\sigma_0} = -\frac{a}{8} - \frac{a^2}{8} - \dots \quad (8)$$

This shows that the molecules rotating about the axis perpendicular to the magnetic axes produce the diamagnetism. Since  $\frac{1}{2}K\omega^2$  in the expression for  $a$  is the rotational energy of the molecules, its value is of the same order of magnitude as  $\tau T$ . Hence even in the strongest field available at the present day, the value of  $a$  is very small, unless the temperature is very low. Hence we may safely neglect  $a^2$  in comparison with  $a$ , and we obtain

$$\frac{\sigma}{\sigma_0} = -\frac{a}{8}. \quad (9)$$

8. Having thus found the magnetic effects of the gas molecules for the two cases above mentioned, we now proceed to consider the case of the magnetization of an actual gas. Since the molecules are always rotating about axes whose directions are continuously changing, the axis of rotation and the magnetic axis do not generally coincide with each other. Now, at any given instant, among  $n$  molecules, there is a certain number of molecules

$$dn = \frac{n}{2} \sin \varphi d\varphi,$$

<sup>1</sup> K. Honda & J. Okubo, Sci. Rep., 5, 1916, 328.

whose magnetic axis make with the axis of rotation an angle lying between  $\varphi$  and  $\varphi + d\varphi$ . Resolving the magnetic moment of these molecules in the axial and transverse directions, we get  $M \cos \varphi$  and  $M \sin \varphi$  for these two components respectively. On the one hand, these molecules have then the magnetic moment  $M \cos \varphi$  in the direction of the axis of rotation, and therefore by applying a magnetizing field, their magnetic polarization is paramagnetic. Its intensity of magnetization  $d\sigma$  is obtained from equation (1), since the quantity corresponding to  $a$  is very small; that is,

$$a' = \frac{MH \cos \varphi}{\frac{1}{2}K\omega^2} = a \cos \varphi$$

and

$$d\sigma = M \cos \varphi dn \times \frac{a'}{3} = \frac{Mna}{6} \cos^2 \varphi \sin \varphi d\varphi.$$

$\varphi$  may take any value between 0 and  $\pi$ ; hence integrating this expression for  $\varphi$ , we get the total effect, that is,

$$\frac{\sigma}{\sigma_0} = \frac{a}{6} \int_0^\pi \cos^2 \varphi \sin \varphi d\varphi = \frac{a}{9}. \quad (10)$$

On the other hand, the magnetic effect due to the transverse component of magnetic moment for the molecules is obtained from equation (9). Thus,

$$a' = \frac{HM \sin \varphi}{\frac{1}{2}K\omega^2} = a \sin \varphi$$

and

$$\begin{aligned} d\sigma &= -M \sin \varphi dn \times \frac{a'}{8} \\ &= -\frac{Mna}{16} \sin^3 \varphi d\varphi. \end{aligned}$$

Integrating this expression for  $\varphi$  between 0 and  $\pi$ , we get the total effect, that is,

$$\frac{\sigma}{\sigma_0} = -\frac{a}{16} \int_0^\pi \sin^3 \varphi d\varphi = -\frac{a}{12}. \quad (11)$$

Hence, if the principal moments of inertia are equal to each other, that is, the shape of the molecules is spherical, the resultant moment of  $n$  molecules is the sum of the expressions (10) and (11). That is,

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} \left( \frac{1}{3} - \frac{1}{4} \right) = 0.0278a. \quad (12)$$

Thus the resultant is paramagnetic. This differs quantitatively from the result of Langevin.

It is commonly assumed that the molecules of a monoatomic gas have a spherical shape and therefore thermal impacts cannot produce any rotation of the molecules and they have no energy of rotation. But according to a theory<sup>1</sup> proposed by one of the present writers, the molecules of the monoatomic gas are also continuously rotating with great velocity, which is acquired in a liquid state, but this velocity does not depend on the temperature of the gas. Hence, even for the monoatomic gas, the above result is equally applicable.

If the form of the molecules is a body of revolution about the magnetic axis, we have

$$a = \frac{MHK'}{\frac{1}{2}K^2\omega^2} = \frac{\sigma_0 H K'}{q K}$$

and

$$a' = \frac{MH}{\frac{1}{2}K'\omega'^2} = \frac{\sigma_0 H}{q},$$

where  $K$  and  $K'$  are respectively the moments of inertia about the magnetic axis and the axis perpendicular to it, and

$$q = \frac{n}{2} K \omega^2 = \frac{n}{2} K' \omega'^2$$

is the kinetic energy of rotation, the equipartition of the rotational energy being assumed. Hence,

$$\frac{\sigma}{\sigma_0} = \left( \frac{K'}{3K} - \frac{1}{4} \right) \frac{\sigma_0 H}{3q}. \quad (13)$$

The gas may therefore be paramagnetic or diamagnetic, according as

$$4K' - 3K \gtrless 0.$$

As an example, take an ellipsoidal molecule, whose form is defined by

$$\frac{x^2}{A^2} + \frac{y^2 + z^2}{B^2} = 1$$

and whose major axis  $x$  coincides with that of its magnetic axis; then we have

$$K = \frac{M}{5} \times 2B^2 \text{ and } K' = \frac{M}{5} (A^2 + B^2).$$

Hence the gas is paramagnetic or diamagnetic according as

$$4A^2 - 2B^2 \gtrless 0,$$

or

$$\frac{A}{B} \gtrless 0.7072,$$

or

$$\text{eccentricity} \gtrless 0.707.$$

<sup>1</sup>K. Honda, Sci. Rep., 7, 1918.



It is an interesting fact that the sign of the magnetization of a gas depends only on the shape of the molecules, and not in the least on their the magnetic moment.

9. It is generally admitted that the molecular magnets consist of certain sets of electrons rapidly revolving about positive nuclei, and that this revolution of electrons does not vary with temperature, its velocity always remaining the same, even at absolute zero. This rotational energy may be called the zero point energy; we denote it by  $\frac{1}{2}R\Omega$ , where  $R$  is the gas constant referred to one gram molecule. Besides, at the temperature  $T$ , we have also the axial component of rotation of the molecules as a whole; its energy may be given by  $\frac{1}{2}RT$ . We have then

$$\frac{n}{2} K\omega^2 = \frac{1}{2}RT + \frac{1}{2}R\Omega = \frac{1}{2}R(T + \Omega).$$

Hence equation (13) becomes

$$\frac{\sigma}{\sigma_0} = p \frac{\sigma_0 H}{R(T + \Omega)},$$

where

$$p = \frac{2}{3} \left( \frac{K'}{3K} - \frac{1}{4} \right);$$

$p$  may be positive or negative; or

$$x = \frac{\sigma}{H} = \frac{p\sigma_0^2}{R(T + \Omega)}. \quad (14)$$

Thus, the paramagnetic or diamagnetic susceptibility is proportional to the magnetic field. From the last equation, we have

$$x(T + \Omega) = \frac{p\sigma_0^2}{R} = \text{const.}$$

If  $p > 0$  and  $\Omega$  is very small as compared with  $T$ , we have

$$xT = \text{const.},$$

which is the Curie law for paramagnetic gas. On the other hand, if  $p < 0$  and  $\Omega$  is very large, we have

$$x = \text{const.},$$

Hence for a gas satisfying these conditions, its diamagnetic susceptibility is independent of temperature.

Here the following remarks are to be made. So far we have assumed the molecular magnets to be perfectly rigid; but as shown by Langevin, the motion of the electrons is slightly modified by applying a strong magnetic field, and this change also contributes to the diamagnetism of

the substance. Hence to equation (14), we must add the diamagnetic susceptibility as given by Langevin, that is,

$$\chi = -\frac{N}{12 \times 1800} \left( \frac{e}{m} \right)^2 \Sigma r^2.$$

As an application of the above theory, it is interesting to calculate the magnetic moment of a hydrogen atom or molecule. According to Bohr, the atom of hydrogen consists of an electron revolving about its nucleus. Since Bohr's value for the smallest radial distance of the revolving electron seems to be too small, we take its next value for  $r$  and the corresponding angular velocity, that is,

$$r = 2.2 \times 10^{-8} \text{ cm.}, \quad \omega = 0.755 \times 10^{15} \text{ per sec.}$$

Then the magnetic moment of an atom is

$$M = \frac{1}{2} e r^2 \omega = 2.95 \times 10^{-21}$$

and

$$\sigma_0 = 1806,$$

which is a little greater than half the magnetic moment of the nickel atom.

The zero point energy  $E_0$  of rotation amounts to

$$E_0 = 6.2 \times 10^{11};$$

but at  $27^\circ \text{ C.}$ ,

$$\frac{1}{2} RT = 1.24 \times 10^{10},$$

which is only a small fraction of the zero point energy. Hence, we have

$$\frac{n}{2} K \omega^2 = E_0 + \frac{1}{2} RT = 0.63 \times 10^{11}.$$

In the well known Bohr model of a hydrogen molecule, the two electrons are revolving about the line joining two positive nuclei. Here the moment of inertia  $K$  about the magnetic axis is very small as compared with the moment of inertia  $K'$  about an axis perpendicular to it; hence the ratio  $K'/K$  is very large and therefore the susceptibility of hydrogen must be paramagnetic. This inference, however, contradicts the observed fact. In order therefore that the hydrogen molecule may give the observed diamagnetic susceptibility, the positive nuclei must approach very near to each other.

Let us next calculate the distance  $d$  between two positive nuclei of a hydrogen molecule composed of two atoms, the planes of the electron orbits being parallel to each other, under the condition that the susceptibility of the gas may give the value<sup>1</sup> actually observed, that is,

$$\chi = -1.89 \times 10^{-6}.$$

<sup>1</sup> The measurement of the susceptibility of different gases is now being made in this laboratory by Mr. J. Soné.

In this case, Langevin's diamagnetic susceptibility amounts to

$$x = -7.0 \times 10^{-6}.$$

Hence by equation (13), we have the relation

$$-1.8 \times 10^{-6} = \frac{1}{3} \left( \frac{K'}{3K} - \frac{1}{4} \right) \frac{\sigma_0^2}{q} + xd;$$

or

$$\frac{K'}{3K} = \frac{1}{12} + 5.4 \times 10^{-6} \frac{q}{\sigma_0^2} = 1.17.$$

$$\therefore \frac{r}{d} = 13.12.$$

Hence the two positive nuclei should be only distant from each other by an amount less than one thirteenth of the radius of the electron orbits.

10. The case of solid substances will be next considered. According to the prevailing theory of the solid state, the molecules are arranged according to a certain space-lattice and are continuously making rectilinear oscillations, but not any rotations. But as we have already remarked, the electrons in each atom are rapidly revolving about the centers of mass of the electrons and the positive nuclei; its axis of rotation coincides evidently with that of the magnetic moment. The energy of this rotational motion, that is, the zero point energy, may be expressed by  $\frac{1}{2}\epsilon R\Omega$ , where  $\Omega$  is a constant and  $\epsilon$  a small fraction of unity. With the rise of temperature, the angular velocity of this rotation may probably increase, because of the axial rotation of the molecules as a whole due to thermal impacts. In a solid, this increase of rotation is not independent of the rectilinear vibrations and therefore the increase of energy is far less than that corresponding to the energy of one independent freedom, that is, than  $\frac{1}{2}RT$ ; hence it may be expressed by  $\frac{1}{2}\epsilon RT$ . For the molecules of a solid, a simultaneous rotation of the molecules perpendicular to the above rotation can not possibly exist.<sup>1</sup> Hence assuming the molecules to form a numerous number of minute groups, in each of which the axis of magnetic molecules take the same direction, but the magnetic axes of these groups are uniformly distributed in all directions, the expression for the susceptibility of a solid substance is given by equation (1). Since

$$\frac{n}{2} K\omega^2 = \frac{\epsilon}{2} R(T + \Omega),$$

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} = \frac{2}{3} \frac{\sigma_0 K' H}{K R \epsilon (T + \Omega)};$$

<sup>1</sup> K. Honda, Sci. Rep., 7, 1918.

$$x = \frac{\sigma}{H} = \frac{2}{3} \frac{\sigma_0^2 K'}{K R \epsilon (T + \Omega)}; \quad (15)$$

$$\therefore x(T + \Omega) = \frac{2}{3} \frac{\sigma_0^2 K'}{K R \epsilon} = \text{const.}$$

There is a considerable number of paramagnetic substances, whose susceptibility satisfies the above relation. Strictly speaking, to the above value of susceptibility, we must also add the small diamagnetic susceptibility given by Langevin.

According to the theory of fusion<sup>1</sup> proposed by one of the present writers, the rotational motion of the molecules during fusion begins to take place with the energy corresponding to the degree of freedom at the melting point, so that the liquid molecules are making axial and transverse rotations like those of a gas. Hence during fusion, the susceptibility of a substance discontinuously changes from a value expressed by equation (15) to that expressed by equation (14), and may in some cases change its sign. The remarkable fact<sup>2</sup> that during melting, the susceptibility of tin changes from paramagnetic susceptibility to the diamagnetic, is explained in this way.

II. The case of a ferromagnetic substance comes next for our consideration. According to our theory of magnetism,<sup>3</sup> which explains the observed fact in a very satisfactory manner, the molecules of a ferromagnetic substance are arranged in a definite space-lattice and exert mutual magnetic action upon one another. In the light of the present theory, this means that the energy of rotation about the magnetic axis, that is  $\frac{1}{2} K \omega^2$ , is very small; for in this case, the gyrostatic action of the molecules is negligibly small in comparison with the mutual magnetic action, and the molecules are now in equilibrium under the action of this magnetic force, a condition required in our theory.

Now, take equations (1) and (1') as our starting point; it appears at first sight that the expression

$$\frac{\sigma}{\sigma_0} = \frac{a}{3} \text{ for } a < 1,$$

$$\frac{\sigma}{\sigma_0} = 1 - \frac{1}{a} + \frac{1}{2a^2} \text{ for } a > 1,$$

where

$$a = \frac{\sigma_0^2 H K'}{\frac{n}{2} K^2 \omega^2},$$

<sup>1</sup> K. Honda, Sci. Rep., 7, 1. c.

<sup>2</sup> K. Honda, Sci. Rep., 1, 1911, 1.

<sup>3</sup> K. Honda and J. Okubo, Sci. Rep., 5, 1. c.

may be looked on as the law of magnetization for a ferromagnetic substance, provided we consider  $\sigma$  to be a function of  $H$  and  $(n/2)K\omega^2$  to be a very small quantity depending on the temperature of the substance. The form of the  $\sigma_1 H$  curve is the same as curve 1 in Fig. 2. The curve possesses two characteristic points of the curve of magnetization, that is, the proportionality between the initial magnetization and the field, and the gradual approach of magnetization to saturation; yet it is only a rough representation of the fact; moreover it cannot explain the important phenomenon of hysteresis. These discrepancies are the consequences of the present theory, which neglects the mutual magnetic action and takes only the axial rotation of the molecules into account. As we have shown in our former papers, in the case of ferromagnetic substances, we must consider the mutual magnetic action as the most important factor in determining the law of magnetization, and therefore equations (1) and (1'), which take only the gyrostatic action of the axially rotating molecules into account, cannot be considered as the true law of magnetization.

The effect of temperature on magnetization will be next considered. The temperature affects magnetization in two opposite ways; the first effect, which is especially conspicuous in weak fields, but which completely vanishes in strong fields, is to increase the magnetization, while the second is always to decrease the magnetization. In our former paper,<sup>1</sup> these two effects are explained on the basis of small vibratory motions of the molecular magnets, about their mean orientations. That the magnetization in weak fields is considerably increased by the thermal vibrations of the molecules, is very satisfactorily explained; but the explanation of the second effect by the same vibrations is met with the great difficulty that if the half-amplitude of this vibration exceeds about  $131^\circ$ , the magnetization in a strong field is reversed; this, however, is an abnormal result, which has never been observed.

In view of the present theory, the following explanation for the temperature-effect in question will be very natural and agrees satisfactorily with the observed facts: The molecules of a ferromagnetic substance are continually making axial rotations about their magnetic axis, and at the same time, small vibratory motions about the axes perpendicular to it. The first motion explains the diminution of magnetization, while the second motion explains its increase in weak fields, as shown in our former paper. At ordinary temperature, the energy of the axial rotation is considered to be very small, but as the temperature rises, it rapidly increases, its rate of increase becoming always greater. At the critical

<sup>1</sup> Sci. Rep., 5, 1916, 325.

point, the energy is considered to be greatly increased, but it has still a small fraction of the value  $\frac{1}{2}RT$  corresponding to one degree of freedom at the temperature  $T$ . The energy-content of the axial rotation is then given by an expression of the form

$$\frac{n}{2} K\omega^2 = \frac{m}{2} RT,$$

where  $m$  is a function of temperature increasing rapidly with the latter, the zero point energy being neglected. An expression  $m$  of the form

$$m = \frac{\epsilon}{\sqrt{1 - \frac{T}{\theta} + \epsilon}},$$

where  $\theta$  is the critical temperature;  $\epsilon$  and  $\epsilon_1$  being very small quantities, will accord with the observed facts very satisfactorily. The law of variation of magnetization with temperature is then given by equations (1), (1') and

$$a = \frac{\sigma_0^2 HK'}{\frac{1}{2} K m R T}.$$

In order to show that these expressions are a good representation of the

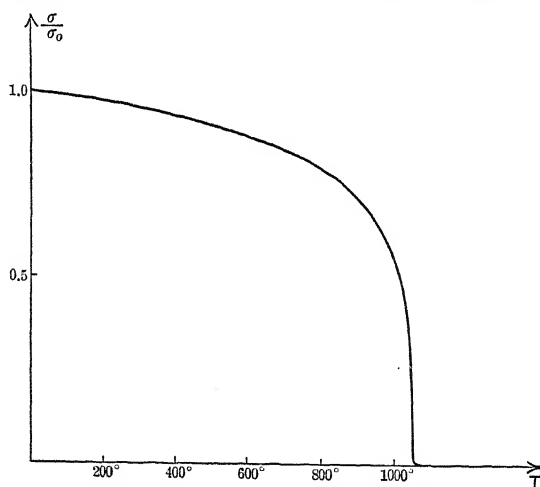


Fig. 3.

observed facts, we have calculated the constants entering into the expression of  $a$  for iron and obtained the following results:

$$\theta = 790^\circ \text{ C.},$$

$$\epsilon = 0.431 \times 10^{-5},$$

$$\epsilon_1 = 4.96 \times 10^{-8}.$$

The curve is also given in Fig. 3; we see that it is very similar to the  $I_1T$  curve in a strong field. Hence we conclude that the effect of temperature, which diminishes the magnetization for all fields, is due to the increase of the axial rotational energy of the molecules. This inference is also confirmed by the fact that during heating or cooling through the critical range of magnetic transformation, the heat is absorbed or evolved, that is, through the same range, the specific heat abnormally changes.

If the above view be correct, the rotational energy gained during heating through the range of magnetic transformation up to the critical point, is equal to the total heat absorbed during the transformation. If we neglect a small rotational energy at room temperature, the total energy  $q$  gained is equal to  $(n/2)K\omega^2$ . Hence, for one gram-molecule of a ferromagnetic substance, we have

$$x = \frac{2}{3} \frac{K'\sigma_0^2}{nK^2\omega^2} \text{ or } = \frac{K'\sigma_0^2}{3Kq};$$

$$q = \frac{K'\sigma_0^2}{3KX}.$$

This relation admits of the calculation of the heat absorbed or evolved during heating or cooling, provided the shape of the molecules be known. But at present, we have no data for determining the ratio  $K'/K$  for ferromagnetic substances, and therefore the verification of the above relation is left for a future occasion.

The following remark deserves also to be made:—The magnetic (or  $A_2$ ) transformation of iron has been the subject of a long dispute among metallurgists, viz. as to the nature of this transformation.<sup>1</sup> The present theory gives, however, an exact mechanism of this transformation; that is, the  $A_2$  transformation is not a change of molecular configuration or a change in the molecules, as is the case in an allotropic change; but it is the process of acquiring the rotational energy of the molecules, whose amount is a definite function of temperature. Hence the transformation cannot be considered as an allotropic change, as has repeatedly been stated by one of the present writers.<sup>1</sup>

10. Lastly let us consider the susceptibility of a ferromagnetic substance above its critical point. In this region, the kinetic energy of axial rotation, that is,  $(n/2)K\omega^2$ , is very large in comparison with that in the magnetic region, but it is still far less than the energy corresponding to one degree of freedom at the temperature under consideration, that is,  $\frac{1}{2}RT$ , which value is only attained by the fusion of the substance. We may express the above fact by the expression

$$\frac{1}{2}K\omega^2 = \frac{1}{2}RT - \frac{1}{2}R\Omega$$

<sup>1</sup> K. Honda, Sci. Rep., 4, 1915, 169; 6, 1917, 213.

and therefore equation (1) becomes

$$\frac{\sigma}{\sigma_0} = \frac{1}{3}a = \frac{2\sigma_0 HK'}{KR(T - \Omega)},$$

or

$$x(T - \Omega) = \frac{2\sigma_0^2 K'}{RK} = \text{const.} \quad (16)$$

This is the relation, which holds good approximately above the critical point. This relation was first obtained by Prof. P. Weiss from his theory of molecular field and given in his paper as a confirmation of his theory. But as was shown by one of the present writers, it can also be obtained from another theory,<sup>1</sup> which has no connection with the molecular field. The above deduction of the same relation affords also a similar example.

The relation (16) may also be written as

$$I(T - \Omega) = \text{const.} \times H.$$

This represents a hyperbola, whose parameter is  $H$ ; curves 1, 2, 3, in Fig. 4 are those corresponding to successively increasing fields. Curie,

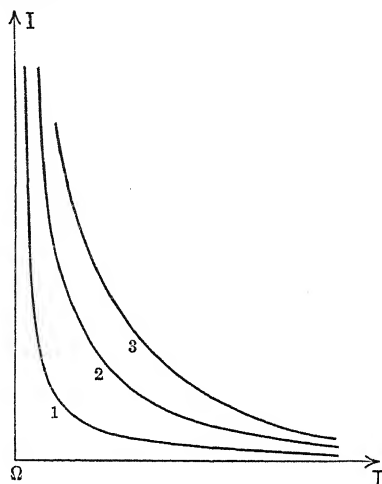


Fig. 4.

in his experimental research on the susceptibility of iron at high temperatures, first noticed this result and remarked that the course of the curves resembles the  $p, v$  curve of a gas near its critical point. In recent years, this remark has attracted the attention of many physicists<sup>1</sup>; they have attempted to explain the relation between ferromagnetic and paramagnetic states by the analogy of the relation of the liquid state to the gaseous. But as we have shown above, the fact is a natural consequence from the present theory, and we think it has not the important significance which other physicists assign to it.

<sup>1</sup> P. Weiss & P. N. Beck, Jour. de Phys., 7, 1908, 249. Ashworth, Phil. Mag., 27, 1915, 357; 30, 1915, 711; 33, 1917, 334. E. Bruins, Phil. Mag., 34, 1917, 380.



## ON THE THEORY OF SUPERPOSED DIFFRACTION-FRINGES.

BY CHANDI PRASAD.

IN the Proceedings of the American Philosophical Society for 1913 (pp. 276-282) C. F. Brush<sup>1</sup> has published various interesting observations of diffraction phenomena which he has attempted to group together under a common explanation by regarding them as "superposed fringes." Among the effects observed by him were those due to the diffraction of light by a cylindrical edge which have recently been further investigated and explained in an entirely different manner by N. Basu.<sup>2</sup> In the present paper, I am concerned with what were perhaps the most interesting and original observations recorded by Brush, that is, the diffraction-effects produced by a row of straight edges placed in echelon order. The arrangement adopted by Brush is shown diagrammatically in Fig. 1. A row of Gillette razor-blades (sometimes as many as twenty-four) were clamped together, so that their edges lay as nearly as possible in one plane which was placed very obliquely in the path of the train of light waves. The phenomena in the immediate neighborhood of the system were observed through a microscope. Brush noticed that the fringes showed contrasts between the maxima and minima of illumination which were much more marked than in the diffraction fringes of the Fresnel type due to a single straight edge, and explained this as due to the superposition of the fringe-systems produced by the successive edges. I propose in the present paper to show how the principle of superposition suggested by Brush may be formulated mathematically and its validity tested in experiment. The value of the principle is that it simplifies the treatment of the problem of diffraction by a succession of edges which

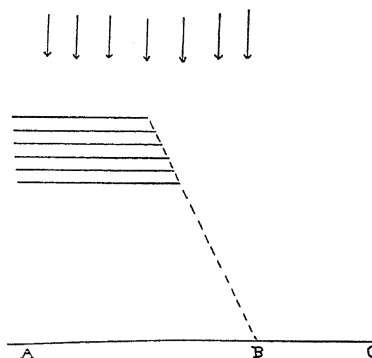


Fig. 1.

<sup>1</sup> See also Science Abstracts (1913), No. 1810.

<sup>2</sup> Phil. Mag., Jan., 1918, p. 79.

would otherwise be very laborious.<sup>1</sup> Incidentally I also describe some observations of my own which appear to be of interest from the point of view of the general theory of diffraction.

Sommerfeld has shown in his well-known investigation on the mathematical theory of diffraction<sup>2</sup> that the diffraction fringes due to a semi-infinite screen may be regarded as due to the interference of a system of a series of cylindrical waves emitted by the edge of the screen with the incident plane waves. In the case of a number of semi-infinite screens placed parallel to each other in echelon order, it would obviously *not* be correct to assume that in the region to the rear of the system, each of the edges emits cylindrical waves, the amplitude of which, in any direction is the same as if the other screens were absent. The assumption would certainly be incorrect in respect of the region lying on one side of the plane containing the edges (*AB* in Fig. 1); for, in this region each of the screens would obviously intercept and cut off the radiations emitted by the edges in front of it. But in the region lying on the other side (*BC* in Fig. 1), an assumption of the kind stated above would be justified as a first approximation, provided each edge in the echelon is sufficiently in advance of the preceding edge in relation to its distance from it. For,

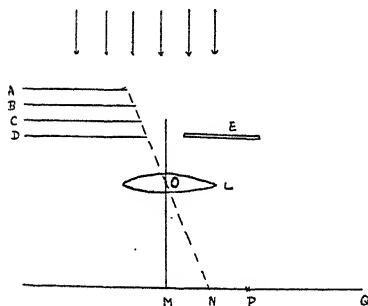


Fig. 2.

the illumination in the region at and near each edge would then be practically the same as that due to the incident waves alone, and the edge of each screen therefore emits a radiation, the amplitude of which in the region considered is practically the same as in the absence of the other screens. By superposing the effects of the cylindrical waves emitted by the edges upon that due to the incident waves, a mathematical treat-

ment of the phenomena observed by Brush would be possible.

In order to test the preceding views and to obtain a definite confirmation, I have devised the experiment shown diagrammatically in Fig. 2 in which the effects due to the echelon of edges may be examined separately from that due to the incident waves.

A row of razor edges is placed on an optical bench, the distances between successive edges being about 10 cm. The edges are brought carefully into parallelism with each other and placed in echelon order.

<sup>1</sup> On this point, see Mascarat's *Traite de Optique*, Vol. I., p. 287.

<sup>2</sup> *Math. Annalen*, Vol. XLVII., 1896, p. 317.

The incident plane waves of light are diffracted by the edges and pass through the lens  $L$ , the observations being made in the focal plane. The superfluous light was cut off by the screen  $E$ . The lens may be dispensed with, if the fringes are examined at a great distance behind the system. Some interesting effects are then observed. First, we have the direct light which comes to a focus at or near the point  $M$ , on either side of which we have the diffracted light due to the limitation of the aperture of the lens. In the region  $NPQ$ , we have another system of fringes which are due to the superposition of the light diffracted by the series of edges. These fringes appear only to the right of the plane containing the edges,  $ON$  being the line drawn through the center of the lens parallel to this plane. The center of this system of fringes (as seen in white light) is at  $P$  where  $MN = NP$ . Fig. 3A and Fig. 3B reproduce photographs of the system of fringes observed with an echelon of two edges only, the second edge being further in advance of the first edge in the latter case than in the former. It will be seen that the fringe-widths are asymmetrical, that is, the fringe width decreases continuously as we move from one side of the system to the other. The positions of the fringes are approximately the same as that due to the interference of two cylindrical sources of light placed respectively at the two diffracting edges. This is shown by the following measurements of the width of the fringes on either side of the center  $P$ .

	Positions of the Maxima Read on a Micrometer Scale.	Fringe Widths Observed.	Fringe Widths Calculated.
First Maxima....	3.283	.125	.124
Second Maxima...	3.158	.105	.098
Third Maxima....	3.053	.094	.084
Fourth Maxima...	2.959	.071	.075
Fifth Maxima....	2.888	.069	.068
Sixth Maxima....	2.819 (the central fringe)	.062	.063
Seventh Maxima..	2.757	.056	.058
Eighth Maxima...	2.701	.051	.054
Ninth Maxima....	2.650	.051	.052
Tenth Maxima...	2.599	.045	.052
Eleventh Maxima.	2.554	.045	.052
Twelfth Maxima..	2.509		

The figures in the last column were calculated from the formula

$$a(\cos \theta - \cos \psi) = \pm n\lambda,$$

where  $a$  is the distance between the two edges;  $\theta$ ,  $\psi$ , are the small angles made by the incident and diffracted rays, respectively, with the plane containing the edges.

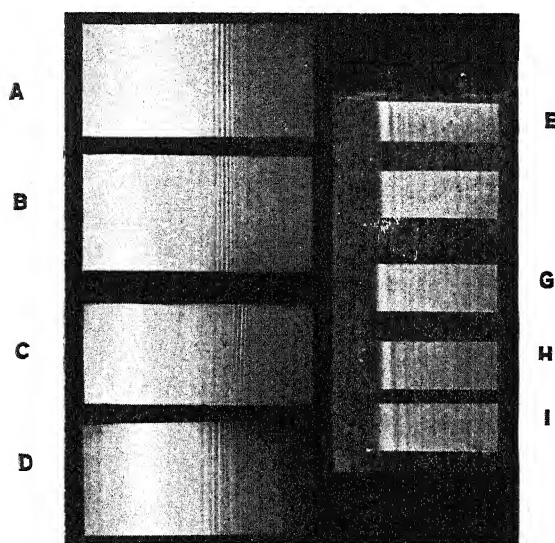
Fig. 3C reproduces a photograph of the fringes obtained with an echelon of three edges and Fig. 3D reproduces one obtained with four edges. It will be seen that in the former case, we have one faint secondary maxima between each pair of primary maxima, and in the latter case, we have two secondary maxima between the principal maxima. In fact, the results are somewhat though not completely analogous to those due to a diffraction grating composed of a small number of elements. The difference in this case is that the light is diffracted by simple edges, while in an ordinary diffraction grating we are concerned with either a corrugated surface or the strips of a plane surface. Photographs of the diffraction spectra formed by a very obliquely held surface consisting of two or three reflecting strips in a plane have recently been published by Mitra,<sup>1</sup> who has observed that the corresponding bands on either side of the pattern are of unequal brightness, the wider fringes on one side being of very feeble intensity in comparison with the narrow fringes on the other. The photographs of the diffraction effects produced by a row of straight edges published in the present paper (Figs. 3A to D) do not show this effect, thus indicating an essential difference between the two cases. In fact, in the diffraction spectra due to a row of straight edges, the broader fringes on one side are, if anything, actually brighter than the narrow fringes on the other.

With a view to further study of the effects noticed by Brush (in which the radiations from the edges are superposed on the transmitted waves), I have also measured and photographed the diffraction-fringes of the Fresnel type due to two edges lying in nearly the same line as the direction of propagation of the incident light. Four photographs are reproduced in Figs. 3 (E to H), the second edge being in advance of the first edge by different distances in the four pictures. The diffraction-fringes due to a single edge are shown in Fig. 3I for comparison. Some of the minima of illumination due to the superposition of the effects of the two edges are seen in the photographs to be much darker than the minima in the fringes of the Fresnel type due to a single edge. If we assume as a first approximation that the effects due to the two edges are practically additive, the illumination at any point in the fringe system may be calculated mathematically as follows:

The problem of the diffraction of light by a semi-infinite screen has been solved by Sommerfeld whose solution of the equation

$$\frac{\partial^2 s}{\partial t^2} = a^2 \left( \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} \right)$$

<sup>1</sup> Philosophical Magazine, Jan., 1918, Plate V.



SUPERPOSED DIFFRACTION FRINGES.

FIG. 3.

C. PRASAD.



is

$$s = \frac{e^{i(nt+\pi/4)}}{\sqrt{\pi}} \left[ e^{-ikr \cos(\phi-\phi')} \int_{-\infty}^{\sqrt{2kr} \cos(\phi-\phi')/2} e^{-iT^2} dT \right. \\ \left. \pm e^{-ikr \cos(\phi+\phi')} \int_{-\infty}^{\sqrt{2kr} \cos(\phi+\phi')/2} e^{-iT^2} dT \right].$$

This solution may be simplified by expanding the integrals in a semi-convergent series, as shown by Sommerfeld. In the region of light where  $\pi - \phi' < \phi < \pi + \phi'$ , and provided  $\sqrt{2kr} \cos(\phi - \phi')/2 > 1$ , we have  $s = \cos(kr \cos \phi - \phi' + nt)$

$$+ \frac{1}{4\pi} \sqrt{\frac{\lambda}{r}} \cos\left(kr - nt + \frac{\pi}{4}\right) \left\{ \frac{\pm}{\cos \frac{\phi + \phi'}{2}} - \frac{1}{\cos \frac{\phi - \phi'}{2}} \right\}.$$

In the case of normal incidence,  $\phi' = \pi/2$ , and writing  $\phi' = 3\pi/2 - \delta$ , where  $\delta$  is small, we find on reduction and simplification

$$s = \cos(kd - nt) + \frac{\sqrt{\lambda d}}{2\pi x} \cos\left(kd - nt + \frac{kx^2}{2d} + \frac{\pi}{4}\right),$$

which gives the usual maxima and minima when

$$\frac{kx^2}{2d} + \frac{\pi}{4} = 2n\pi \quad \text{or} \quad (2n + 1)\pi,$$

respectively. Now, if another edge were placed at a distance  $a$  cm. and projecting  $b$  cm. from the line of the geometrical shadow of the first edge, and its effect is added to that of the latter,

$$s = \cos(kd - nt) - \frac{\sqrt{\lambda d}}{2\pi x} \cos\left(kd - nt + \frac{kx^2}{2d} + \frac{\pi}{4}\right) \\ - \frac{\sqrt{\lambda(d-a)}}{2\pi(x-b)} \cos\left(kd - nt + \frac{kx-b^2}{2(d-a)} + \frac{\pi}{4}\right).$$

This gives for the intensity of illumination at any point

$$J = 1 + \frac{\lambda d}{4\pi^2 x^2} + \frac{\lambda(d-a)}{4\pi^2(x-b)^2} - \frac{\sqrt{\lambda d}}{\pi x} \cos\left(\frac{kx^2}{2d} + \frac{\pi}{4}\right) \\ - \frac{\sqrt{\lambda(d-a)}}{\pi(x-b)} \cos\left(\frac{k(x-b)^2}{2(d-a)} + \frac{\pi}{4}\right) \\ + \frac{\lambda \sqrt{d(d-a)}}{2\pi^2 x(x-b)} \cos \frac{k}{2} \left( \frac{x^2}{d} - \frac{x-b^2}{d-a} \right).$$

Of these six terms, the second, third and sixth are small compared with the first, third and fourth and may in calculating the position of the maxima and minima, be left out of account.

To test the results obtained from the formula, measurements have been made of the position of the maxima and minima in several cases. The experimental data and the theoretical values are shown for comparison in the table, the latter being taken from the illumination curves plotted from the formula. Fig. 4 shows three illumination curves and illustrates the general effect of the superposition of the fringes due to two edges.

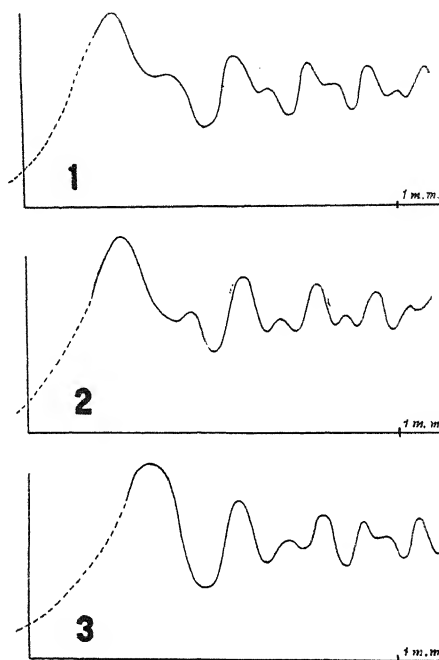


Fig. 4.

Illumination curves for two edges.

1.  $b = .000$  G.m.      2.  $b = .002$  G.m.      3.  $b = .014$  G.m.

I would like to express my indebtedness to Professor C. V. Raman for the help and encouragement he gave me during the course of this investigation, which was carried on last summer in the Laboratory of the Indian Association for the Cultivation of Science, Calcutta.



*Positions of the maxima and minima of illumination (measured from the geometrical edge of the shadow of the edge nearest the plane of observation.*

1.  $d = 20$  cm.,  $a = 10$  cm. and  $b = .002$  cm.

Bands.	Observed.	Calculated.
First maximum.....	+.024	.025
First minimum.....	+.040	.040
Second maximum.....	+.046	.044
Second minimum.....	+.052	.050
Third maximum.....	+.058	.058
Third minimum.....	+.065	.065
Fourth minimum.....	+.073	.073
Fifth minimum.....	+.083	.083
Sixth minimum.....	.....	.089
Seventh minimum.....	+.096	.098
Eighth minimum.....	+.108	.105

2.  $d = 20$  cm.

$a = 10$  cm.

$b = .014$  cm.

First minimum.....	+.0378	.034
Second minimum.....	+.0526	.051
Third minimum.....	+.0617	.060
Fourth minimum.....	+.0712	.072
Fifth minimum.....	.....	.081
Sixth minimum.....	+.0892	.088
Seventh minimum.....	+. 102	...

3.  $d = 20$  cm.

$a = 10$  cm.

$b = .022$  cm.

First minimum.....	.021	.019
Second minimum.....	.032	.032
Third minimum.....	.042	.042
Fourth minimum.....	.051	.050
Fifth minimum.....	.059	.059

The agreement is fairly satisfactory.

QUEEN'S COLLEGE,  
BENARES, INDIA,  
March 28, 1918.

# ON THE MECHANICAL AND ELECTRODYNAMICAL PROPERTIES OF THE ELECTRON.

BY MEGH NAD SAHA.

THE object of the present paper is to extend Minkowski's method<sup>1</sup> of four-dimensional analysis to the investigation of the mechanical and electrodynamical problems connected with the electron. As is well-known, Minkowski's four-dimensional analysis is based on the principle of relativity, and we have thereby to abandon two time-honored concepts of physics, *i. e.*, absolute independence of time and space, and the constancy of mass. The correctness of these two principles is no longer a matter of hypothesis, but is founded on experiments. It is therefore to be hoped that the results of these investigations will be helpful to us for the elucidation of the mechanical and electrical problems connected with the electron, though sometimes difficulty may be encountered in putting proper interpretation on these results.

The notation is the same as that adopted by Minkowski, and for the convenience of the reader, it is explained at the very outset.

## I.

$(x, y, z, l \pm ict)$  denotes the space and time coördinates of any point in the four-dimensional world

$$(\omega_1, \omega_2, \omega_3, \omega_4) = \sqrt{1 - \frac{u^2}{c^2}} \left[ \frac{u_1}{c}, \frac{u_2}{c}, \frac{u_3}{c}, i \right]$$

denotes the velocity four-vector of the point.

We put

$$ds^2 = - (dx^2 + dy^2 + dz^2 + dl^2)$$

therefore we have

$$(\omega_1, \omega_2, \omega_3, \omega_4) = \left( \frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}, \frac{dl}{ds} \right), \text{ and } \sqrt{-} (\omega_1, \omega_2, \omega_3, \omega_4)$$

<sup>1</sup> Minkowski's method of four-dimensional analysis is expounded in two papers: (1) *Raum und Zeit*, published in the *Phys. Zeits.*, and (2) *Die Grundgleichungen für die Electromagnetischen Vorgänge in bewegten Körpern*-Gött. Nach, 1908. These two papers have been translated by me, and are being published by the Calcutta University.

denote the direction cosines of the four-dimensional tangent to the path of the particle.  $c$  = velocity of light in space.

We put

$$(f_{23}, f_{31}, f_{12}) = (Hx, Hy, Hz),$$

the components of the magnetic field, and

$$(f_{14}, f_{24}, f_{34}) = -i[Ex, Ey, Ez],$$

the components of the electric field. Minkowski has shown that  $f$  constitutes a six-vector.

$$(a_1, a_2, a_3, a_4) = [F, G, H, l\phi],$$

are the components of the potential four-vector;  $(F, G, H)$  are the vector potentials  $\Phi$  is the scalar potential.

$\rho$  = electrical space-density;

$$\rho \left[ \frac{u_1}{c}, \frac{u_2}{c}, \frac{u_3}{c}, i \right] = \rho_0(\omega_1, \omega_2, \omega_3, \omega_4)$$

are the components of the stream four-vector  $s$ ;

$$\rho_0 = \rho \sqrt{1 - \frac{u^2}{c^2}}$$

is known as the rest-density of electricity.

The vector operator

$$\square = \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} + \frac{\partial}{\partial l} \right),$$

is known as the lor and the scalar operator

$$\square^2 = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial l^2} \right)$$

is known as the generalized D'alembertian.

The equations of electrodynamics can be written in the forms

$$\begin{aligned} \text{lor } f &= s, & \text{lor } f^* &= 0, \\ f &= \text{Curl } a; & \square^2 a &= -s, & \square a &= 0. \end{aligned}$$

## 2. THE SCALAR AND VECTOR POTENTIALS OF A MOVING ELECTRON.

Lienard,<sup>1</sup> and almost simultaneously Wiechert<sup>2</sup> showed that the scalar and vector potentials are given by the expressions

$$\phi = -\frac{e}{r \left( 1 - \frac{ur}{c} \right)}, \quad (F, G, H) = -\frac{e(u_1, u_2, u_3)}{cr \left( 1 - \frac{ur}{c} \right)}, \quad (1)$$

<sup>1</sup> Lienard, L'eclairage electrique, 16 (1898), pp. 5, 53 and 106.

<sup>2</sup> Wiechert, Arch. Neérl., (2), 5 (1900).

If  $P$  be the point at which the potentials are calculated at the time  $t$  and  $M$  be the position of the electron at the time  $t_0$ , where  $MP = c(t - t_0)$ , the distance  $MP$  is denoted by  $r$  and  $[u]$  denotes the velocity in the position  $M$ , and  $(ur)$  its component in the direction of  $r$ .

The formulæ are deduced from the theory of retarded potential and do not involve the principle of relativity.

Several investigators<sup>1</sup> have shown that the formulæ can also be deduced from the theory of relativity and can be thrown into the compact form

$$a = \frac{e[\omega]}{[R\omega]},$$

$R$  being the four-vector joining the two points,  $[R \cdot \omega]$  denoting the scalar product of  $R$  and  $\omega$ .

It is quite clear that the forms (1) and (2) are quite equivalent.

In a paper published elsewhere, it has been shown that from Minkowski's four-dimensional analysis we obtain

$$a = \frac{e[\omega]}{P}. \quad (3)$$

In this formula,  $(x, y, z, l)$  denote the time-space coördinates of the electron ( $A$ ),  $(\omega_1, \omega_2, \omega_3, \omega_4)$  its velocity-components,  $(x' y' z' l')$  denote the space-time coördinates of the point  $B$  at which the potentials are estimated,

$P$  denotes the four-dimensional perpendicular distance of  $B$  from the axis of motion of ( $A$ ); since the direction-cosines of this axis are  $-i(\omega_1, \omega_2, \omega_3, \omega_4)$ , we have

$$\begin{aligned} P^2 &= (x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 \\ &\quad + [(x - x')\omega_1 + (y - y')\omega_2 + (z - z')\omega_3 + (l - l')\omega_4]^2 \\ &= R^2 + [R\omega]^2. \end{aligned}$$

Now if we make the assumption that the time coördinates are so chosen that

$$\begin{aligned} R^2 &= (x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 = 0 \\ \text{i. e.,} \quad c^2(t - t')^2 &= r^2, \\ \text{i. e.,} \quad c(t - t') &= r, \end{aligned} \quad (4)$$

the formula (3) becomes the same as (2) and therefore (1). Also the assumption which we make here about the interval between the time-coördinates is identical with the premises of Lienard and Wiechert.

<sup>1</sup> Sommerfeld, Über die Relativitäts-theorie, Ann. d. Physik, Vols. 32 and 33.

I am not quite certain whether this assumption (4) which is made here is at all essential. I am inclined to think that it is not essential, but necessary only for the interpretation of the result to those three-dimensional beings whose senses are not sharpened enough to enable them to grasp a result expressed in four-dimensional figures.

### 3. THE ELECTRIC AND MAGNETIC FIELDS DUE TO A MOVING ELECTRON.

If  $a$  denote the potential four-vector, the components of the six-vector  $f$  giving the electric and magnetic fields are given by

$$f = \text{Curl } a = \begin{vmatrix} \frac{\partial}{\partial x'} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z'} & \frac{\partial}{\partial t'} \\ a_1 & a_2 & a_3 & a_4 \end{vmatrix}.$$

Thus

$$f_{12} = \frac{\partial a_2}{\partial x'} - \frac{\partial a_1}{\partial y'} = -\frac{e}{P^3} [\omega_3 \alpha_2 - \omega_2 \alpha_3], \text{ etc.} \quad (5)$$

where

$$\alpha_1 = P \frac{\partial P}{\partial x'}, \quad \alpha_2 = P \frac{\partial P}{\partial y'} \dots \text{etc.}$$

we can easily verify that if we put  $c(t - t') = r$ , we have

$$Hx = \frac{e\beta^2}{r^3\lambda^3} \left[ \frac{u_2}{c} (z - z') - \frac{u_3}{c} (y - y') \right],$$

where

$$\begin{aligned} \lambda &= \left( 1 - \frac{Ur}{c} \right), \\ &= \frac{e\beta^2}{r^3\lambda^3} [\mathbf{u} \times \mathbf{r}]. \end{aligned} \quad (6)$$

The electric forces are given by

$$\begin{aligned} f_{14} &= -iEx = \frac{\partial a_4}{\partial x'} - \frac{\partial a_1}{\partial t'}, \\ &= -\frac{e}{P^3} [\omega_1(l - l') - \omega_4(x - x')], \\ &= -i \frac{e\beta^2}{r^3\lambda^3} \left[ r \frac{u_1}{c} - (x - x') \right], \end{aligned} \quad (7)$$

and generally

$$Ex = \frac{e\beta^2}{r^3\lambda^3} \left[ (x - x') - \frac{ru_1}{c} \right], \quad E = \frac{e\beta^2}{r^3\lambda^3} \left[ \mathbf{r} - \frac{r\mathbf{u}}{c} \right].$$

These values are widely different and simpler than the values obtained

from the older theories, for example, compare the values given by Crehore.<sup>1</sup>

The discrepancy is due to the fact that in these older theories, we always assume that the equation

$$(x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 = 0,$$

is an essential condition. But in performing differentiations with regard to  $(x', y', z', l')$  we here assume that they are quite independent of  $(x, y, z, l)$ . I am not quite definite as to which of these two standpoints is correct but I am inclined to think that my standpoint is more in accordance with Minkowski's ideas of time and space. However it is preferable to keep an open mind on this point.

### 3. MAXWELL'S STRESSES, POYNTING-VECTOR, ETC.

Minkowski has shown that if we multiply  $f$  by its own matrix, we obtain a matrix

$$ff = \begin{vmatrix} S_{11} - L^2 & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} - L^2 & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} - L^2 & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} - L^2 \end{vmatrix}$$

where

$$S_{11} = \frac{1}{2}[f_{23}^2 + f_{34}^2 + f_{42}^2 - f_{12}^2 - f_{13}^2 - f_{14}^2],$$

$$S_{12} = [f_{13}f_{32} + f_{14}f_{42}],$$

$$L = \frac{1}{2}[f_{23}^2 + f_{32}^2 + f_{12}^2 + f_{14}^2 + f_{24}^2 + f_{34}^2],$$

and the matrix

$$\frac{1}{4\pi} \begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{vmatrix} = \begin{vmatrix} X_x & X_y & X_z \\ Y_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{vmatrix}$$

denote the Maxwellian stresses,  $i(S_{14}, S_{24}, S_{34})$  denote the components of the Poynting vector, and  $S_{44}$  is the energy function. We have generally

$$X_x = \frac{1}{8\pi} [f_{23}^2 + f_{34}^2 + f_{42}^2 - f_{12}^2 - f_{13}^2 - f_{14}^2], \quad (8)$$

$$X_y = \frac{1}{4\pi} [f_{13}f_{32} + f_{14}f_{42}],$$

etc.

Now on the standpoint taken up by me, it is quite easy to calculate these quantities. It can be shown that

$$X_x = \frac{e^2}{8\pi P^2} [-P^2(1 + 2\omega_1^2) + \alpha_1^2], \quad X_y = \frac{e^2}{8\pi P^6} [-\omega_1\omega_2 P^2 + \alpha_1\alpha_2], \quad (9)$$

<sup>1</sup> PHYS. REV., July, 1917, p. 448.

The Poynting-vector

$$(X_t, Y_t, Z_t) = \frac{e^2}{8\pi P^6} [(-\omega_1\omega_4P^2 + \alpha_1\alpha_4), (-\omega_2\omega_4P^2 + \alpha_2\alpha_4), (-\omega_3\omega_4P^2 + \alpha_3\alpha_4)]$$

and the energy function

$$S_{44} = L = \frac{e^2}{8\pi P^6} [-P^2(1 + 2\omega_4^2) + \alpha_4^2]$$

where

$$\alpha_1 = P \frac{\partial P}{\partial x'}, \quad \alpha_2 = P \frac{\partial P}{\partial y'}, \quad \text{etc.}$$

and

$$\alpha^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 = P^2.$$

#### 4. THE LAW OF ATTRACTION BETWEEN TWO MOVING ELECTRONS.

We can now proceed to find out the attraction which one moving electron exerts upon another moving electron.

According to Lorentz's theorem the components of the force acting on an electron (4) moving in any electromagnetic field are

$$\begin{aligned} X &= e[\omega_2 f_{12} + \omega_3 f_{13} + \omega_4 f_{14}], \\ Y &= e[\omega_1 f_{21} + \omega_3 f_{23} + \omega_4 f_{24}], \\ Z &= e[\omega_1 f_{31} + \omega_2 f_{32} + \omega_4 f_{34}], \end{aligned} \quad (10)$$

and we can also add the fourth or the time component

$$L = -\frac{ie}{c}[Xu_1 + Yu_2 + Zu_3], \quad \beta = \sqrt{1 - \frac{v^2}{c^2}}$$

which is proportional to the rate at which work is done by the moving charge,—we have

$$L = e[\omega_1 f_{41} + \omega_2 f_{42} + \omega_3 f_{43}].$$

In this case, the field is due to the second electron (charge  $e'$ , position  $x' y' z' l'$ , velocity components  $\omega_1' \omega_2' \omega_3' \omega_4'$ ).

According to the last section, the potential four-vector

$$a = \frac{e'[\omega']}{P'}, \quad \text{where } P'^2 = R^2 + [R\omega']^2.$$

We have now, since  $f = \text{Curl } a$ ,

$$\begin{aligned} X &= ee' \left[ \omega_2 \left\{ \frac{\partial}{\partial x} \left( \frac{\omega_2'}{P'} \right) - \frac{\partial}{\partial y} \left( \frac{\omega_1'}{P'} \right) \right\} + \omega_3 \left\{ \frac{\partial}{\partial x} \left( \frac{\omega_3'}{P'} \right) - \frac{\partial}{\partial z} \left( \frac{\omega_1'}{P'} \right) \right\} \right. \\ &\quad \left. + \omega_4 \left\{ \frac{\partial}{\partial x} \left( \frac{\omega_4'}{P'} \right) - \frac{\partial}{\partial l} \left( \frac{\omega_1'}{P'} \right) \right\} \right] \\ &= ee' \left[ \frac{\partial}{\partial x} \left( \frac{\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4'}{P'} \right) \right. \\ &\quad \left. - \left( \omega_1 \frac{\partial}{\partial x} + \omega_2 \frac{\partial}{\partial y} + \omega_3 \frac{\partial}{\partial z} + \omega_4 \frac{\partial}{\partial l} \right) \left( \frac{\omega_1'}{P'} \right) \right], \end{aligned}$$

Now putting

$$\Phi = ee'(\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4')/P',$$

we find that

$$X = \frac{\partial \Phi}{\partial x} - \frac{d}{ds} \left( \frac{\partial \Phi}{\partial \frac{dx}{ds}} \right),$$

where  $\partial/\partial x$  denotes differentiation in which  $x$  is explicitly involved, similarly with

$$\frac{\partial}{\partial \omega_1} = \frac{\partial}{\partial \frac{dx}{ds}}.$$

$$\frac{d}{ds} = \left( \omega_1 \frac{\partial}{\partial x} + \omega_2 \frac{\partial}{\partial y} + \omega_3 \frac{\partial}{\partial z} + \omega_4 \frac{\partial}{\partial l} \right),$$

as is easily seen. We have similarly

We have similarly

$$\begin{aligned} Y &= \frac{\partial \Phi}{\partial y} - \frac{d}{ds} \left( \frac{\partial \Phi}{\partial \frac{dy}{ds}} \right), & Z &= \frac{\partial \Phi}{\partial z} - \frac{d}{ds} \left( \frac{\partial \Phi}{\partial \frac{dz}{ds}} \right), \\ L &= \frac{\partial \Phi}{\partial l} - \frac{d}{ds} \left( \frac{\partial \Phi}{\partial \frac{dl}{ds}} \right). \end{aligned} \quad (11)$$

We can say that  $\Phi$  is the kinetic-potential of the electron (A) in the field of the electron (B). Similarly if  $\Phi'$  denote the Kinetic-Potential of the electron (B) in the field of (A),

$$\Phi' = ee'(\omega_1\omega_1' + \omega_2\omega_2' + \omega_3\omega_3' + \omega_4\omega_4')/P,$$

$$P^2 = R^2 + (R\omega)^2.$$

and we have similarly

$$X' = \frac{\partial \Phi'}{\partial x'} - \frac{d}{ds'} \left( \frac{\partial \Phi'}{\partial \frac{dx'}{ds'}} \right). \quad (12)$$

Let us now interpret the results in three dimensions. We have

$$X = \frac{ee'\beta'^2}{r^2\lambda^3\beta'}(x - x') \left( 1 - \frac{uv \cos \theta}{c^2} \right) - \frac{ee'\beta'^2}{r^2\lambda^3\beta'c} \left( 1 - \frac{Ur}{c} \right) u_1', \quad (13)$$

where

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}, \quad \beta' = \sqrt{1 - \frac{v'^2}{c^2}}, \quad \lambda = \left( 1 - \frac{Ur'}{c} \right).$$



In three dimensions, the forces are equivalent to a force of repulsion

$$\frac{ee'\beta'^2}{r^3\lambda^3\beta} \left( 1 - \frac{uv \cos \theta}{c^2} \right) \mathbf{r},$$

in the direction of the line joining the two points, and a force

$$\frac{ee'\beta'^2}{r^2\lambda^3\beta c} \left( 1 - \frac{Ur}{c} \right) \mathbf{u}', \quad (14)$$

in the direction of the velocity of the second or the attracting point.

We thus perceive that the force which comes out in a very simple form in four dimensions takes a very complicated form in three dimensions.

The kinetic potential

$$\phi = \frac{ee' \left( 1 - \frac{vv' \cos \theta}{c^2} \right)}{r \left( 1 - \frac{U'r}{c} \right) \beta}. \quad (15)$$

This kinetic potential is practically coincident with the kinetic potential assumed by Clausius in order to find out the law of attraction between two moving charges of electricity; Clausius has shown that this kinetic potential leads us to the celebrated electrodynamic laws of Ampere. A short résumé of the work done in this connection is given below for the purpose of comparison. The problem was first enunciated by Gauss in the year 1835, and was called by him the fundamental keystone of electrodynamics.<sup>1</sup>

1. Gauss (1835): The forces are the derivatives with regard to  $(x, y, z)$  of the potential function,

$$\phi = \frac{ee'}{r} \left( 1 - \frac{3}{2c} \frac{d^2 r}{dt^2} \right).$$

2. Weber (1843) takes the potential function

$$\phi = \frac{ee'}{r} \left( 1 - \frac{1}{c^2} \frac{dr^2}{dt} \right).$$

Both of these forms have been long discredited. Later writers have pointed out that the force cannot be simply the derivations with regard to  $(x, y, z)$  of some potential function, but are the Lagrangian derivatives of a certain kinetic-potential. We give the form of this kinetic potential according to different investigators.

<sup>1</sup> For the literature on the subject, see Maxwell, *Electricity and Magnetism*, Vol. 2, Chap. XXIII., and J. J. Thomson, *Application of Dynamics to Problems of Physics and Chemistry*, pp. 35.

## 1. Clausius (1881);

$$\phi = \frac{ee'}{r} \left( 1 - \frac{uu' \cos \theta}{c^2} \right),$$

where  $v$  and  $u'$  are the velocities of the two electrons, and  $\theta$  is the angle between their lines of motion. In two papers communicated to the Crelle's journal,<sup>1</sup> Clausius deduces Ampere's laws of electrodynamical action between two currents from this law.

## 2. J. J. Thomson

$$\phi = \frac{ee'}{r} \left( 1 - \frac{\mu}{3c^2} uu' \cos \theta \right) \quad (\mu = \text{magnetic permeability} = 1).$$

Crehore<sup>2</sup> has calculated the forces components according to J. J. Thomson's theory.<sup>3</sup> He finds that the forces are equivalent to

$$F_1 = \frac{ee'}{r^2}, \text{ a repulsion along the line joining the centers.}$$

$$F_2 = \frac{ee'}{c^2 r^2} uu' \cos \theta, \text{ an attraction along the same line.}$$

$$F_3 = \frac{ee'}{r} u', \text{ a force in the direction opposite to the acceleration of the second charge.}$$

$$F_4 = ee' u' \frac{d}{dt} \left( \frac{1}{r} \right), \text{ a force in a direction opposite to the velocity of the second charge.}$$

3. Sommerfeld<sup>4</sup> has also calculated the ponderomotive forces, assuming that the value of the potential four-vector

$$a = \frac{e[\omega]}{(R\omega)},$$

and using the condition

$$(x - x')^2 + (y - y')^2 + (z - z')^2 + (l - l')^2 = 0$$

in course of differentiation. Their forms are a bit too complicated.

## 5. EQUATIONS OF MOTION OF THE ELECTRON.

Minkowski<sup>5</sup> deduces the equations of motions of a ponderable particle by means of a variational process in which the function

$$\int m_0 c^2 ds, \text{ where } ds^2 = - (dx^2 + dy^2 + dz^2 + dl^2) = c^2 dt^2 \left( 1 - \frac{u^2}{c^2} \right)$$

is used instead of the three-dimensional  $\int T dt$ .

<sup>1</sup> Vols. 82 and 83.

<sup>2</sup> It will be seen that forces  $F_1$ ,  $F_2$ ,  $F_4$  are, but for some minor details, represented in our formula. Force  $F_3$  does not occur at all.

<sup>3</sup> Phil. Mag., 1913.

<sup>4</sup> Ann. d. Phys., vols. 32 and 33, Über die Relativitäts theories.

<sup>5</sup> Minkowski, *loc. cit.* Anhaup, Mechanics.

He obtains

$$m_0 c^2 \frac{d^2 x}{ds^2} = X, \quad m_0 c^2 \frac{d^2 y}{ds^2} = Y, \quad m_0 c^2 \frac{d^2 z}{ds^2} = Z, \quad m_0 c^2 \frac{d^2 l}{ds^2} = L. \quad (16)$$

Now we have

$$X = e[\omega_2 f_{12} + \omega_3 f_{13} + \omega_4 f_{14}],$$

according to Lorentz's theorem. We have also

$$\begin{aligned} \frac{d^2 x}{ds^2} &= \left( \omega_1 \frac{\partial}{\partial x} + \omega_2 \frac{\partial}{\partial y} + \omega_3 \frac{\partial}{\partial z} + \omega_4 \frac{\partial}{\partial l} \right) \omega_1 \\ &= \omega_1 \frac{\partial}{\partial x} \left[ \frac{1}{2} (\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \right] + \omega_2 \left[ \left( \frac{\partial \omega_1}{\partial y} - \frac{\partial \omega_2}{\partial x} \right) \right. \\ &\quad \left. + \omega_3 \left( \frac{\partial \omega_1}{\partial z} - \frac{\partial \omega_3}{\partial x} \right) + \omega_4 \left( \frac{\partial \omega_1}{\partial l} - \frac{\partial \omega_4}{\partial x} \right) \right] \\ &= -(\omega_2 \Omega_{12} + \omega_3 \Omega_{13} + \omega_4 \Omega_{14}), \text{ putting } \Omega_{hk} = \frac{\partial \omega_k}{\partial x^h} - \frac{\partial \omega_h}{\partial x^k}. \end{aligned}$$

Hence we have the four equations, putting  $\mu = c^2 m_0 / e$ ,

$$\left. \begin{aligned} \omega_2(f_{12} + \mu \Omega_{12}) + \omega_3(f_{13} + \mu \Omega_{13}) + \omega_4(f_{14} + \mu \Omega_{14}) &= 0 \\ \omega_1(f_{21} + \mu \Omega_{21}) + \omega_3(f_{23} + \mu \Omega_{23}) + \omega_4(f_{24} + \mu \Omega_{24}) &= 0 \\ \omega_1(f_{31} + \mu \Omega_{31}) + \omega_2(f_{32} + \mu \Omega_{32}) + \omega_4(f_{34} + \mu \Omega_{34}) &= 0 \\ \omega_1(f_{41} + \mu \Omega_{41}) + \omega_2(f_{42} + \mu \Omega_{42}) + \omega_3(f_{43} + \mu \Omega_{43}) &= 0 \end{aligned} \right\}.$$

Of these, only three are independent; the fourth can be deduced from the first three.

We have now identically

$$\begin{vmatrix} f_{12} + \mu \Omega_{12} & f_{13} + \mu \Omega_{13} & f_{14} + \mu \Omega_{14} \\ f_{21} + \mu \Omega_{21} & f_{23} + \mu \Omega_{23} & f_{24} + \mu \Omega_{24} \\ f_{31} + \mu \Omega_{31} & f_{32} + \mu \Omega_{32} & f_{34} + \mu \Omega_{34} \\ f_{41} + \mu \Omega_{41} & f_{42} + \mu \Omega_{42} & f_{43} + \mu \Omega_{43} \end{vmatrix} = 0,$$

i. e.,

$$\begin{aligned} (f_{12} + \mu \Omega_{12})(f_{34} + \mu \Omega_{34}) + (f_{23} + \mu \Omega_{23})(f_{14} + \mu \Omega_{14}) \\ + (f_{31} + \mu \Omega_{31})(f_{24} + \mu \Omega_{24}) = 0. \end{aligned} \quad (17)$$

The condition is evidently satisfied if we put

$$-\mu = \frac{f_{12}}{\Omega_{12}} = \frac{f_{23}}{\Omega_{23}} = \frac{f_{31}}{\Omega_{31}} = \frac{f_{14}}{\Omega_{14}} = \frac{f_{24}}{\Omega_{24}} = \frac{f_{34}}{\Omega_{34}}. \quad (18)$$

If of these equations, any three are satisfied the remaining three come out automatically from the equations of motion. But we cannot possibly be sure of the authenticity of these relations unless it can be deduced from an independent source. For this purpose let us take the original variational equations.

Let  $(X, Y, Z, L)$  represent the components of the force four-vector at any point, which is subjected to a virtual displacement  $\delta x, \delta y, \delta z, \delta l$ .

Then

$$\delta W = X\delta x + Y\delta y + Z\delta z + L\delta l,$$

i. e., if we call

$$W = \frac{\partial A}{\partial s}, \quad A = \int W ds,$$

$$\begin{aligned} \delta A &= \int \delta W ds = \int (X\delta x + Y\delta y + Z\delta z + L\delta l) ds \\ &= e \int [f_{12}(dy\delta x - \delta y dz) + f_{23}(dz\delta y - \delta z dy) + f_{31}(dx\delta z - \delta x dz) \\ &\quad + f_{14}(dl\delta x - \delta l dx) + f_{24}(dl\delta y - \delta l dy) + f_{34}(dl\delta z - \delta z dl)]. \end{aligned}$$

Now the function  $\int m_0 c^2 ds$  can also be subjected to a variational process. Since

$$ds = \omega_1 dx + \omega_2 dy + \omega_3 dz + \omega_4 dl,$$

we find

$$\begin{aligned} \delta \int m_0 c^2 ds &= -m_0 c^2 \int [\Omega_{12} \delta S_{12} + \Omega_{23} \delta S_{23} + \Omega_{31} \delta S_{31} \\ &\quad + \Omega_{14} \delta S_{14} + \Omega_{24} \delta S_{24} + \Omega_{34} \delta S_{34}], \end{aligned}$$

where

$$\delta S_{hk} = dx_k \delta x_h - \delta x_k dx_h.$$

Thus from the variational equation

$$\delta \int m_0 c^2 ds + \int \delta W \cdot ds = 0;$$

i. e., from the principle of least action, keeping the initial and final points fixed, we obtain the original equation

$$\int [(f_{12} + \mu \Omega_{12}) \delta S_{12} + \dots] = 0.$$

The relations (18) thus seem to be borne out by independent evidence.

Difficulty is encountered here about the interpretation of the terms ( $\Omega_{12}$ ,  $\Omega_{23}$ , ...) in three dimensions  $\Omega$  is evidently a six-vector being the four-dimensional curl of the velocity four-vector. The components [ $\Omega_{23}$ ,  $\Omega_{31}$ ,  $\Omega_{12}$ ] are evidently connected with rotations

$$\left[ \frac{\partial u_3}{\partial y} - \frac{\partial u_2}{\partial z}, \quad \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}, \quad \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \right]$$

and [ $\Omega_{14}$ ,  $\Omega_{24}$ ,  $\Omega_{34}$ ] are connected with the accelerations

$$\left[ \frac{d^2 x}{dt^2}, \quad \frac{d^2 y}{dt^2}, \quad \frac{d^2 z}{dt^2} \right]$$

but the exact interpretation in three dimensions has not yet been obtained. We can style  $\Omega$  as the acceleration six-vector.

On a future occasion, I hope to communicate the result of my investigations on the orbits of the electron under different conditions.

In conclusion, I wish to express my best thanks to my friend, Mr. S. N. Basu, for much help, and useful criticism.

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May 11, 1918.

A NEW EXPERIMENTAL DETERMINATION OF THE  
BRIGHTNESS OF A BLACK BODY, AND OF THE  
MECHANICAL EQUIVALENT OF LIGHT.

BY EDWARD P. HYDE, W. E. FORSYTHE AND F. E. CADY.

PART I. BRIGHTNESS OF A BLACK BODY AS A FUNCTION OF ITS  
TEMPERATURE.

THE brightness or candlepower per  $\text{cm}^2$  of a black body at various temperatures has long been a matter of interest. Lummer and Kurlbaum,<sup>1</sup> in connection with their work on radiation, determined the relation between brightness and temperature for radiating platinum and expressed the opinion that the exponential relation found would hold at least approximately for a black body. Subsequently<sup>2</sup> Lummer and Pringsheim determined the brightness of a black body at three temperatures ( $1448^\circ \text{K.}$ ,  $1598^\circ \text{K.}$  and  $1708^\circ \text{K.}$ ) and extrapolated their observed curve to give the brightness up to  $2073^\circ \text{K.}$ , at which temperature they ascribed the brightness of 100 Hefner candles per  $\text{cm}^2$ .

A couple of years later Nernst,<sup>3</sup> starting from one of the extrapolated points given by Lummer and Pringsheim, found a brightness of 91 Hefner candles per  $\text{cm}^2$  at the temperature of melting platinum (taken by him as  $2018^\circ \text{K.}$ ), and a brightness of 1210 Hefner candles per  $\text{cm}^2$  at the temperature of melting iridium, which latter he calculated to be  $2621^\circ \text{K.}$  Subsequently<sup>4</sup> Nernst undertook a new set of measurements on the brightness of a black body, carrying his determinations over a range of temperature extending from about  $1460^\circ \text{K.}$  to about  $2280^\circ \text{K.}$  His temperature measurements of the black body were made with a Wanner pyrometer, and were based on a temperature of  $1337^\circ \text{K.}$  as the melting point of gold, and a value of  $14600 \mu \text{ deg.}$  for the constant  $C_2$  in the Planck equation.

From the observed data Nernst computed the values of the constants  $A$  and  $B$  in the equation

$$\log K = -\frac{A}{T} + B,$$

<sup>1</sup> Verh. d. Deut. Phys. Gesel., 2, p. 90, 1900.

<sup>2</sup> Phys. Zeit., 3, p. 97, 1901.

<sup>3</sup> Ibid., 4, p. 733, 1903.

<sup>4</sup> Ibid., 7, p. 380, 1906.

which had first been given by Rasch<sup>1</sup> and subsequently deduced on what Nernst considers more justifiable theoretical grounds by Haber<sup>2</sup> and Lucas<sup>3</sup> as expressing the relation between brightness,  $K$ , and temperature,  $T$ . Nernst apparently relies on the values deduced from this equation as more accurate than the individual direct determinations. With the use of this equation he computes the brightness of the black body at the temperature of melting platinum, taken as 2018° K., to be 63.4 Hefner candles per cm<sup>2</sup>, rather than 91 as given in his original paper. He offers an explanation of the cause of error in the earlier work.

It should be noted in passing that Rasch's equation is deducible from the Planck equation as the variation in brightness in some one wave-length, and so is the theoretical foundation for the method of heterochromatic photometry proposed some time earlier by Crova.<sup>4</sup> But it has been recognized for a long time that Crova's method is only approximate over more than a small temperature interval, since the effective wave-length varies with the temperature.<sup>5</sup>

Eisler<sup>6</sup> in 1904, assuming Planck's equation for the distribution of energy in the spectrum of a black body and using Langley's original data on visibility computed the relative brightnesses of a black body at different temperatures, obtaining a relation of brightness with temperature from which he computed the exponent  $x$  in the equation

$$\frac{H_1}{H_2} = \left( \frac{T_1}{T_2} \right)^x,$$

where  $H_1$  and  $H_2$  are the relative brightnesses at the two respective temperatures  $T_1$  and  $T_2$  taken close together. The curve of  $x$  as a function of temperature passes fairly closely through the points plotted from the observations of Lummer and Pringsheim.

Various computations<sup>7</sup> after the method of Eisler have been made in recent years, using visibility data obtained in more accurate ways and unquestionably much nearer the truth than the original data of Langley, but these computations are founded on the original experimental brightness values of Lummer and Pringsheim, and the subsequent values of Nernst, and there is reason to believe that these experimental values are subject to some error, and that, moreover, in each case there is some question about the relative computed brightnesses on account either of

<sup>1</sup> Ann. d. Phys., 14, p. 193, 1904.

<sup>2</sup> Thermodynamik technisches Gasreactionen (Munich), 1905.

<sup>3</sup> Phys. Zeit., 6, p. 19, 1905.

<sup>4</sup> Comptes Rendus, 93, p. 512, 1881.

<sup>5</sup> Phys. Rev., 32, p. 320, 1911.

<sup>6</sup> Elek. Zeit., 25, p. 188, 1904.

<sup>7</sup> Elec. World, 57, p. 1565, 1911. Verh. d. Deut. Phys. Gessell., 17, p. 219, 1915.

the temperature scale or of the visibility data assumed. The only recent experimental data available are those of Langmuir,<sup>1</sup> and those of Ives and Kingsbury.<sup>2</sup> The former conducted his measurements on a tungsten filament, and his results are subject to some modification largely on account of the values assumed for the emissive power of tungsten which more precise determinations have shown to be in error. The experimental results of Ives and Kingsbury<sup>3</sup> determined both by the use of a black-body furnace and by the application of the wedge method in measuring black-body brightness at the melting points of platinum and gold respectively, are not very concordant, and are given but little weight by the authors themselves.

It therefore seemed opportune to the authors to undertake a new series of experimental determinations of black-body brightness as a function of temperature, and to compare these experimental results with the relative values computed on the assumption of the most probable visibility curve and of the Planck equation for spectral energy distribution after the manner of Eisler. Within the past few years much progress has been made in the establishment of the high temperature scale, and black-body furnaces may now be operated up to temperatures of at least 2600° K. under reasonably favorable conditions for brightness determinations, and with a considerable degree of confidence in the precision of the temperature measurements. Of course, the temperature scale assumed plays an important rôle. This question will be dealt with in some detail in a subsequent section.

In like manner much progress has recently been made in determining the visibility curve of the average eye, but, in the opinion of the authors, since all the recent determinations had been made by the flicker method, there seemed a necessity for a determination of visibility by the direct comparison method. The findings of the flicker method in the investigations of several observers were not very concordant, and the method itself was open to question. The present investigation was therefore interrupted to make a new determination of the visibility curve by the direct comparison method, as will be discussed in a separate section. Fortunately uncertainties in this function produce only second order errors in the relative brightness curve of a black body, and since the range of temperature that can be employed is not extremely large the errors in the relative computed brightnesses are quite small if the chosen visibility curve even roughly expresses the facts. To what extent these

<sup>1</sup> *PHYS. REV.*, (2), 7, p. 322, 1916.

<sup>2</sup> *PHYS. REV.*, (2), 8, p. 177, 1916.

<sup>3</sup> Since this paper was written, Dr. Ives has published a note in the *Journal of the Franklin Institute* (186, p. 122, 1918), giving data which seem to verify this original value.

considerations are justified will become apparent in the presentation and discussion of the experimental data.

The second part of the paper deals with the evaluation of the mechanical equivalent of light based upon the observed values of black-body brightness and upon the assumption of the constant  $\sigma$  ( $E = \sigma T^4$ ) in the Stefan-Boltzmann law, and of the visibility curve. In this case errors in the visibility curve enter as first order effects and so the precise knowledge of this function becomes of much greater importance.

*Temperature Scale.*—The relation between observed brightness and temperature of a black body obviously depends upon the chosen scale of temperature. It is customary, in establishing a temperature scale, to assume certain fixed points, such as the melting points of gold and palladium, and to determine other points of the scale on the assumption of some radiation law such as the Stefan-Boltzmann law or the Planck equation. The former of these is accepted without reservation, but there is some slight question regarding the latter, though it is generally believed to represent the facts and is assumed as the basis of optical pyrometry. The values of certain of the constants entering into these equations are still, however, the subject of much discussion and investigation. This is particularly true of the constant  $C_2$  in the Planck equation

$$E_\lambda = C_1 \lambda^{-5} \frac{1}{e^{\frac{C_2}{\lambda T}} - 1}.$$

As already stated it is customary to assume certain fixed points, such as the melting-points of gold and palladium as measured with the gas thermometer. The best determinations of these values are those of Day and Sossman,<sup>1</sup> but recent investigations would seem to show that the two melting-point temperatures are not consistent with the most probable value of  $C_2$  in the Planck equation.<sup>2</sup> In order to have a definite temperature scale on which to express measurements of temperature the laboratories of the General Electric Company agreed upon the adoption of the gold point as given by Day and Sossman (1336° K.), and of the value 14350  $\mu$  deg. for the constant  $C_2$ . On this scale the temperature of melting palladium must be changed from 1823° K., the value found by Day and Sossman, to 1828° K. if the earlier measurements of the present authors<sup>3</sup> are correct, since it was found that the value of Day and Sossman was confirmed only upon the assumption of the value of 14460  $\mu$  deg. for  $C_2$ , if the gold point was accepted as 1336° K.

<sup>1</sup> Am. Jour. of Sci., (4), 29, p. 93, 1910.

<sup>2</sup> Gen. Elec. Rev., 20, p. 819, 1917.

<sup>3</sup> Astrophys. Jour., 42, p. 300, 1915.



*Visibility Data.*—As stated in an earlier paragraph, there was much doubt in the minds of the authors regarding the acceptability of the various visibility data available when the present investigation was begun. This doubt arose partly from the inconsistency of the various published results but more largely from an inherent skepticism regarding the applicability of visibility data obtained with the flicker photometer to practical problems in heterochromatic photometry in which the method of direct comparison was employed, as in the present investigation.

It is true that only gross differences in the visibility curve can produce appreciable errors in the computed relative brightnesses of a black body at two temperatures differing by only a few hundred degrees, but uncertainties in the visibility curve produce first-order effects in the evaluation of the mechanical equivalent of light. And when the authors found that even the second-order errors in the computed relative brightnesses of a black body on the assumption of the most recently published visibility data<sup>1</sup> with the flicker photometer, as compared with the experimental values found using the direct-comparison method of photometry, were larger than might readily be justified on the ground of experimental error, it was decided to delay the publication of the present investigation until a new determination of the visibility of radiation, using the direct-comparison method, might be made. The results of this investigation, together with a more complete discussion of the question, have already been published.<sup>2</sup>

The visibility data obtained in that investigation will be used in computing the relative brightnesses of a black body at different temperatures, for comparison with the values experimentally observed, and in evaluating the mechanical equivalent of light. For the extreme red end of the spectrum not included in the investigation referred to, use will be made of the visibility data obtained by two of the present authors in an earlier study<sup>3</sup> of visibility in this region; and for the extreme blue end the data of Hartman<sup>4</sup> obtained also in this laboratory will be employed. The complete visibility curve used is that recommended in appendix II. of the recent paper to which reference has just been made.

*Apparatus and Method.*—The measurements of the brightness of a black body at various temperatures were carried out with two electrically heated black-body furnaces. Most of the measurements were made with a specially designed water-cooled carbon-tube furnace, the remaining ones being made with a platinum-wound porcelain furnace provided with

<sup>1</sup> Bul. Bur. of Stds., 14, p. 167, 1917.

<sup>2</sup> Astrophys. Jour., 48, Sept., 1918.

<sup>3</sup> Ibid., 42, p. 285, 1915.

<sup>4</sup> Ibid., 47, p. 83, 1918.

a Lummer-Kurlbaum black-body tube. The set-up, with the carbon furnace,<sup>1</sup> is shown in Fig. 1, in which *F* is the carbon furnace supplied with a limiting diaphragm. Three different sizes were used with apertures varying from 4 to 5 mm. in diameter. For details of construction and operation of this furnace the reader is referred to a previous paper by the authors on "Color Temperature Scales for Tungsten and Carbon."<sup>2</sup> *P* is a Lummer-Brodhun contrast photometer of special design, in which the contrast is only about 3.5 per cent. on the average and is graduated in either direction from the center—the two features conducing to higher

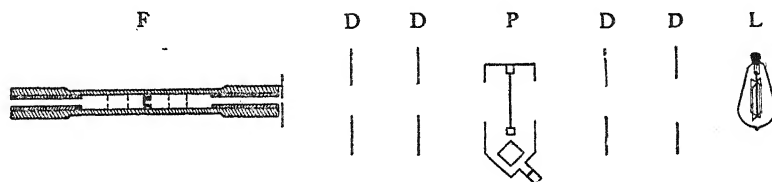


Fig. 1.

Diagrammatic Sketch of Apparatus.

sensibility. *L* is the comparison lamp,—at low temperatures a carbon lamp and at the higher temperatures a vacuum tungsten lamp. A series of black velvet screens *D* with suitable openings shut off stray light from the photometer.

The brightness measurements of the black body were always carried out with the comparison lamp at an approximate color match, except beyond 2400° above which temperature it was considered unsafe to operate the comparison lamp. The calibration of the comparison lamp at the various voltages, involving the difficulties of heterochromatic photometry, was made subsequently. In order to avoid the possible errors arising from individual idiosyncrasies of vision, the candlepower scale chosen was that given for a tungsten lamp by Middlekauff and Skogland.<sup>3</sup> The procedure of calibration consisted in transferring the photometer and comparison lamp to a standard photometer bench, and substituting a tungsten vacuum lamp for the black body. The comparison lamp was then brought successively to the various voltages used against the black body and readings were made against the tungsten lamp. In cases of large color difference blue screens calibrated at the Bureau of Standards were used. The relative candlepowers of the

<sup>1</sup> Not as much weight is attached to the results obtained with the platinum-wound furnace because of the photometric difficulties at the low temperatures and with the necessarily small limiting diaphragm which the dimensions of the furnace demanded.

<sup>2</sup> *PHYS. REV.*, (2), 10, p. 395, 1917.

<sup>3</sup> *Bul. Bur. of Stds.*, 11, p. 483, 1915.

standard tungsten lamp were computed from the data of Middlekauff and Skogland, and hence the candlepowers of the comparison lamp, and so the brightnesses of the black body were obtained on this same scale.

The justification of the Middlekauff and Skogland candlepower scale as representative of the average eye may be arrived at through the inter-comparison of the visibility curves of several members of this group with the average of the 125 observers employed in the investigation of Coblentz on the visibility of radiation. Such a comparison indicates that the mean of the eight observers employed by Middlekauff and Skogland is not markedly different from the mean of the 125 observers used by Coblentz. Other lines of reasoning would indicate that the scale of Middlekauff and Skogland is not far from correct, as representing an average eye, but that if any change were made it would be in the direction of raising the upper region of the candlepower curve slightly with respect to the curve for the lower temperatures. This change would be less than one per cent. and so, in the absence of more definite information, the Middlekauff-Skogland scale has been adopted in the present investigation.

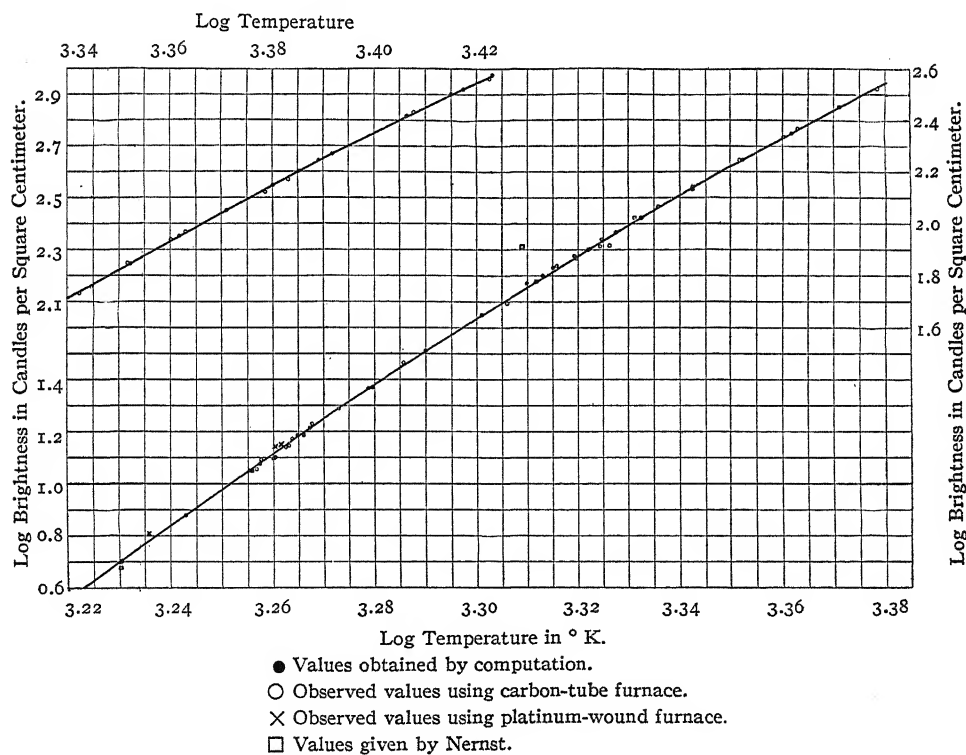


Fig. 2.

Brightness of a Black Body at Various Temperatures.

*Experimental Results.*—The results on black-body brightness expressed in candles per  $\text{cm}^2$  as a function of the temperature in degrees K. are given in a logarithmic form in Fig. 2, and the corresponding numerical results, taken from the curve are given for every  $50^\circ$  in Table I. The circles on the plot are the observed points obtained with the carbon-tube furnace, and the  $\alpha$ 's are the observed points using the platinum-wound

TABLE I.

*Brightness of a Black Body at Various Temperatures.*

Temperature.	Brightness Candles per Square Centimeter.	Temperature.	Brightness Candles per Square Centimeter.
1700° K.	5.0 <sub>8</sub>	2200° K.	137. <sub>6</sub>
1750	7.6 <sub>4</sub>	2250	177.
1800	11.3	2300	226.
1850	16.3	2350	284.
1900	23.1	2400	354.
1950	32.2	2450	438.
2000	44. <sub>3</sub>	2500	537.
2050	60. <sub>0</sub>	2550	651.
2100	80. <sub>1</sub>	2600	785.
2150	105. <sub>7</sub>	2650	939.

furnace. The curve is drawn according to a least-square solution applied to the 46 observed points and assuming the cubic equation,

$$u = a + bt + ct^2 + dt^3,$$

where  $u = \log \text{candles per cm}^2$ ,

$$t = \log T - 3.2,$$

$T = \text{temperature in degrees K.}$

The values of the constants were found to be as follows:

$$a = 0.2626, \quad c = -12.38,$$

$$b = 14.959, \quad d = -1.13.$$

The 46 points are so distributed with respect to the least square curve that 23 points lie on one side and 23 on the other, the maximum deviation of any point, as seen from the curve, being about 8 per cent. The two points furthest from the curve by Chauvenet's criterion could be discarded. Thirty points lie within 2.5 per cent. of the curve, the average deviation of all points being about 2.4 per cent. The upper one of the three points obtained by Lummer and Pringsheim is within about 3.5 per cent of the curve (the other two points are at lower temperatures than any included in our observations) and the later observations of Nernst, reduced to the present temperature scale, are indicated by  $\square$ 's.

The dots are the points computed on the basis of the authors' recently

published data on the visibility of radiation, as discussed above. These computed brightnesses are, of course, only relative, but the value at  $2077^{\circ}$  K., which is in the neighborhood of the color temperature of the 4-wpc carbon standard lamp, was taken from the least square solution, and the other points were plotted in terms of that. It is seen that the computed points lie on the observed curve within an error of less than 1 per cent. except at the extreme ends of the temperature range. This is better than might be expected owing to the inherent difficulties in the present experiment. Points computed from Coblentz's visibility data show a consistent difference from the observed curve, though the difference is nowhere large.

The accuracy indicated in the brightness measurements suggests the advisability of adopting tentatively the value  $70.2$  candles per  $\text{cm}^2$  as the brightness of a black body at the temperature  $2077^{\circ}$  K. and as a value for a primary standard of light. The temperature chosen is approximately the color temperature of the standard 4-wpc carbon lamps.<sup>1</sup>

## PART II. THE MECHANICAL EQUIVALENT OF LIGHT.

This term has come to be used to mean the watts per lumen of the monochromatic light of greatest visibility, corresponding, in the recent experiments of the authors, to a wave-length  $\lambda = 0.556 \mu$ . It is a physical constant of theoretical interest, and also serves as a means of expressing the absolute efficiency of illuminants. It depends on the brightness of a black body, or other source with known emissive power, on the temperature scale chosen, also on the constant  $\sigma$  in the Stefan-Boltzmann law ( $E = \sigma T^4$ ), and finally on the curve of relative visibility of radiation,—i. e., provided it is evaluated in the way to be followed in this paper. It may be determined directly by measuring the radiant flux and the luminous flux from some monochromatic source in a wave-length at or near that of maximum visibility. Determinations have been carried out by this method,<sup>2</sup> but in the opinion of the authors the experimental difficulties are too great to justify much confidence in the results.

In the present investigation the mechanical equivalent  $M$ , following the usual procedure, has been evaluated in the following way. Let  $B_0 \equiv$  normal brightness of the black body at any temperature  $T$  expressed in candles per  $\text{cm}^2$ .

<sup>1</sup> The data given above are somewhat different from those published in the advance note in the Journal of the Franklin Institute (Jour. of Franklin Inst., 181, p. 420, 1916.) These differences are due to the assumption of a different temperature scale and to the addition of several new observed points which produced a small change in the curve, particularly at the higher temperatures.

<sup>2</sup> PHYS. REV., (2), 5, p. 269, 1915.

$E_\lambda$  = the radiant flux per  $\text{cm}^2$  per unit wave-length at the wave-length  $\lambda$ .  
 $V_\lambda$  = the relative visibility of radiation in any wave-length  $\lambda$  in terms of the maximum visibility taken equal to unity.

Then

$$(1) \quad B_0 = \frac{1}{\pi M} \int_0^\infty E_\lambda V_\lambda d\lambda$$

or

$$(2) \quad M = \frac{1}{\pi B_0} \int_0^\infty E_\lambda V_\lambda d\lambda.$$

In the present investigation the visibility function is assumed as extending from  $0.40 \mu$  in the blue to  $0.76 \mu$  in the red, the values assigned in the two extreme ends of the spectrum being taken from the previously published data of two of the present authors<sup>1</sup> in the red, and of Hartman<sup>2</sup> in the blue.<sup>3</sup> Beyond the chosen limits the added luminosity area would be entirely negligible.

The method employed in evaluating the integral in equation (1) was that of summing the product  $E_\lambda V_\lambda \Delta\lambda$  for steps  $\Delta\lambda = 0.01 \mu$ , and correcting for the errors of summation by the one third rule. The value of  $E_\lambda$  and therefore that of  $M$  depends upon the constant  $C_1$  in Planck's equation, or its equivalent  $\sigma$  in the Stefan-Boltzmann equation, since these two constants are definitely related if  $C_2$  is known. The latter is taken as  $14350 \mu \text{ deg.}$ , as discussed in the first part of the paper, and for  $\sigma$  the value  $5.7 \times 10^{-12} \frac{\text{watts}}{\text{cm}^2 \text{ deg}^4}$ , as found by Coblentz, has been assumed as the most probable value. On the basis of these values,  $C_1$  comes out as  $3.72 \times 10^{-12} \text{ watts cm}^2$ .

TABLE II.

*Mechanical Equivalent M for Various Temperatures.*

Temperature.	M in Watts per Lumen.	Temperature.	M in Watts per Lumen.
1700° K.	0.00147 <sub>8</sub>	2200° K.	.00149 <sub>6</sub>
1800	.00149 <sub>1</sub>	2300	.00149 <sub>7</sub>
1900	.00149 <sub>8</sub>	2400	.00149 <sub>7</sub>
2000	.00149 <sub>8</sub>	2500	.00150 <sub>2</sub>
2100	.00149 <sub>7</sub>	2600	.00151 <sub>1</sub>
Average.....			0.001496

<sup>1</sup> *Loc. cit.*

<sup>2</sup> *Loc. cit.*

<sup>3</sup> The complete visibility curve used is given in Appendix II. of the paper by the present authors on "The Visibility of Radiation," *Astrophys. Jour.*, Sept., 1918.

TABLE III.

Investigators.	Recommended Values of the Mechanical Equivalent of Light in Watts per Lumen.	Visibility Data.	Constants $\sigma$ in $\frac{\text{Watts}}{\text{Cm.}^2 \text{ Deg.}^4}$ $C_2$ in $^\circ \text{ Deg.}$	Method.
P. G. Nutting	$0.00120 \pm 0.00005$	Nutting	$\sigma = 5.7 \times 10^{-12}$ (Presumably)	Measurements of total radiant flux and total luminous flux from acetylene. Coblentz' energy curve for acetylene used.
H. E. Ives	$0.00160 \pm 0.00003$	Ives & Kingsbury Curve from equation	$\sigma = 5.7 \times 10^{-12}$	(1) Measurement of total radiant flux and total luminous flux from a monochromatic source. (2) Measurement of radiant flux through luminosity filter and integral luminous flux direct from the source.
	Secondary value $0.00154$	Ives & Kingsbury	$\sigma = 5.7 \times 10^{-12}$ $C_2 = 14350$	(3) Measurement of brightness of a platinum-wedge black body at the melting point of platinum taken as $2037^\circ \text{ K.}$
W. W. Coblentz	$0.00162 \pm 0.00005$  Note: If in method (1) the final data of Hyde, Forsythe and Cady were used, the value derived by this method would be increased by approximately $1\frac{1}{2}$ per cent.	Coblentz and Emerson	$C_2 = 14350$ Melting points as given by Day and Sossman	Several methods using observations by himself and others. (1) Computation using preliminary data on brightness of a black body by Hyde, Forsythe and Cady. (2) Direct measurement on monochromatic radiation, by Coblentz, Ives and Kingsbury. (3) Measurements on incandescent lamps by Coblentz and Emerson.
Hyde, Forsythe and Cady	$0.00150 \pm 0.00005$	Hyde, Forsythe and Cady	$\sigma = 5.7 \times 10^{-12}$ $C_2 = 14350$ Melting point of gold $1336^\circ \text{ K.}$	Direct measurement of brightness of carbon and platinum tube black bodies.

Substituting in equation (5) the brightness of the black body at any temperature, as given in the first part of this paper, a value of the mechanical equivalent  $M$  for any temperature is found. If the observed and computed curves of black-body brightness are the same throughout,

then the value of  $M$  would be found to be the same for all temperatures, as might be expected. The differences between the two curves as presented in the first part of this paper are so small that the values for  $M$  computed for every  $100^\circ$ , as given in Table II., are sensibly constant, except at the extreme ends of the curve, the total range being only 2.2 per cent., and the range from  $1800^\circ$  to  $2400^\circ$  K. being only 0.5 per cent.

The average value from the present experiments is given in Table III., in comparison with the published values of other observers. In each case so far as is possible from the published reports, the method used, the visibility curve employed (if any) and the temperature scale involved are given.

#### SUMMARY.

1. A new set of experimentally determined values of the brightness of a black body from  $1700^\circ$  to  $2600^\circ$  K. are given on the assumption of a temperature scale based upon Planck's equation taking the gold point as  $1336^\circ$  K. and the constant  $C_2$  as  $14350 \mu$  deg.

2. These values, as plotted in a curve obtained by a least-square solution, are compared with the relative computed values on the assumption of the visibility curve recently published by the authors.

3. A value of  $70.2$  candles per  $\text{cm}^2$  as the brightness of a black body at a temperature of  $2077^\circ$  K. (color match with 4-wpc standard carbon lamps) is proposed tentatively as an absolute standard of light.

4. The mechanical equivalent of light for the wave-length of maximum visibility ( $\lambda = 0.566 \mu$ ) is computed to be  $0.00150 \pm 0.00005$  watts per lumen.

#### APPENDIX I.

It is frequently desired to know the rate of change of candlepower or of brightness of a black body at any temperature with a change in temperature or a change in watts. Since the data presented in the body of this paper are somewhat different from other published data it seems advisable to compute these coefficients and at the same time to evaluate the Crova wave-length<sup>1</sup> at different points on the brightness-temperature scale.

Differentiating equation (1) for the brightness of a black body with reference to temperature, the following final equation is obtained:

$$(3) \quad \frac{dB_0}{dT} \frac{T}{B_0} = \frac{C_2}{T} \frac{\int_0^\infty \frac{1}{\lambda} E_\lambda V_\lambda d\lambda}{\int_0^\infty E_\lambda V_\lambda d\lambda}.$$

<sup>1</sup> *Loc. cit.*



If now the summation method employed in the body of the paper in computing the mechanical equivalent of light be used the integral in the denominator is at once obtained; and if the respective terms at any temperature are multiplied by the reciprocals of the wave-length and the summation made, the numerator is also obtained. The ratio of these two integrals at any temperature, multiplied by  $C_2/T$  gives the desired percentage change in brightness corresponding to a change of 1 per cent. in temperature. These results are shown in Fig. 3. Since

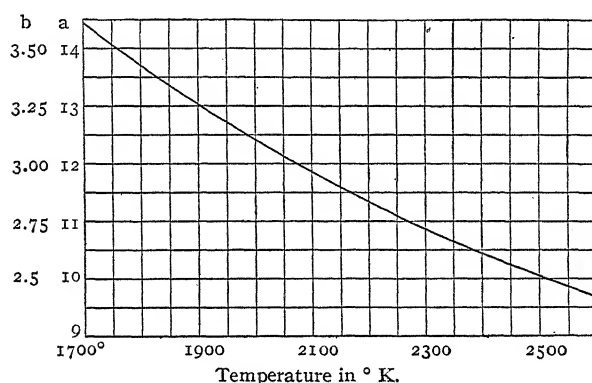


Fig. 3.

Percentage Changes in Brightness of a Black Body at Various Temperatures for (a) one per cent. change in temperature and for (b) one per cent. change in watts.

$E = \sigma T^4$  expresses the relation between temperature and total watts radiated it follows that if the ordinates of the curve are divided by 4 the resultant curve will show the percentage change of brightness corresponding to a change of 1 per cent. in total watts radiated.

Moreover it follows from equation (3) that if  $\lambda_0$  is the wave-length such that at some temperature the ratio of the two integrals in equation (3) is  $1/\lambda_0$ , then at that temperature

$$(4) \quad \frac{dB_0}{dT} \cdot \frac{T}{B_0} = \frac{C_2}{\lambda_0 T},$$

or, the percentage change in brightness corresponding to a change of 1 per cent. in temperature is equal to  $C_2/\lambda_0 T$ .

But it follows immediately from Wien's equation that

$$(5) \quad \frac{dE_{\lambda T}}{dT} \cdot \frac{T}{E_{\lambda T}} = \frac{C_2}{\lambda T},$$

or, the percentage change in watts radiated by a black body at any wave-length  $\lambda$  and at any temperature  $T$ , corresponding to a change of 1 per

cent. in temperature is  $C_2/\lambda T$ . Hence if the particular value  $\lambda_0$  be taken for  $\lambda$ , then from equations (4) and (5) it follows that the percentage change in brightness at any temperature, and the percentage change in watts emitted at that temperature in the wave-length  $\lambda_0$ , corresponding to a change of 1 per cent. in temperature, will be the same. By definition, therefore,  $\lambda_0$  is the Crova wave-length for that temperature, and hence is readily evaluated since the two integrals in equation (3) are now known. The values of the Crova wave-length thus obtained are given in Fig. 4.

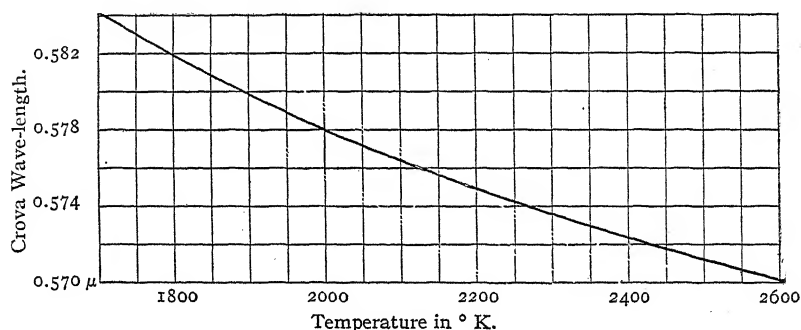


Fig. 4.

Crova Wave-length at Various Temperatures for Black Body Radiation.

It will be seen by comparison with the data published by Kingsbury<sup>1</sup> and by Coblentz<sup>2</sup> that the authors' values for the Crova wave-length are somewhat smaller. This difference is for the most part due to the difference in visibility curves since those obtained by the flicker method show larger values in the longer wave-lengths of the visible spectrum.

NELA RESEARCH LABORATORY,  
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,  
NELA PARK, CLEVELAND, OHIO,  
September, 1918.

<sup>1</sup> PHYS. REV., (2), 7, p. 167, 1916.

<sup>2</sup> Bul. Bur. of Stds., 14, p. 255, 1918.

IONIZATION AND RESONANCE POTENTIALS FOR  
ELECTRONS IN VAPORS OF ARSENIC,  
RUBIDIUM, AND CÆSIUM.

BY PAUL D. FOOTE, O. ROGNLEY AND F. L. MOHLER.

WHEN electrons are accelerated in a metallic vapor, at least two types of inelastic impact between an electron and atom occur. The first of these results in an orbital shift of the electrons bound in the atom and the second in the complete removal of an electron or ionization of the atom. The potential differences through which an electron must fall to obtain sufficient velocity or kinetic energy for these two types of collision are known as the resonance and the ionization potentials for the particular metal in question. The resonance and ionization potentials of a series of metals have been determined in this laboratory.<sup>1</sup> The present paper is a report upon the work with arsenic, rubidium and cæsium.

The arrangement of apparatus for arsenic is shown in Fig. 1. Arsenic was vaporized at the bottom of the pyrex glass tube and the vapor, after passing through the superheated ionization chamber, condensed in the upper half of the tube. The ionization chamber was supported entirely by steel rods running from the top of the tube. The source of electrons was a hot tungsten wire cathode, *A*, of low resistance, coated with lime. Surrounding the cathode was a cylinder of iron net, *B*, and outside and coaxial with this a cylinder of sheet iron, *C*. The apparatus was evacuated by means of a Langmuir and Stimson condensation pump to a gas pressure of about  $5 \cdot 10^{-6}$  cm. Hg as registered by a McLeod gage. Sufficient vapor was obtained by heating the arsenic to about 400 to 500° C.

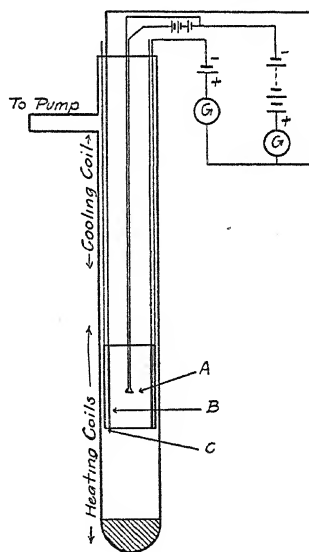


Fig. 1.

Diagram of apparatus used for  
arsenic.

<sup>1</sup> Na, Tate and Foote, *J. Wash. Acad. Sci.*, 7, p. 517, 1917. Cd, Tate and Foote, *Bur. Standards Sci. Paper No. 317*. Na, Cd, K, Zn, Tate and Foote, *Phil. Mag.*, 36, p. 64; 1918. Mg, Tl, Foote and Mohler, *Phil. Mag.* (in press).

A trap containing cadmium-tin alloy chips immersed in liquid air was used between the pumps and the ionization tube in order to preclude the possibility of the presence of mercury in the ionization chamber.

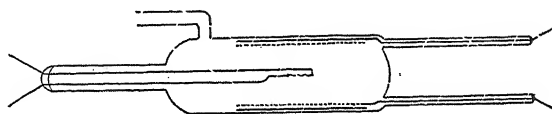


Fig. 2.

Ionization tube used for rubidium and caesium.

The experimental procedure consisted in applying a constant retarding potential of 0.8 to 1.5 volts between the net and the outside cylinder and measuring both the total current from the hot wire, and that portion of the current which reached the outside cylinder against the retarding field,

as functions of a variable accelerating potential applied between the hot wire and net. The apparatus used for rubidium and caesium is illustrated by Fig. 2. This was constructed of soda glass, as it was found that the heated vapors of these metals do not noticeably attack soda glass while the reaction with lead glass is very pronounced. The net and cylinder were of nickel, and a platinum hot wire coated with lime served as the cathode. A retarding potential of about 0.5 volt was applied between the net and cylinder. Metallic rubidium from Kahlbaum was used and several grams of caesium were obtained by heating metallic calcium and Kahlbaum caesium chloride to about 400° C. at a very low pressure. The metal was distilled into a small test-

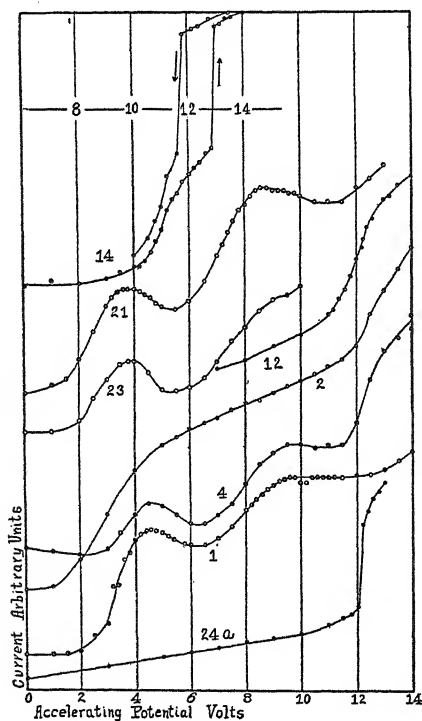


Fig. 3.

Total and partial current curves for arsenic.

tube, covered with a drop of benzol, and inserted into the pump intake, after which the apparatus was sealed and evacuated. The metal was then redistilled into the ionization chamber. During the observations the

entire ionization tube was uniformly heated to about 150° C., and the metal gradually distilled out, condensing in the exhaust tube. When necessary the metal was redistilled back into the ionization tube.

TABLE I.

*Ionization and Resonance Potentials for Electrons in Arsenic Vapor. [Curves 1 to 12; Cadmium-tin alloy trap and liquid-air trap between pump and ionization tube for several hours before and during the time of observation. Curves 21 to 24a: All apparatus reconstructed and both traps used continuously from the time the apparatus was assembled.]*

Curve.	Resonance.			Applied Potential at Ionization.	Initial Potential.	Ionization Potential. <sup>1</sup>	Resonance Potential. <sup>2</sup>	
	a.	b.	c.				b-a.	c-b.
1	3.6	8.4	...	...	..	...	4.8	..
2	..	..	...	10.4	1.0	11.4	..	..
3	..	..	...	10.5	1.0	11.5	..	..
4	3.8	8.4	12.6	...	..	...	4.6	4.2
4a	3.8	8.4	12.8	...	..	...	4.6	4.4
5	..	..	...	10.6	1.0	11.6	..	..
10	3.6	8.8	...	...	..	...	5.2	..
12	..	..	...	10.6	1.1	11.7	..	..
21	2.8	7.7	...	...	..	...	4.9	..
22	..	..	...	10.0	1.8	11.8	..	..
23	3.0	7.8	...	...	..	...	4.8	..
23a	3.2	..	...	...	..	...	..	..
24	..	..	...	9.6	1.6	11.2	..	..
24a	..	..	...	10.0	1.6	11.6	..	..

Fig. 3 represents some of the data obtained for arsenic. A complete analysis of these and other curves is given in Table I. The partial-current curves show pronounced changes in curvature at successive points *a*, *b*, *c*, which differ in potential by a constant amount. This constant difference gives the resonance potential directly, eliminating any consideration of initial velocities. The total-current curves show a rapid increase in slope at a point for which the potential, corrected for the initial velocity (potential) of the electrons as obtained from the partial-current curves, gives the ionization potential. The initial potential is the difference between the resonance potential and the applied potential corresponding to the first point of resonance. A detailed discussion of this method for determining the ionization potential has been given elsewhere.<sup>3</sup> Curve 14, Fig. 3, is of interest as an example of a discontinuous total-current curve. On increasing the accelerating potential the total current increases gradually up to about 13 volts. At this point the current increases discontinuously and upon decreasing the

<sup>1</sup> Mean ionization potential = 11.5 volts.

<sup>2</sup> Mean resonance potential = 4.7 volts.

<sup>3</sup> Tate and Foote, J. Wash. Acad. Sci., 7, p. 520, 1917.

accelerating potential the current decreases discontinuously at about 12 volts. Similar curves have been obtained for mercury, sodium and rubidium. Curves of this type may be secured when the vapor density and the electronic current are high. In the present instance a measurement of the initial velocity of the electrons was not made but, in general, these discontinuous curves afford as accurate a determination of the ionization potential as those obtained with a smaller vapor density and initial velocity. The discontinuous change occurs considerably above the ionization potential and is probably due to a potential redistribution in the ionization chamber. As the potential is gradually increased the most intense part of the discharge, at least for rubidium, sodium, etc., for which metals the glow discharge is visible, moves suddenly at the point of discontinuity from the hot wire to the net.

The final results with arsenic gave 4.7 volts for the resonance potential and 11.5 volts for the ionization potential. These values are quite similar to the corresponding voltages for mercury, namely, 4.9 volts and 10.3 volts, and for this reason it was at first suspected that mercury from the pumps and gage was present in the ionization chamber. Although both liquid air and cadmium-tin alloy were used as a mercury trap in the first series of experiments here described (curves 1 to 12) the apparatus was entirely reconstructed with every precaution again being taken to avoid contamination by mercury. The results of this second series of experiments (curves 21 to 24a) confirmed those of the first series. In work at this laboratory with eleven different metals no trace of mercury has ever been observed, when the metals are volatilizing rapidly, in the characteristic ionization and resonance curves, although heretofore we have never used either cadmium-tin or liquid-air traps. In the case of the gases and non-metallic vapors, however, considerable precaution must be taken to avoid the presence of mercury. It was therefore thought possible that contamination by mercury might still be present and show up prominently in the somewhat non-metallic arsenic vapor. The following experiment was accordingly performed.

It was noted that, contrary to our experience with other metals, no glow discharge could be detected in arsenic at the ionization point. A special discharge tube was constructed for observing the arsenic arc more carefully. A steel disk anode and a tungsten hot-wire cathode were mounted about 1 cm. apart in a glass tube lined with iron net in electrical contact with the hot wire. The tube was mounted in a special electric furnace having a small side opening through which the arc could be viewed. Above the distillation point of arsenic the electron current indicated marked ionization as before at the ionization potential

of 11.5 volts. With a current of 30 milliamperes no arc or glow discharge whatever was visible. A few drops of mercury were then placed in the ionization tube. A visible arc was obtained with a current of 0.02 milliampere, and with a current of 20 milliamperes the arc was very intense. Thus the fact that a current of 0.02 milliampere is sufficient to show the presence of mercury as an impurity of arsenic, and that a current of 30 milliamperes in supposedly pure arsenic vapor does not show a trace of any glow whatever, must be taken as a proof of the purity of the vapor. It is of further interest to note that when arsenic is heated with a small amount of mercury the ionization due to the mercury will appear  $200^{\circ}$  C. below the point where arsenic vaporizes sufficiently to show ionization, and after prolonged heating at  $500^{\circ}$  C. the mercury is driven out of the tube so that finally the ionization of arsenic alone is observed and the glow discharge disappears.

It has been shown in the earlier papers referred to that for many metals the ionization and resonance potentials are determined by the quantum relation  $h\nu = eV$  where  $h$  is Planck's constant of action,  $e$  the electronic charge,  $V$  the resonance or ionization potential, and  $\nu$  a frequency determinable from spectroscopic data. The value assigned to  $\nu$  appears to depend upon the column of the periodic table to which the particular metal under consideration belongs. Thus, for Group I. of the periodic table,  $\nu$  is the frequency of the first line of the principal doublet series for resonance potential and the convergence of this series for ionization potential. For Group II. the resonance potential is determined by the first line of the combination series of single lines  $1.5 S - mp_2$ , and the ionization potential is determined by the convergence of this series. Arsenic is the first metal in Group V. so far investigated. The quantum relation may be written  $\lambda = 12334/V$  on substituting the proper values for  $h$  and  $e$ , where  $\lambda$  is the wave-length in Ångstrom units corresponding to the potential  $V$  expressed in volts. Thus, the resonance potential of 4.7 volts and the ionization potential of 11.5 volts correspond to the wave-lengths  $\lambda = 2620$  Å. and  $\lambda = 1070$  Å., respectively. No series having its first line at  $\lambda = 2620$  Å. or its convergence at  $\lambda = 1070$  Å. is known for arsenic. In fact, all the series relations so far observed in arsenic have been quite unsatisfactory, which is true also of the other metals in this family. Accordingly, the present work very likely indicates an undiscovered series in arsenic having its first term (or terms if a doublet series) near  $\lambda = 2620$  Å. and converging near  $\lambda = 1070$  Å., but as to the character of the series nothing can be inferred.

Fig. 4 represents the partial-current curves and Fig. 5 the total-current curves obtained with rubidium. As seen from Table II., the mean

value of the resonance potential is 1.6 volts. This value and the first point of resonance collision,  $a$ , permit the determination of the initial velocities or equivalent potentials shown in column 9. The total-current curves 1, 6, 11, 12 were taken under the same experimental conditions as the partial-current curves 3, 4, 5, so that the initial potential,  $V_0$ , to be added to the observed applied potential at ionization to give the ionization potential, is 1.2 volts. Curve 12 corresponds to curve 7 with an initial potential of 1.4 volts, the cathode being operated

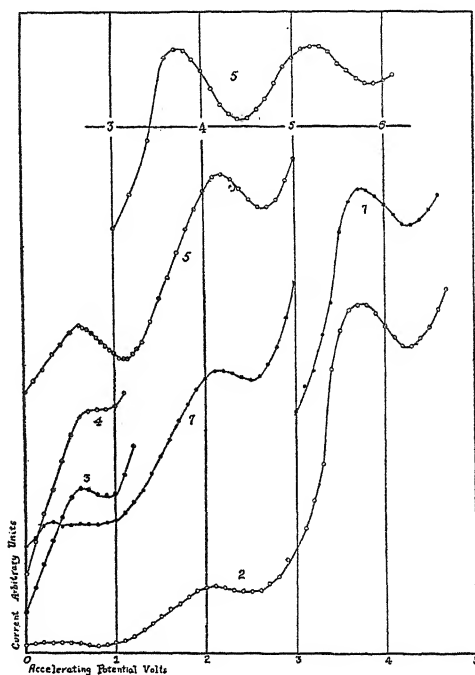


Fig. 4.

Partial-current curves for rubidium.

at a higher temperature when these two curves were obtained. In the case of rubidium and caesium the total current did not show saturation below the ionization potential, so that the point at which ionization begins is not indicated by as sharp a break as would otherwise occur. The following method for determining the break point proved quite satisfactory.

Langmuir<sup>1</sup> has derived an equation showing the relation between the thermionic current and voltage for the case of a hot wire surrounded by a cylindrical anode. If account is taken of the initial velocity of the

<sup>1</sup> *Phys. Rev.*, 2, p. 457, 1913.



electrons, an additional constant appears in the equation derived by Langmuir. The kinetic energy of an electron at a distance  $r$  from the hot wire is:

$$(1) \quad \frac{1}{2}mv^2 = (V + V_0)e,$$

where  $V_0$  is the initial potential and  $V = f(r)$  is the applied accelerating potential. Defining  $i$  as the thermionic current per unit length of the hot wire, one obtains directly from Poisson's equation:

$$(2) \quad r \frac{d^2 V}{dr^2} + \frac{dV}{dr} = i \sqrt{\frac{2m}{e(V + V_0)}},$$

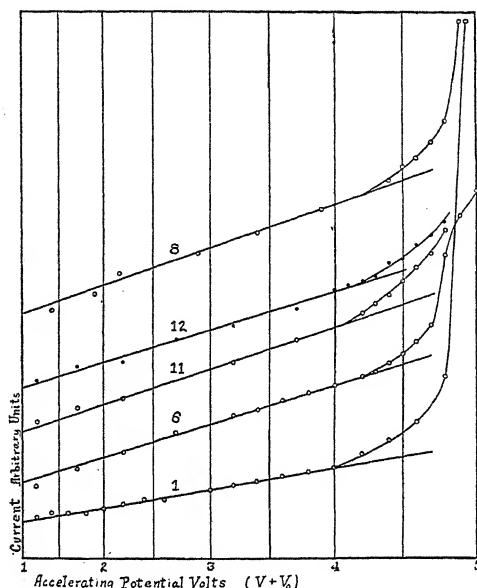


Fig. 5.

Total-current curves for rubidium.

the solution of which may be represented by the following relation where  $A$  is a constant for any definite configuration of the ionization chamber and temperature of the hot wire.

$$(3) \quad i = A(V + V_0)^{3/2}.$$

Hence, the relation between  $i$  and  $(V + V_0)^{3/2}$  is linear for the purely thermionic current below its saturation value. When ionization begins the current increases more rapidly so that the ionization potential is given by the value of  $(V + V_0)$  at the point where the curve  $i$  plotted against  $(V + V_0)^{3/2}$  deviates from a straight line. Fig. 5 represents the

total-current curves for rubidium plotted in this manner. The values of the applied potential were corrected by adding the initial potential  $V_0$  as given in Table II. The linear relation is well satisfied except possibly at the lowest potentials. The agreement is even better than

TABLE II.

*Ionization and Resonance Potentials for Electrons in Rubidium Vapor.*

Curve.	Resonance.				Resonance Potential. <sup>1</sup>			Initial Potential.
	a.	b.	c.	d.	b-a.	c-b.	d-c.	
2.....	..	1.65	3.35	...	...	1.70	...	} 1.2
3.....	0.4	...	...	...	...	...	...	
4.....	.4	...	...	...	...	...	...	
5.....	.4	1.85	3.45	4.90	1.45	1.60	1.45	
7.....	.15	1.70	3.50	...	1.55	1.80	...	1.4
					Mean = 1.6 volts.			
	$V_0$ .	$V+V_0$ . <sup>2</sup>						
1.....	1.2	4.0						
6.....	1.2	4.2						
8.....	1.4	4.2						
11.....	1.2	4.1						
12.....	1.2	4.2						

might be expected from a more careful consideration of the applicability of equation (3). Strictly considered, this equation should not hold exactly for two reasons: (1) Because of inelastic collision of some of the

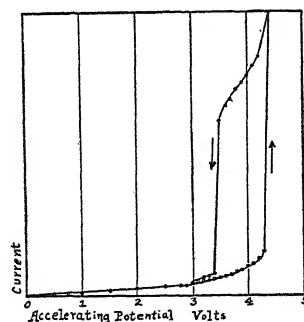


Fig. 6.  
"Discontinuous" total-current  
curve for rubidium.

electrons at the resonance potential, and (2) because of the potential drop along the hot wire. Thus, equation (3) states that when  $V = 0$ ,  $i = A V_0^{3/2}$ , but, although  $V_0$  is comparable with  $V$ , the observed current is practically zero when the applied field is zero. The reason for this is that the electrons with no accelerating field between the hot wire and net fall along the hot wire from points of lower to higher potential or along one of the leading-in wires. A measurable current is observed only when the applied potential is comparable with

the electrostatic field along the wire. This fact accounts in part for the high initial velocities observed. Also it may be noted that our method of selecting the point of resonance gives greatest weight to the electrons

<sup>1</sup> Resonance potential = 1.6 volts.

<sup>2</sup> Ionization potential = 4.1 volts.

of highest speed. A point of further interest is that practically the same values for the initial velocity are obtained when the partial-current curves are plotted directly and when they are plotted on the rectified coördinate system used for the total-current curves.

The mean value of the ionization potential of rubidium obtained from the curves shown in Fig. 5 is 4.1 volts. Fig. 6 illustrates a type of discontinuous total-current curve obtained when the density of the rubidium vapor is high.

TABLE III.

*Ionization and Resonance Potentials for Electrons in Cesium Vapor.*

Curve.	Resonance.		Resonance Potential. <sup>1</sup>	Initial Potential.
	<i>a.</i>	<i>b.</i>		
3	0.60	2.10	1.50	0.88
5	.55	2.00	1.45	.93
7	.50	2.00	1.50	.98
8	.55	2.00	1.45	.93
10	.55	2.00	1.45	.93
11	.55	2.00	1.45	.93
12	.40	1.90	1.50	1.08
14	.40	1.90	1.50	1.08
Mean .....			1.48	1.0
	$(V + 1.0)^2$ .	Ionization Potential. <sup>2</sup>		
1	8.5	4.2		
4	7.7	3.9		
6	7.6	3.9		
9	7.1	3.7		
13	7.4	3.8		

Fig. 7 shows the partial-current curves and Fig. 8 the total-current curves for cesium vapor. The method of determining the ionization and resonance potentials was that described above for rubidium. The final values, summarized in Table III., are 1.48 volts for the resonance potential and 3.9 volts for the ionization potential.

From the behavior of sodium and potassium one would expect the resonance and ionization potentials of rubidium and cesium to be determined by the relation  $V = 12334/\lambda$  where  $\lambda$  is the first line and convergence wave-length, respectively, of the principal series of doublets for each metal. The following table, to which for completeness have been added the data on the other alkali metals, confirms this fact.

<sup>1</sup> Resonance potential = 1.48 volts.

<sup>2</sup> Ionization potential = 3.9 volts.

TABLE IV.

*Resonance and Ionization Potentials for the Alkali Metals.*

Metal.	$\lambda$ (Å). ( $\nu=1.55-2\mu$ ).	Resonance Potential Volts.		$\lambda$ (Å). ( $\nu=1.5 s$ ).	Ionization Potential Volts.	
		Computed.	Observed.		Computed.	Observed.
Lithium....	6707.85 <sup>1</sup>	1.839	....	2299.67 <sup>2</sup>	5.363	....
Sodium....	5895.94 <sup>1</sup>	2.092	} 2.12 <sup>3</sup>	2412.63 <sup>4</sup>	5.112	5.13 <sup>5</sup>
	5889.97 <sup>1</sup>	2.094				
Potassium..	7699.01 <sup>1</sup>	1.602	} 1.55 <sup>3</sup>	2856.69 <sup>5</sup>	4.318	4.1 <sup>3</sup>
	7664.94 <sup>1</sup>	1.609				
Rubidium..	7947.64 <sup>1</sup>	1.552	} 1.6 <sup>6</sup>	2968.40 <sup>5</sup>	4.155	4.1 <sup>6</sup>
	7800.29 <sup>1</sup>	1.581				
Cæsium....	8943.46 <sup>1</sup>	1.379	} 1.48 <sup>6</sup>	3184.28 <sup>5</sup>	3.873	3.9 <sup>6</sup>
	8521.12 <sup>1</sup>	1.447				

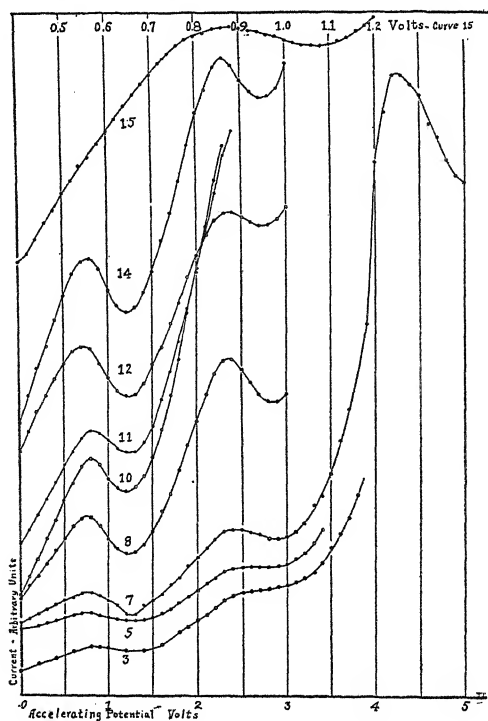


Fig. 7.

Partial-current curves for caesium.

<sup>1</sup> Meggers, Bur. Standards Sci. Paper No. 312.<sup>2</sup> Dunz Tübingen Thesis, 1911.<sup>3</sup> Tate and Foote, Phil. Mag., 36, p. 75, 1918.<sup>4</sup> Wood and Fortrat, Astrophys. Jour., 43, p. 73, 1916.<sup>5</sup> Watts, Phil. Mag., 29, p. 775, 1915.<sup>6</sup> Foote, Rognley, and Mohler, this paper.

The first pair of doublets in the principal series of both rubidium and caesium is rather widely separated. In the case of caesium the separation is equivalent to about 0.07 volt. If resonance collision occurs at both 1.379 and 1.447 volts, corresponding to each member of the caesium doublet, the partial-current curves should show two sets of maxima differing by 0.07 volt at the first point of resonance, by 0.14 volt at the second point of resonance, etc. None of the curves of Fig. 7 show double

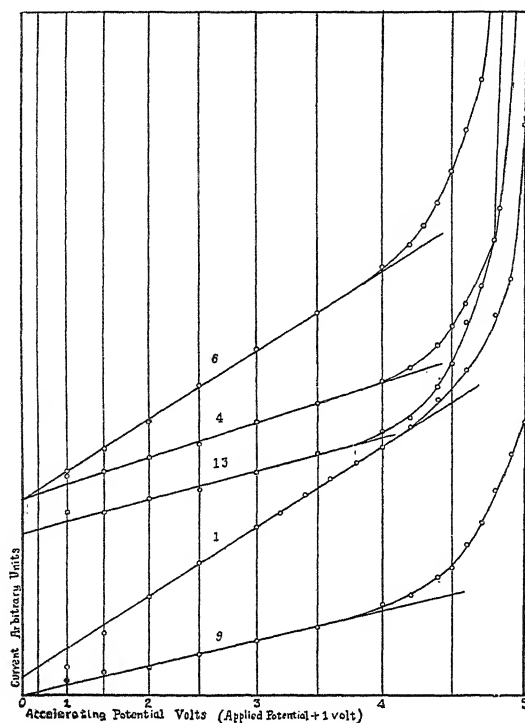


Fig. 8.

Total-current curves for caesium.

maxima and for curve 15, observations were made at every 0.02 volt. It is possible that the presence of double maxima could not be detected on account of the initial velocity distribution from the hot wire. However, it has been found by Foote and Mohler<sup>1</sup> that in the case of thallium the shorter wave-length member of the first doublet of the principal series appears to determine the resonance potential. Hence it seems quite likely that the proper theoretical values for the resonance potentials of rubidium and caesium are 1.581 and 1.447 volts, respectively. This possibility suggests an analogy to the general behavior of the *K*

<sup>1</sup> *Phys. Rev.*, II, p. 487, 1918.

series in the X-ray spectrum of metals as has been pointed out by Foote and Mohler.<sup>1</sup> The present work indicates that the "single line" spectra of rubidium and caesium are the doublets  $\lambda = 7949$  and  $7803 \text{ \AA.}$  and  $\lambda = 8950$  and  $8528 \text{ \AA.}$ , respectively. At the ionization potential the principal series, the first subordinate series, and the second subordinate series appear. The following doublets were observed visually in the case of caesium when the accelerating potential was equal to the ionization potential: Principal series,  $m = 3$ ; first subordinate series,  $m = 5, 6, 7, 8, 9, 10$ ; second subordinate series,  $m = 4.5$ .

#### SUMMARY.

Two types of inelastic collision between the electron and atom occur in vapors of arsenic, rubidium and caesium. The potential differences through which an electron must fall to attain sufficient energy or velocity to produce these collisions are known as the resonance and ionization potentials for the particular metal in question. For rubidium and caesium the resonance potential is determined by the quantum relation  $h\nu = eV$  where  $\nu$  denotes the frequency corresponding to the first doublet of the principal series. A similar relation holds for the ionization potential where  $\nu$  denotes the convergence frequency of the principal series. Experimentally determined values of the resonance and ionization potentials for rubidium were 1.6 volts and 4.1 volts, respectively; for caesium, 1.48 volts and 3.9 volts, respectively; for arsenic, 4.7 volts and 11.5 volts, respectively. By applying the quantum relation to the results obtained with arsenic we are able to predict the presence of an undiscovered spectral series in arsenic having its first term (or terms if a doublet series) near  $\lambda = 2620 \text{ \AA.}$  and converging near  $\lambda = 1070 \text{ \AA.}$  A Wehnelt tube discharge in arsenic possesses no luminosity indicating that the important series lines lie in the ultra-violet. At the ionization potential of rubidium and caesium the principal, first, and second subordinate series appear. No ionization whatever could be detected at the resonance potentials of rubidium, caesium, and arsenic.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.,  
August 16, 1918.

<sup>1</sup> J. Wash. Acad. Sci., Sept. 19, 1918.

ENERGY OF THE CHARACTERISTIC X-RAY EMISSION  
FROM MOLYBDENUM AND PALLADIUM AS A  
FUNCTION OF APPLIED VOLTAGE.

BY BENJAMIN ALLEN WOOTEN.

INTRODUCTION.

SINCE the introduction of the X-ray spectrometer by W. H. and W. L. Bragg as an instrument for studying the various properties of X-radiation, much work has been done. Most of this work concerns itself with the general radiation and there is relatively little published matter on the intensity of the characteristic rays. In a recently published article<sup>1</sup> D. L. Webster finds that the rays of the *K* series of rhodium are produced only at voltages above that required by the Planck quantum hypothesis to excite general radiation of a frequency slightly greater than that of the  $\gamma$  line. He finds that above that potential all *K* lines increase in the same ratio for a given increase in potential. The rates of increase of intensity are continuously accelerated.

The following work was undertaken with a view of studying the relation between the energy of the lines of the *K* series of molybdenum and palladium and the voltage producing them. The spectrometer that was used was similar in general to the one used by Dr. Clayton T. Ulrey<sup>2</sup> in his work on the general radiation. Voltages were measured by means of an electrostatic voltmeter and energy was supplied for the X-ray tubes by a Kenotron rectifying set. During the progress of the work points concerning the probability of new lines, absorption coefficients, etc., presented themselves. They were dealt with and will be discussed.

DESCRIPTION OF APPARATUS.

Current at 125 volts and 500 cycles was taken from a motor generator set and was stepped up by a three-kilowatt oil-immersed transformer. The high voltage alternating current from the transformer was rectified by means of two General Electric Kenotrons and an oil-immersed glass-plate condenser. The energy supplied to the X-ray tube was taken from this condenser. Under normal conditions the voltage generated by the

<sup>1</sup> D. L. Webster, *PHYS. REV.*, June, 1916, pp. 599-613.

<sup>2</sup> C. T. Ulrey, *PHYS. REV.*, May, 1918.

set was constant to one and one half per cent. The voltage was measured by an electrostatic voltmeter that was built in the laboratory. It consisted of a pair of fixed spheres and a pair of movable ones hung on a steel strip suspension and pivoted at the bottom on a needle. The four spheres were charged to the same potential, and the force of repulsion between the fixed and movable ones caused the instrument to show a deflection.

The voltmeter was calibrated by means of a standard spark gap. This gap consisted of 12.5 cm. balls. The relation between sparking distance and voltage was obtained from published tables. The calibration curve of the voltmeter was very regular, almost approaching a straight line at high voltages. The voltmeter kept its adjustment perfectly.

Currents to the X-ray tube were measured on a Weston direct current milliammeter in all the work except that on absorption. In that part of the work the currents were read on a Leeds and Northrup portable galvanometer properly shunted. An accident to the milliammeter made this change necessary.

The X-ray tube that was used in the work on molybdenum was one that was kindly supplied by Dr. W. D. Coolidge, of the General Electric Company. The Palladium tube was built up in the laboratory. The cathode was of the Coolidge type and it was also furnished by Dr. Coolidge. The anode consisted of a cylinder of iron with a small piece of palladium set in it. The target face was inclined at an angle of 26 degrees with the plane perpendicular to the elements of the cylinder. The tube was evacuated by means of a diffusion pump of the Langmuir type, the mercury trap of which was cooled with a mixture of carbon dioxide snow and ether. This pump was in series with a Gaede rotary pump and a piston pump. The tube was baked at 300° Centigrade while the preliminary evacuation was in progress and the target was heated to dull redness from time to time by discharges at 25 kilo-volts. It was found necessary to keep the pump in operation whenever observations were being made.

The spectrometer was peculiar only in the fact that the ionization chamber was very long. Its length was 128 cm., and complete absorption through a long range of wave-lengths was possible with its use. This chamber contained the saturated vapor of methyl iodide. Methyl iodide was used because it does not show abnormal absorption at any point throughout the range of wave-lengths investigated.<sup>1</sup> The chamber and electrometer were freed of water vapor by the use of phosphorous anhydride. The gold-leaf electrometer was attached directly to the end

<sup>1</sup> "X-rays," G. W. C. Kaye, p. 147.



of the ionization chamber, and the image of the leaf was projected on a ground glass screen with a scale marked on it. This screen was securely fastened to the ionization chamber. The leaf was charged negatively to 400 volts by means of a small dynamotor. The X-ray tube was surrounded by a heavy lead screen and a beam was taken off at about 26 degrees from the face of the target, this beam coming as nearly as possible from the center of the focal spot. After passing through an aperture in the screen this beam was passed through an adjustable slit, the jaws of which were of lead, and then reflected by the crystal. All circuits to the tubes, voltmeter, etc., carrying high potential current returned through ground. The Coolidge cathodes were heated by current from a small storage battery.

The crystal used was of Montana calcite and was kindly lent to us by Professor A. J. Moses, of the Department of Mineralogy. Photographs of the back slit taken with the reflected beam showed remarkably clear and perfect lines, and it was not possible to detect any irregularities in them.

The natural leak of the electrometer when properly connected to the ionization chamber was such as to cause the leaf to move across the interval of the scale that was used in taking readings in about 625 seconds. This of course varied, but it was usually more than 600 seconds. The readings of X-ray intensity were taken in the usual way by allowing the reflected beam to pass into the chamber through an adjustable slit, and noting with a stop watch the time taken for the leaf to pass across a fixed part of the scale. The same scale interval was used in all the readings taken. The time taken for a reading varied from less than one second to 550 seconds. The projected image of the leaf on the ground glass screen was clear and sharp and readings could be repeated with an accuracy of one per cent.

#### RESULTS.

The apparatus was adjusted and lined up so that a beam of X-rays from the back slit would pass over the center of the spectrometer circle. The position of the ionization chamber, when this beam was entering its slit was read on the circle and taken as its position for direct rays.

The crystal was then put in place and adjusted so that its reflecting face was as nearly as possible parallel to the beam of direct rays and over the axis of the spectrometer. The crystal was moved in steps of  $\theta$  and the ionization chamber in steps of  $2\theta$  until strong reflection was observed. The crystal was then held stationary while the chamber was moved in small steps across the reflected beam. From the readings of intensity

obtained for each step and the position of the chamber at each step a curve showing a sharp peak at the position of the chamber for maximum intensity was plotted. The position of the chamber corresponding to this peak was taken as its position when the reflected beam was passing correctly through its slit.

In obtaining the values of intensity from the readings of the rate of fall of the leaf, this rate was first corrected for natural leak and then reduced to what it would have been for a current of one milli-ampere in the tube.

As the nature of the work undertaken was not such as to demand extreme accuracy in wave-length measurements, the angular readings were not corrected for depth of penetration of the beam into the crystal. The natural leak was always taken when the apparatus was in operation with the chamber well out of line. The walls of the ionization chamber were thick enough to shield out all the secondary radiation at the highest voltages that were used in this work, and no correction for it was necessary.

#### VOLTAGE INTENSITY CURVES.

After properly lining up the apparatus data was taken on intensity for different wave-lengths at constant voltages up to and including 50 KV. The readings were taken at intervals of three minutes of arc and the intensities were corrected for natural leak and reduced to values for unit current. Curves were taken for different voltages and their general shapes noted for both molybdenum and palladium. Fig. 1 is such a curve taken for palladium and it is shown as being typical of the ones obtained. These curves were taken in order to locate the more prominent lines in the *K* series and to show the general nature of the spectra of the metals.

In order to study the intensity of the characteristic radiation it was necessary to subtract the intensity of the general radiation of the same wave-length as a given line from the intensity of the line as measured by the spectrometer. With a number of the constant voltage, intensity and wave-length curves at hand, a point in the general radiation on each side of a line ( $\alpha$  or  $\beta$ ) was taken. These points were chosen so that a straight line through them would pass through the line at the same place that a continuation of the curve of general radiation would cut it. It was found necessary to take a great many curves in order to choose these points correctly.

Having chosen these points for the  $\alpha$  and  $\beta$  line of each of the metals, data was taken in the following manner: The spectrometer was set for

the wave-length of one of the chosen points, care being taken to have the setting correct. In order to make certain of this, the crystal was set on the proper reading as determined from the wave-length intensity curves and the ionization chamber moved past the reflected beam in steps as described above. The results of reflected intensity were plotted against angular displacement of the chamber and the chamber was set at the reading corresponding to the peak of this intensity curve. In all settings

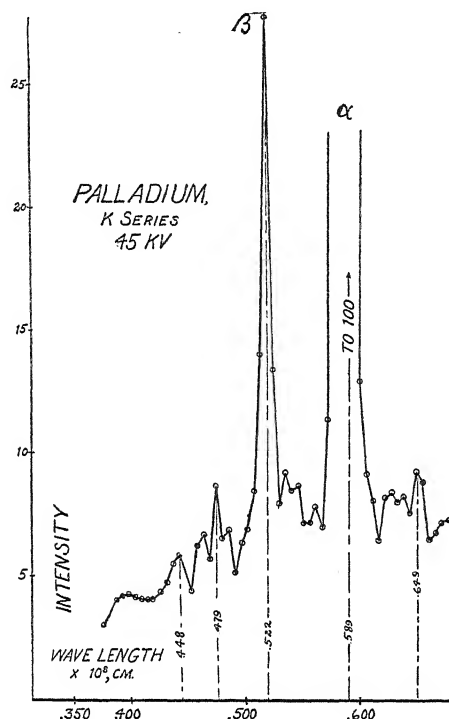


Fig. 1.

made in this part of the work this procedure was gone through. The voltage on the tube was increased in steps to 50 KV., and with each change of potential the current through the tube was adjusted so as to keep the energy supplied to the tube constant. Determinations were made of intensity of reflection for each value of voltage and all intensities were reduced to unit current conditions. The spectrometer was then set for the wave-length of the other point and similar readings taken for it, giving the same values to potential and current that were used before, for each step. Then the spectrometer was set for the line under consideration and values for its intensity for the voltages and currents that

were used in the determinations on the two wave-lengths in the general radiation were obtained. This data was determined for each line that was studied.

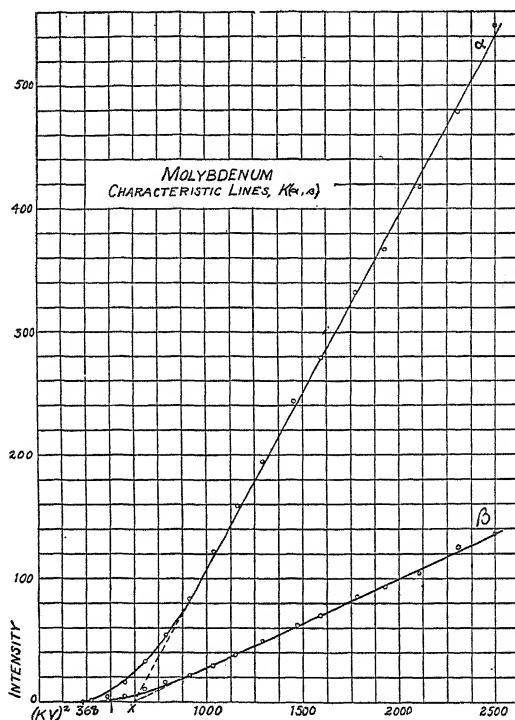


Fig. 2.

The three intensities for any potential, determined in this way give three points which determine a triangle with its acute angle very small. The distance between the vertex of this acute angle and the base line, as measured along the bisector of the angle, gives the intensity of the radiation of the line corrected for the general radiation of the same wave-length. The shape of the triangle thus formed is such as to make the length of this bisector sensibly equal to the length of either one of its sides except for low voltages.

It was found that the  $\alpha$  and  $\beta$  lines for each metal appeared together at a certain minimum voltage. In the case of molybdenum this voltage was found to be 19.2 KV. and for palladium it was 24 KV. Below these potentials the wave-length curves were smooth and they intersected the wave-length axis at points corresponding to longer wave-lengths than the wave-length of the  $\beta$  line. It was shown by Duane and Hunt<sup>1</sup> that

<sup>1</sup> Duane and Hunt, American Physical Society Proc., April, 1915, PHYS. REV., August, 1915.

this point of intersection of the curve with the wave-length axis represented the minimum wave-length that the potential at which the curve was taken could give by the quantum relation. This was found to be true for these curves.

Curves were plotted between the kilo-volts squared to the tube and the intensity obtained as above of the lines for each metal. These curves are shown in Figs. 2 and 3 for molybdenum and palladium re-

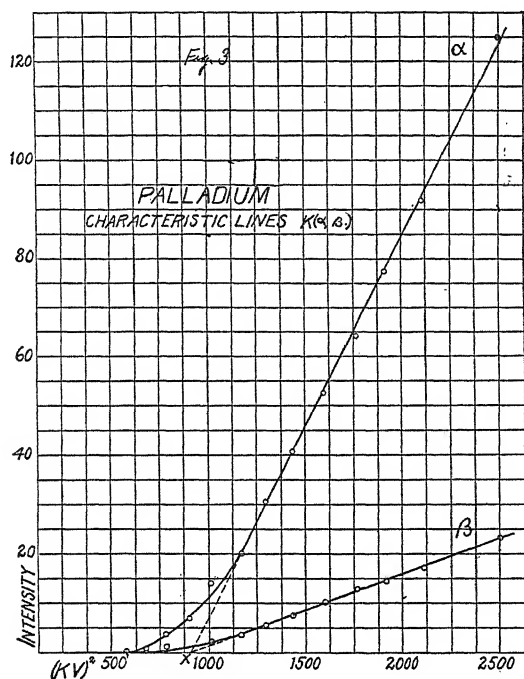


Fig. 3.

spectively. It will be noticed that the two sets of curves, each set consisting of a curve for the  $\alpha$  and one for the  $\beta$  line of a metal, are of the same general shape and character. In each set the  $\alpha$  and  $\beta$  curves rise from a common point on the voltage squared axis corresponding to the square of the lowest potential that will excite the line or the series, and quickly become straight lines. The intensity of the radiation in the  $\alpha$  and  $\beta$  lines for the two metals thus appears to vary as the square of the voltage applied for voltages not too near the critical voltage; and the ratio of the intensity of the  $\alpha$  to the intensity of the  $\beta$  line in each case quickly becomes constant for voltage variations.

Below is a table (Table I.) of the results of various measurements

taken in determining these curves, together with other data concerning them.

TABLE I.

	Molybdenum.	Palladium.
Atomic number.....	42	46
Wave-length of $\alpha$ line (cm. $\times 10^8$ ).....	.716	.589
Wave-length of $\beta$ line (cm. $\times 10^8$ ).....	.641	.522
Minimum potential to produce K series, in kilo-volts	19.2	24.0
Point of intersection of straight line parts of curves with (voltage) <sup>2</sup> axis, (kilo-volts) <sup>2</sup> .....	625	930
Ratio of intensity of $\alpha$ to $\beta$ line as measured from data.....	3.99	5.28
Ratio of intensity of $\alpha$ to $\beta$ line corrected for absorp- tion in glass.....	5.55	6.25

## ABSORPTION COEFFICIENTS.

A theoretical explanation of these voltage intensity curves has been developed by Professor Bergen Davis<sup>1</sup> so it was decided to correct the curves for absorption in the glass of the X-ray tubes and to compare the experimentally determined results with the curves obtained from his equation. An investigation of the absorption coefficients of the  $\alpha$  and  $\beta$  lines of molybdenum and palladium in glass and in molybdenum and palladium respectively was undertaken.

The radiation was taken from a Coolidge tube with a tungsten anti-cathode and the wave-lengths desired were reflected from the same crystal that was used in the other part of the work. The voltage on the tube was made greater than the critical voltage of either metal for the  $K$  series and it was held constant. The collimator slit and the slit on the ionization chamber were narrowed to .2 mm. and the crystal and chamber were set at the proper angles to give the desired wave-length. An aluminum screen .3 mm. thick was placed between the crystal and the slit on the ionization chamber so that the reflected beam would pass through it. It was found necessary to use this screen in order to get consistent results. It absorbed practically all of the secondary radiation from the crystal, and thus allowed the reflected ray to pass into the ionization chamber as an homogeneous beam. This filtering out of the X-rays of long wave-length that came from the crystal was important.

The reflected beam after having passed through the aluminum filter was passed directly into the ionization chamber and its intensity determined. This value was used as the initial intensity in the computations. Then thin sheets of glass were placed in front of the slit in such a way that

<sup>1</sup> Bergen Davis, PHYS. REV., June, 1918.

the reflected beam passed through them before it entered the ionization chamber. Determinations of intensity were made with the rays passing through various thicknesses of glass and the absorption coefficient calculated by the well-known exponential equation. The results obtained with different thicknesses of glass were averaged for the true absorption coefficient.

Measurements were made in this way for the  $\alpha$  and  $\beta$  lines of both molybdenum and palladium in glass. In addition absorption coefficients of the  $\alpha$  and  $\beta$  line of molybdenum in molybdenum were determined, as were the absorption coefficients of the  $\alpha$  and  $\beta$  lines of palladium in palladium.

The aluminum filter was kept in place in all this work and conditions were the same as regards voltage, current, etc., in all the determinations that were made. In one set of observations the potential across the tube was 35 KV. and in another it was held at 45 KV. The results shown in this paper were those obtained at 45 KV. The sheets of glass that were used were .022 cm. in thickness and the sheets of palladium were .0065 cm. thick. The molybdenum was .016 cm. thick and only two thicknesses were used. The absorption coefficients obtained are given in Table II.

TABLE II.

Line.	Wave-length Cm. $\times 10^8$ .	Absorption Coeff. in Glass.	Absorption Coeff. Mo.	Absorption Coeff. Pd.
Mo. $\alpha$ .....	.716	14.50	210	
Mo. $\beta$ .....	.641	11.65	170	
Pd. $\alpha$ .....	.589	7.79		138
Pd. $\beta$ .....	.522	5.37		100

## WAVE-LENGTH INTENSITY CURVES.

During the early part of the work a great many curves were obtained between wave-length and intensity of radiation for molybdenum. The readings were taken for intervals of three minutes of arc on the spectrometer circle and in no case was the  $\alpha$  line divided. The  $\alpha$  line as obtained is wider than the  $\beta$ , and this widening at the base suggests that the  $\alpha$  line consists of two lines. It has been shown by Professor Duane<sup>1</sup> that the  $\alpha$  line of rhodium is a doublet consisting of an  $\alpha_1$  and an  $\alpha_2$  line. In this work no distinction is made between the  $\alpha_1$  and the  $\alpha_2$  lines and the radiation designated here as that of the  $\alpha$  line for either metal consists of the combined intensities of the two lines of the  $\alpha$  doublet.

In order to correct the intensities of the lines that were studied ( $\alpha$  and

<sup>1</sup> Not yet published.

$\beta$  lines) for absorption in the glass of the X-ray tubes, the thickness of the glass was determined. A tube was placed under a microscope and the microscope was focused on an object on the inner surface of the glass and then on an object on the outer surface. From these readings and the index of refraction of the glass the thickness of the glass was determined. Determinations were made of the thickness of both tubes used and the results were:

Thickness of glass of molybdenum tube.....1125 cm.

Thickness of glass of palladium tube.....0706 cm.

Using these thicknesses and the absorption coefficients in Table II., the intensities within the tubes of the lines studied were found to be

Mo $\alpha$  .....5.11 times the observed intensity.

Mo $\beta$  .....3.67 " " " "

Pd $\alpha$  .....1.73 " " " "

Pd $\beta$  .....1.46 " " " "

The curves between intensity of radiation and voltage squared (Figs. 2 and 3) show that for the straight line parts of the curves the ratio of the intensity of the  $\alpha$  radiation to that of the  $\beta$  radiation is, in the case of molybdenum, 3.99; and in the case of palladium 5.28. When the intensities of the lines are corrected for absorption in the glass these ratios are changed for molybdenum to 5.55 and for palladium to 6.25. The curves in Figs. 2 and 3 are the experimentally determined curves and they are not corrected for this absorption.

#### THE RATIO OF THE ENERGY IN THE $\alpha$ AND $\beta$ RADIATION AS EMITTED FROM THE ATOM.

The corrected values give the intensities of the  $\alpha$  and  $\beta$  radiation inside the tube before coming out through the glass. *A still more important matter is the ratio of the intensities of these frequencies actually emitted by the atom.* The theory developed by Professor Bergen Davis<sup>1</sup> makes the calculation of this ratio possible. The equation resulting from a mathematical treatment of this theory is

$$I_{\alpha} = E_{\alpha} (h\nu_{\alpha}) \frac{BN}{b} \left[ \frac{b}{cu} \left( 1 - e^{-\frac{cu}{b}(V^2 - V_0^2)} \right) - 2V_0(V - V_0) + \frac{2}{3} \left( \frac{cu}{b} \right) V_0 \{ 2V^3 - V_0(V^2 - V_0^2) \} \right]$$

<sup>1</sup> Loc. cit.



$$- \frac{V_0}{15} \left( \frac{cu}{b} \right)^2 \{ 8(V^5 - V^4 V_0) - 4V^2 V_0 (V^2 - V_0^2) - 3V_0 (V^2 - V_0^2)^2 \} ]$$

when expanded to four terms.

$V_0$  = minimum voltage required to produce the characteristic radiation.

$u$  = the absorption coefficient of a line in the metal that produces it.

$c$  = the ratio of the path of a ray through the metal to the depth of penetration of the electron producing it.

$b$  = a constant corresponding to  $a$  in Whiddington's law.<sup>1</sup> This constant is expressed for use here in terms of potential.

$V$  = any voltage applied to the tube (Kilo-volts).

$I$  = intensity of the line (as written above it means the intensity of the  $\alpha$  line).

$N$  = the number of electrons striking the target per second.

$B$  = the number of impacts of an electron with the atomic nuclei per unit distance.

$(E_a h n_a)$  is proportional to the energy emitted by one atom.

It will be noticed that the constants in the equation, with the exception of  $E_a h n_a$  and  $u$  have the same values for all the lines from a particular metal. The absorption coefficients were determined experimentally for the  $\alpha$  and  $\beta$  lines of both palladium and molybdenum and their values are given in Table II. The constants  $E_a$  and  $E_\beta$  are the fractions of the effective impacts that result in producing the corresponding radiations.

The combined constants  $E_a h n_a$  and  $E_\beta h n_\beta$  are proportional to the energies emitted as radiation of each frequency from one atom. The ratio  $E_a h n_a / E_\beta h n_\beta$  is the ratio of the energy of the  $\alpha$  frequency to that of the  $\beta$  frequency emitted by a single atom.

The experimentally determined values of  $I$  for the  $\alpha$  and  $\beta$  lines of molybdenum and palladium, after having been corrected for absorption in the glass of the tubes, were plotted. Values of  $I$  for different voltages were computed for the  $\alpha$  and  $\beta$  lines of both metals and plotted to the same scale. From these computations and from the experimental results the data in Table III. were obtained.

TABLE III.

Metal.	Observed $I_a/I_\beta$ .	Inside Tube $I_a/I_\beta$ .	Ratio of Intensities Emitted by Atom $E_a h n_a / E_\beta h n_\beta$ .
Mo.....	3.99	5.55	5.70
Pd.....	5.28	6.25	7.18

<sup>1</sup> Proc. Roy. Soc., Vol. 86, 1912.

Professor Bergen Davis's equation can be written:

$$I_a = (E_a h n_a) f_a(V)$$

and therefore

$$\frac{I_a}{I_\beta} = \frac{E_a h n_a f_a(V)}{E_\beta h n_\beta f_\beta(V)}.$$

The computations made in determining the points for the curves give the ratio  $f_a(V)/f_\beta(V)$  so that ratio  $E_a h n_a/E_\beta h n_\beta$  can be determined. This ratio, the ratio of the energy emitted by an atom as  $\alpha$  radiation to that emitted as  $\beta$  radiation, determined in this way is contained with other data pertaining to the comparison of the curves in Table III.

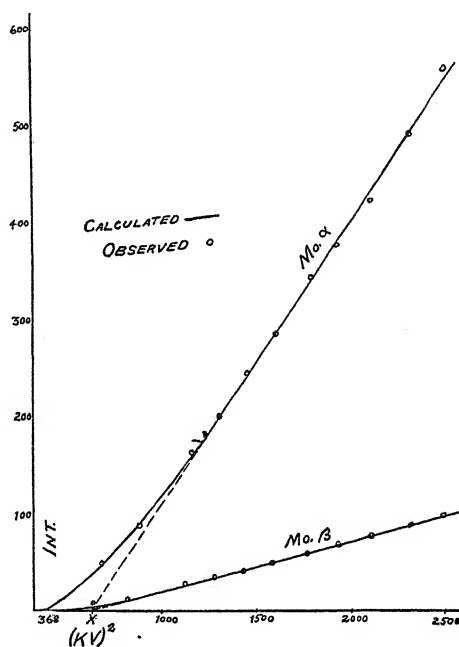


Fig. 4.

The curves given by Professor Bergen Davis's equation plotted to the same scale with the experimentally determined points corrected for absorption, are shown in Figs. 4 and 5. The very close agreement as to shape between the calculated curves and the experimental ones indicates that the assumptions underlying the theory are sound.

In these figures the calculated curves are given in full lines, and the experimentally determined points are marked with circles.

The above comparison is only true in so far as one may consider the reflecting power of the crystal the same for two frequencies not differing

more than the  $\alpha$  and  $\beta$  frequencies differ. If the reflecting power of a crystal does depend to a marked degree on the wave-length of the radiation it will be necessary to correct the above ratios for such variation in reflectivity when the same shall be experimentally determined.

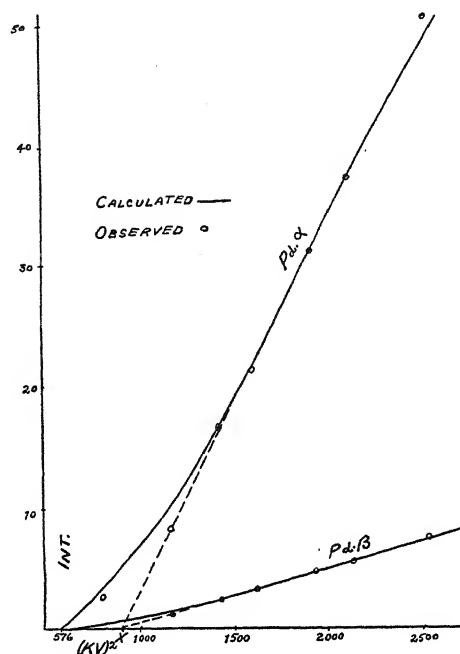


Fig. 5.

## DISCUSSION OF RESULTS.

In all the work that was done it was assumed that the crystal that was used reflected all wave-lengths with equal efficiency. W. H. Bragg<sup>1</sup> showed that the reflecting power of a crystal for a given wave-length varied with the angle of reflection. He varied the angle of reflection of the beam by taking the different orders of reflection. His results show that the reflecting power of a crystal varies inversely as the square of the order of reflection. This is the only experimental work that has been published as far as the writer knows, and the lack of published matter dealing experimentally with the subject of reflecting power in any single order of different wave-lengths has made the above assumption necessary.

Reference to Figs. 2 and 3 shows that the intensity of the radiation for the  $\alpha$  and  $\beta$  lines of the metals used varies directly as the square of the voltage for voltages considerably higher than the critical voltage. In a

<sup>1</sup> W. H. Bragg, Philosophical Magazine, 27, 1914, 881.

paper recently published by Webster and Clark<sup>1</sup> it was shown that the lines of the *K* series of rhodium vary approximately as the  $3/2$  power of the difference between the potential applied to the tube and the critical potential of the series. This relation was tried with the data obtained in these experiments but it was found that the curves obtained did not fit the points as well as the ones in Figs. 2 and 3 fit them.

In these curves it will be seen that the linear parts of the curves when produced back cut the voltage squared axis at the same point. In each set of curves this point is marked *X*. It was found on computation that the following equation is true.

$$\frac{N_{\text{Pd}}^2}{N_{\text{Mo}}^2} = \frac{V_{0\text{Pd}}}{V_{0\text{Mo}}} = \frac{\sqrt{X_{\text{Pd}}}}{\sqrt{X_{\text{Mo}}}},$$

where

$N_{\text{Pd}}$  = atomic number of palladium = 46.

$N_{\text{Mo}}$  = atomic number of molybdenum = 42.

$V_{0\text{Mo}}$  = critical voltage for *K* series of molybdenum = 19.2 Kv.

$V_{0\text{Pd}}$  = critical voltage for *K* series of palladium = 24.0 Kv.

$X_{\text{Mo}}$  = point on axis where straight lines intersect = 625.

$X_{\text{Pd}}$  = point on axis where straight lines intersect = 930.

The square root of the value in Kv. squared represented by the point *X* then, bears the same relation to the metal producing the characteristic radiation that the critical voltage does, *i. e.*, it is proportional to the square of its atomic number.

The method that was used in determining the absorption coefficients is the same as that of the Braggs<sup>2</sup> except for the aluminum filter which was placed at the crystal to absorb its secondary radiation. They find the mass absorption coefficient for the  $\alpha$  line of palladium in palladium to be 11.4. The coefficient given in Table I., when divided by the density of palladium gives 12.1. Their work shows that absorption coefficients in a given substance vary as the  $5/2$  power of the wave-length of the radiation. A. W. Hull and Marion Rice<sup>3</sup> find that absorption coefficients when corrected for scattering vary as the cube of the wave-length, and C. D. Miller<sup>4</sup> finds that they vary for long wave-lengths as the 2.77 power of the wave-length. Miller uses a beam that has passed through a great thickness of absorbing material and in that way absorbs out all but the

<sup>1</sup> D. L. Webster and E. Clark, Nat. Acad. of Science Proc., 3, pp. 181-185, March, 1917.

<sup>2</sup> W. H. and W. L. Bragg, X-rays and Crystal Structure, pp. 175-206.

<sup>3</sup> A. W. Hull and Marion Rice, Proc. Amer. Phys. Soc., Phys. Rev., Sept., 1916, pp. 326-328.

<sup>4</sup> C. D. Miller, Phys. Rev., Oct., 1916, pp. 329-343.

shortest wave-length. The absorption coefficient is determined for this end beam.

The results obtained here, while uncorrected for scattering in the thin sheets of absorbing material that were used, follow the cube of the wave-length law more consistently than they do either of the others. Their agreement with this law is fairly good. The coefficients for the molybdenum  $\alpha$  and  $\beta$  radiation in molybdenum are not as trustworthy as the others, because it was not possible to use more than two thicknesses of molybdenum in the screen.

On substituting the observed values of the critical voltage in Planck's equation,

$$Ve = hn$$

in the case of molybdenum, the value of  $n$  is such as to correspond to a wave-length of  $.643 \times 10^{-8}$  cm. The wave-length of the  $\beta$  line as measured is  $.641 \times 10^{-8}$  cm. The measured voltage required to excite the  $K$  series characteristic radiation is therefore almost exactly the same as that given by the Planck relation for the  $\beta$  line of the  $K$  series of this metal.

For palladium the calculated result, using the observed critical voltage, is a slightly shorter wave-length than that of the  $\beta$  line of the  $K$  series of palladium as measured. The measured wave-length of the  $\beta$  line is  $.552 \times 10^{-8}$  cm. while the calculated wave-length for the critical voltage is  $.516 \times 10^{-8}$  cm.

In a paper published by Uhler<sup>1</sup> it is shown that while Moseley's law does not completely represent the X-ray series over the entire range of wave-lengths, it represents the series for wave-lengths corresponding to the atomic numbers of molybdenum and palladium very closely. Placing  $\tau_1 = 2$  in Bohr's equation to determine the wave-length of the  $\alpha$  line, the value obtained is, for molybdenum  $.713 \times 10^{-8}$  cm., and for palladium  $.581 \times 10^{-8}$  cm. The measured wave-length for the molybdenum  $\alpha$  radiation is  $.716 \times 10^{-8}$  cm. and for palladium  $\alpha$  radiation it is  $.589 \times 10^{-8}$  cm. In each case the agreement between calculated and observed wave-length is as close as Moseley's law agrees with the observed wave-lengths according to Uhler.

#### SUMMARY.

The  $K$  series characteristic X-radiation of molybdenum and palladium has been studied.

It was found that in each metal the  $\alpha$  and  $\beta$  radiation appeared at a certain voltage. For molybdenum this voltage was found to be 19.2 Kv. and for palladium it was 24 Kv. For molybdenum this measured

<sup>1</sup> H. S. Uhler, PHYS. REV., April, 1917, pp. 325-335.

critical voltage is almost exactly that required by the Planck hypothesis to produce the  $\beta$  line. For palladium the measured critical voltage is that required by the same hypothesis to produce a wave-length slightly shorter than the  $\beta$  line.

Curves were plotted between the voltage squared and the intensities of the  $\alpha$  and  $\beta$  radiation for both metals, and these curves show that the intensities of these radiations varied directly as the square of the voltage for voltages not too near the critical voltage. The ratio of the intensity of the  $\alpha$  line to that of the  $\beta$  line was found to become constant as the voltage was increased, for each metal.

The straight line parts of the curves, if produced back cut the voltage squared axis in the same point, and this point was found to bear the same relation to the atomic number of the metal of the anti-cathode that the point at which the curves themselves cut the axis bears, that is, the square root of the coördinate represents a voltage proportional to the square of the atomic number of the radiator.

Absorption coefficients for wave-lengths of the  $\alpha$  and  $\beta$  lines of molybdenum and palladium in glass and in molybdenum and palladium respectively were determined, and they were found to vary approximately as the cube of the wave-length of the radiation. The observed intensities of the  $\alpha$  and  $\beta$  lines of each metal were corrected for absorption in the glass of the X-ray tube.

By application of the theory of Professor Bergen Davis it was possible to calculate the ratio of the intensity of the  $\alpha$  radiation to that of the  $\beta$  radiation as given by a single atom.

The curves obtained by plotting the theoretical equation were found to agree excellently with the experimentally determined curves.

#### CONCLUSION.

In conclusion the writer wishes to thank Professor Bergen Davis, who suggested the research, for the active help and constant advice that he has furnished throughout the work. He is deeply indebted to Columbia University for the grant of the Tyndall Fellowship for the prosecution of the research.

He is also grateful to Dr. W. D. Coolidge for supplying the molybdenum tube, and to Dr. C. S. Brainin for the loan of the sheet palladium that was used.

PHENIX PHYSICAL LABORATORY,  
COLUMBIA UNIVERSITY,  
May 1, 1918.

NEW BOOKS.

*The Science and Practice of Photography.* By JOHN R. ROEBUCK. New York, D. Appleton and Co., 1913. Pp. xiv + 298.

This book was written for class room use in a course given by the author in the Department of Physics at the University of Wisconsin. The work consists of two parts; the first is a general treatise on photography, while the second part, about one fifth of the book, is a manual of experiments to be performed by the student. The book is planned for students who have had elementary courses in physics and chemistry; no mathematics beyond elementary algebra is needed.

While the book was written primarily as a text for students, many who are interested in photography will find it very instructive reading, especially those who are interested in a more scientific treatment than is usually given in books on this subject. Many references to recent researches are freely given and the results are well summarized. Most of this material, because it was so scattered through the literature of photography, was usually not available to the amateur. This book should find many eager readers.

O. M. S.

*Radio Communication, Theory and Methods.* By JOHN MILLS. New York, McGraw-Hill Book Co., 1917. Pp. xi + 205.

This book is based on a course of lectures given soon after the United States entered the war, to employees of the Western Electric Company who were being trained for radio work. Since then it has been adopted as a text by the War Department for those men who have had some training in electrical engineering.

Methods used in radio work have been improved so rapidly that the older treatises do not cover modern methods. This book contains much that cannot be found elsewhere except by going through a great many original papers. For example the explanation of the use of the vacuum tube both as a detector and as an amplifier of alternating currents without distortion, is not contained in any of the standard treatises.

The author tries to adapt the book to those who have not had calculus by using the vector operator  $j$  and the differential operator  $p$  and using only algebraic methods. However, short-cut mathematical processes are of doubtful value to the beginner and certainly one who has had calculus and knows something about the vector methods of solving differential equations as used by engineers will more quickly grasp the processes used. The author has pur-

posely left some of the most important chapters, on the vacuum tube, on detection, on undamped waves, and on radio telegraphy and telephony almost entirely free from mathematical methods.

The author has aimed throughout to "present fundamental principles and methods rather than detailed instruction as to apparatus and its operation." And this aim is fulfilled excellently. In an appendix there is a chapter on transmission over wire circuits and some graded problems for the use of the student.

This interesting book on one of the most interesting branches of electricity will certainly be welcomed by those physicists and and engineers who have not had the time or opportunity to keep in touch with the principles involved in the modern methods.

O. M. S.



## THE PHYSICAL REVIEW.

### WITH FURTHER REFERENCE TO THE FORCE BETWEEN TWO REVOLVING ELECTRONS AT A GREAT DISTANCE.

BY ALBERT C. CREHORE.

IN a paper<sup>1</sup> published in June, 1917, reasons were given for the conclusion that the current form of electromagnetic equations requires modification in a fundamental manner. Briefly, it appears from a consideration of the mechanical force between two revolving electrons in small circular orbits, the centers being a great distance apart, that the force between two bodies of gross matter at a great distance apart must be immensely greater than the known force between them, namely the gravitational force, assuming, of course, that there was no error in the somewhat involved processes of applying the electromagnetic equations to the case.

In a recent paper<sup>2</sup> G. A. Schott has examined in much detail the investigation in the paper above referred to, and says in his opening paragraph "hence (Crehore) concludes, quite legitimately if his result be correct, that the fundamental equations of the accepted electron theory require substantial modification" and "It is obviously imperative that Crehore's result be either verified or disproved." In the second paragraph he says: "The following investigation is based on Crehore's equations for the electric part of the mechanical force,"<sup>3</sup> "which have been verified, except some obvious misprints,<sup>4</sup> *e. g.*,  $a_2$  for  $a_1$  in the last term of (49)." Above in the same paragraph Schott remarks: "Thus it is desirable to take the magnetic effect into account *ab initio*, although it will be found to be inappreciable on the average for an amorphous medium but not necessarily so for a crystal."

<sup>1</sup> A. C. Crehore, *PHYS. REV.*, Sec. Ser., Vol. IX., p. 445, June, 1917.

<sup>2</sup> G. A. Schott, *PHYS. REV.*, Sec. Ser., Vol. XII., p. 23, July, 1918.

<sup>3</sup> *Loc. cit.*, pp. 453, 454—this referring to the June, 1917, paper above mentioned.

<sup>4</sup> It is hoped that Dr. Schott will publish all of the errors in the expression for the instantaneous force that he alludes to, in order that this important equation may be available in its complete form, having been verified.

In reply to this last criticism it should be stated that the complete instantaneous force, including the magnetic component, had been obtained and averaged in a similar manner to the electric component prior to the time of publishing the June, 1917, paper, but it was not published partly because of its length, and partly because each magnetic term in the final average was affected by a common factor in powers of  $\beta$  higher than the square. So long as terms were being obtained from the electric component having the greater factor  $\beta^2$ , they were large compared with any terms obtained from the magnetic component. The quantity,  $\beta_1\beta_2^3$ , is a common factor of the equation (48), p. 36 of the Schott paper,<sup>1</sup> which is his final expression for the time average of the magnetic component. The importance of the equation for the instantaneous force with no terms whatever omitted is so great that the equation for the magnetic component, supplementing the electric already published in equations (48), (49) and (50), June, 1917,<sup>2</sup> is given below in an appendix. In the case of coaxial rings of electrons, where the instantaneous and the average forces are identical, this equation has a wider application than to possible long range gravitational forces. As evidence that the complete force had been obtained, equation (92), p. 469, of the June, 1917, paper may be cited. This gives the complete force, including all terms from both the electric and the magnetic parts exerted upon any selected electron in a ring of electrons due to any other electron in the same ring.

The present paper is offered as a preliminary reply to the Schott paper referred to. It is submitted before there has been opportunity to make a careful study of the suggested corrections as to averaging, which at best is a laborious process, and is submitted principally because the deductions that Schott draws from his own formula fall short of the mark. In his concluding remarks, p. 37,<sup>3</sup> Schott states that "from an experimental point of view, however, the deviation is not of much moment, for it is easily seen by expanding the function of  $\beta_2$  in (50) that the first power which occurs is the fourth, so that, if  $\beta_2$  be as large as .01, the relative deviation is less than the hundred millionth. For this reason I shall not pursue the matter further here." This statement refers to the previous sentence, in which it is pointed out that the average force of the second electron acting upon the first differs from that of the first acting upon the second, the law of action and reaction not holding true, as Schott states, except to a first approximation.

The only interpretation that I can give to these remarks is that Schott

<sup>1</sup> *Loc. cit.*

<sup>2</sup> *Loc. cit.*

<sup>3</sup> *Loc. cit.*

is making a comparison between the magnitude of the terms containing  $\beta$ , which vary with the value of  $\beta$ , and the constant term, namely the electrostatic force, which does not vary with any change of speed. (See his equation for the average force reproduced in (1) below.) It is manifestly improper to make such a comparison when applying these results to the *atoms* of matter, as was done in the June, 1917, paper referred to. The obtaining of the force between two revolving *electrons* is but one step in the process of finding the force between two *atoms*, supposing these atoms to consist of rings of electrons revolving around a positive nucleus located at the center of the orbits, and having a charge equal to the sum of the charges on the electrons but of opposite sign, as in a neutral atom.

To place the equation under discussion before us, Schott's result, his equation (50) p. 37,<sup>1</sup> is reproduced here, after making an obvious correction in the denominator of the last term under the logarithm, making  $\beta^2$  read  $\beta_2$ , namely

$$(\mathbf{F} \cdot \mathbf{r})/r = -\frac{e_1 e_2}{r^2} \left\{ 1 + \beta_2^2 \left( \frac{1}{1 - \beta_2^2} - \frac{1}{2\beta_2} \log \frac{1 + \beta_2}{1 - \beta_2} \right) \right\}. \quad (1)$$

Adopting Schott's suggestion, and expanding this in powers of  $\beta_2$ , we find

$$\frac{1}{2\beta_2} \log \frac{1 + \beta_2}{1 - \beta_2} = 1 + \frac{1}{3}\beta_2^2 + \frac{1}{5}\beta_2^4 + \frac{1}{7}\beta_2^6 + \dots \quad (2)$$

and

$$\frac{1}{1 - \beta_2^2} = 1 + \beta_2^2 + \beta_2^4 + \beta_2^6 + \dots, \quad (3)$$

and, finally, the complete force equivalent to (1) is

$$(\mathbf{F} \cdot \mathbf{r})/r = -\frac{e_1 e_2}{r^2} \left( 1 + \frac{2}{3}\beta_2^4 + \frac{4}{5}\beta_2^6 + \frac{6}{7}\beta_2^8 + \dots \right). \quad (4)$$

When the effect of the positive nuclei are taken into the account in summing up the forces between the various component parts of two *atoms*, all the electrostatic terms cancel out, and require no further consideration, the whole force at great distances between such atoms being due solely to the terms involving the speed of the revolving electrons, the  $\beta$ -terms in (1) or (4) above.

Referring again to the paper of June, 1917, top of page 456, it is stated that the electrostatic terms were omitted in taking the average force for the reason that they contribute nothing to the inverse square of the distance terms under discussion, when the effect of the positive charge of the atoms is included so as to get the force between atoms instead of

<sup>1</sup> *Loc. cit.*

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Lwing Christian College,  
Allahabad.

electrons only. Schott, however, has carried these electrostatic terms through the averaging process from the beginning, and they give rise to the constant term,  $-e_1e_2r^{-2}$ , in (1) above, which should now be omitted in making any deductions about atoms or gross matter. Making this omission, and also making a sufficiently close approximation to the value of the force by omitting all powers of  $\beta$  higher than the fourth, we simplify Schott's result, as applied to a pair of electrons in neutral atoms, from (4) to be

$$(\mathbf{F} \cdot \mathbf{r})/r = -\frac{2}{3}e^2\beta_2^4r^{-2}. \quad (5)$$

Comparing this with the result obtained in June, 1917, the difference is principally in the sign and in the substitution of  $\beta_2^4$  for  $\beta_2^2$ . As far as the law of action and reaction is concerned, the difference is greater with  $\beta_2^4$  than with  $\beta_2^2$ . If we substitute  $\beta_1$  for  $\beta_2$ , and take the ratio of the forces, it is evidently equal to  $(\beta_1/\beta_2)^4$ . If  $\beta_1$  is nearly twice as large as  $\beta_2$ , the ratio is nearly 16 to 1, and the forces of Action and Reaction are not only not equal and opposite to a first approximation, as Schott states, but they are not even of the same order of magnitude. The ratio between them is greater than it would be if  $\beta^2$  appeared in the formula instead of  $\beta^4$ .

An important criticism of the formula, however, is connected with the magnitude of the force between two pieces of matter at great distances. If we had  $\beta^2$  instead of  $\beta^4$ , as formerly, the magnitude of the force at great distances exceeds the known force of gravitation in an immense ratio, about  $10^{31}$ . The change from  $\beta^2$  to  $\beta^4$ , assuming  $\beta^2$  to be of the order of  $10^{-4}$ , only reduces this ratio from  $10^{31}$  to about  $10^{27}$ , the two figures being so nearly alike that there is little choice between them. Whether the formula calls for a repulsive force or an attraction makes no difference in this argument as to the magnitude. We know that no such force exists, and the argument for a revision of the fundamental electromagnetic equations remains valid, taking Schott's results as to averaging to be correct.

This matter of the magnitude of the force may, perhaps, best be shown by a special example. In order not to raise the question of synchronism between the two electrons, let us select two atoms, the one having a single ring of two electrons and the other of four. If the two atoms were supposed to be alike, it is reasonable to take the speeds the same for each, but Schott does not claim that his formula applies to two synchronous electrons. It may be remarked, however, on this point I have found that, when complete rings of equally spaced electrons are concerned, the formula for synchronous electrons reduces to precisely the same form as for incommensurable velocities, since all the terms involving the

phase angles disappear when rings are taken, and we obtain the same result as without synchronism. Not to raise the question, however, let us calculate the force on atom *A*, a two electron single ring atom, due to atom *B*, a four electron single ring atom, the speed of each electron in *A* being  $\beta_1$ , and in *B*,  $\beta_2$ . According to (5) the force of the four electrons in *B* upon one electron in *A* is 4 times the expression in (5). Similarly, the force of the four upon the second electron in *A* is 4 times (5), making a total of 8 times (5).

On the other hand, the force of the two electrons in *A* upon one in *B* is two times (5), writing  $\beta_1$  instead of  $\beta_2$ . The total force of the two in *A* upon the four electrons in *B* is then eight times (5) writing  $\beta_1$  instead of  $\beta_2$ . We have then

$$\text{Total force of } B \text{ on } A = -\frac{16}{3} e^2 \beta_2^4 r^{-2}. \quad (6)$$

$$\text{Total force of } A \text{ on } B = -\frac{16}{3} e^2 \beta_1^4 r^{-2}. \quad (7)$$

If it is permitted, in making an approximation to the speeds of the electrons, to adopt the approximate formula (3), page 14, PHYSICAL REVIEW, July, 1918, for the speed of any ring of electrons, and a moderate error in the estimated speed cannot affect our argument at all, namely

$$\beta^2 = p \frac{\pi^2 e^4}{c^2 h^2}, \quad (8)$$

we have for the fourth power of the speed of each electron in atom *A*

$$\beta_1^4 = 4 \frac{\pi^4 e^8}{c^4 h^4}, \quad (9)$$

and in atom *B*

$$\beta_2^4 = 16 \frac{\pi^4 e^8}{c^4 h^4}. \quad (10)$$

whence (6) and (7) become

$$F_{BA} = -\frac{256}{3} \frac{\pi^4 e^{10}}{c^4 h^4} r^{-2}, \quad (11)$$

$$F_{AB} = -\frac{64}{3} \frac{\pi^4 e^{10}}{c^4 h^4} r^{-2}. \quad (12)$$

We shall also make a further assumption that the two-electron atom, *A*, represents hydrogen, and the four electron atom, *B*, helium. This assumption also will not affect our argument where such a large ratio as  $10^{27}$  is concerned. According to the law of gravitation the average force between one atom of hydrogen and one of helium is

$$F = km_H m_{He} r^{-2}. \quad (13)$$

The mass of the helium atom is approximately four times that of the hydrogen atom, so that, in terms of  $m_H$ , (13) becomes

$$F = 4km_H^2r^{-2}. \quad (14)$$

Taking the smaller of the two forces, namely that in (12), for the purpose of comparison with the magnitude of the gravitational force, and taking the ratio, we have

Ratio of the calculated force to the actual gravitational force

$$= \frac{16}{3} \frac{\pi^4 e^{10}}{km_H^2 c^4 h^4}. \quad (15)$$

It was pointed out in the paper above referred to, equation (9), p. 15, July, 1918, that a very exact numerical value of the gravitational constant,  $k$ , is given by the expression,

$$k = \frac{p^4}{3K} \frac{m_0}{m_H^2} \frac{\pi^4 e^{10}}{c^4 h^4}, \quad (16)$$

in which  $K = 1$ , is the specific inductive capacity, and  $p = 2$ , the number of electrons in the hydrogen atom.  $m_0$  is the mass of the negative electron at slow velocities. With these values, we have

$$k = \frac{16}{3} \frac{m_0}{m_H^2} \frac{\pi^4 e^{10}}{c^4 h^4}. \quad (17)$$

Upon substitution of this correct numerical value of the Newtonian constant,  $k$ , in (15), the expression reduces to

Ratio of calculated force to the gravitational force

$$= 1/m_0 = 1/90 \times 10^{-27} = 1.11 \times 10^{27}. \quad (18)$$

The ratio would be four times greater than this if we had used the larger of the two forces as given in (11) above. This completes the special example, and shows that the force given by the Schott formula is greater than the gravitational force by a factor of about  $10^{27}$ , as was pointed out before considering the example.

In conclusion, it has, we believe, been clearly shown that the application which Schott has made of his final result, equation (50) of his paper, or (1) above, to gross matter is incorrect. The positive nuclei of the atoms must be taken into the account as well as the revolving electrons. Whether or not the matter has been properly treated above in the simple omission of the constant, which is independent of the speed remains to be seen. It seems that any system of electromagnetic equations that does not make the mechanical force of one revolving electron acting upon a second exactly equal and opposite to that of the other on the one, thus strictly conforming to the law of action and reaction is pretty certain to lead to insuperable difficulties when applied to the accepted forms of atoms. It is difficult to believe that the average mechanical force on

the first electron due to the second is entirely independent of the state of motion of the first electron. It seems as if its motion must in some way alter the average force upon it due to the second electron independent of any change that may be taking place in the motion of the second electron, and yet this is contrary to the Larmor-Lorentz form of theory.

Let us admit for the moment that a form of theory is possible that makes the average mechanical force between the two electrons a function of the product of the two speeds instead of the speed of one of the charges only. Then, when either of the two speeds vanishes the product vanishes, and the force between a revolving negative electron and a stationary positive charge is the same whether we consider the force of the one on the other or the other on the one, and is equal simply to the electrostatic terms. In the case of two revolving electrons the force of the one on the other would be the same as the other on the one, the Law of Action and Reaction being preserved way down to the ultimate constituent parts of the atoms and of gross matter, namely the electrons.

It seems as if the finding of a simple expression for the Newtonian constant given in a former paper, equation (9),<sup>1</sup> has some bearing upon this question, especially because all of the laws of gravitation have been shown to follow from the premises assumed. It is very evident that the least change in the fundamental equations of electromagnetic theory may easily switch the resulting force at great distances about from a repulsion to an attraction, the force being so small that the residual may take either a positive or a negative sign. The chief point that has now been established is that there are existing forces at great distances that vary inversely as the square of the distance, the gravitational law, as demanded by the present unmodified form of theory. Not much attention has been given to these forces in the past, and it seems as if it was not realized that they existed, as demanded by the accepted theory.

<sup>1</sup> *Loc. cit.*

## ON THE SPECULAR REFLECTION FROM ROUGH SURFACES.

BY T. K. CHINMAYANANDAM.

IT is well known that matt surfaces behave nearly in the same way as polished reflectors, when the incident rays are of great wave-length or fall very obliquely on the surface. In a paper published in the PHYSICAL REVIEW for January, 1916, A. F. Gorton has followed up the work of Lord Rayleigh<sup>1</sup> and T. J. Meyer<sup>2</sup> on this subject, and has given several curves showing the relative reflecting power of surfaces of ground

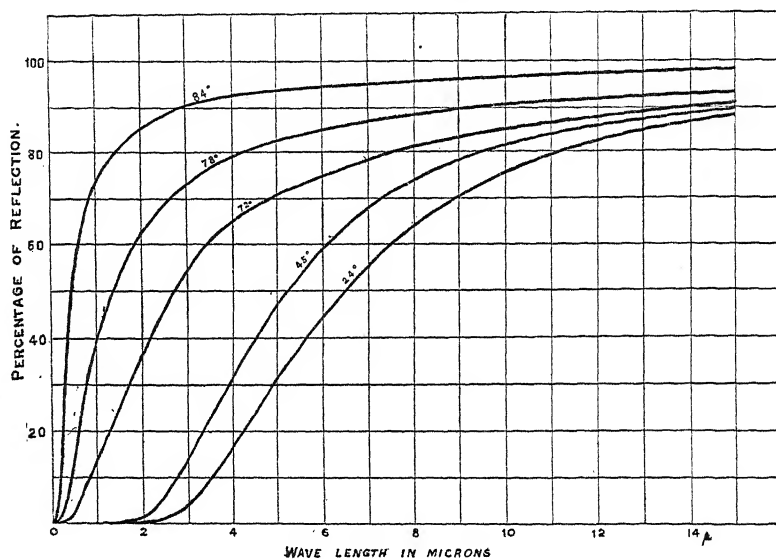


Fig. 1.

glass as compared with that of a polished surface of the same material for various angles of incidence and for various wave-lengths of incident radiation. Though the primary object of Gorton's investigation appears to have been the development of an infra-red screen, the experimental results obtained by him are also of interest from the point of view of the general theory of reflection from rough surfaces. In the present paper, it is proposed to record the results of an attempt made to develop a math-

<sup>1</sup> Rayleigh, *Nature*, 64, p. 385, 1901.

<sup>2</sup> T. J. Meyer, *Verh. Deutsch. Phys. Ges.*, p. 126, Feb., 1914.



ematical formula which would quantitatively express the relative reflecting power of a rough surface over the range of incidences and wavelengths studied by Gorton.

One important feature about the reflection from rough surfaces which is emphasized in Gorton's paper is the absence of decided interference minima in the reflection curves, such as would occur in the case of a plane grating or any surface of regular topography. This result is attributed to the fact that surfaces like those of groundglass are extremely irregular in structure. Starting on this idea, it seems possible, on certain hypotheses, to calculate the relative reflecting power of the surface for a given wave-length and a given incidence. It is convenient in the first place to make the simplifying assumption that the "relative reflecting power" of a rough surface depends only on its topography and not upon the nature of the material of which it is composed. *A priori*, it can be seen that such an assumption would not be very wide of the truth, particularly in the case of surfaces which, if polished, would have a fairly high coefficient of reflection at all incidences.<sup>1</sup> Further, the surface may be assumed to be "ideally rough," in other words, that different elements of it are distributed at different depths according to the probability law, the aggregate area of the elements lying between the distances  $x$  and  $x + dx$  from a mean plane being proportional to  $e^{-ax^2}dx$ . The disturbance due to the reflected radiations is the resultant of the disturbances due to radiation from each of the surface elements, the proper phase of each being taken into account. Taking the mean plane as the reference plane, the relative phase of the disturbance sent out from an element of the surface at a depth  $x$  below it may be taken to be  $2x \cos \theta \cdot 2\pi/\lambda$ , where  $\lambda$  is the wave-length of the incident radiation, and  $\theta$  the angle of incidence. Assuming now that, to a sufficient approximation, the surface is uniformly illuminated by the incident radiation, we get for the resultant disturbance due to the reflected radiation

$$y = \Sigma \cos (\omega t + 2x \cos \theta \cdot 2\pi/\lambda) ds, \quad (1)$$

the summation extending all over the elements. Since the distribution of the elements at different depths is assumed to be according to the probability law,

$$y = \frac{\int_{-\infty}^{\infty} e^{-ax^2} \cos (\omega t + [4\pi x \cos \theta]/\lambda) dx}{\int_{-\infty}^{\infty} e^{-ax^2} dx}. \quad (2)$$

Evaluating the integrals, the intensity of the reflected radiation is found to be given by

<sup>1</sup> See also Gorton's remarks, *PHYS. REV.*, VOL. VII., ser. 2, p. 75.

$$I = e^{-(8\pi^2 \cos^2 \theta) / a\lambda^2}, \quad (3)$$

and this will also determine the relative reflecting power of the surface.

It is easily seen that this formula represents curves of the same general form as those obtained experimentally by Gorton. The curves have an inflection point, the inflectional gradient being proportional to  $a^3/\cos \theta$ , so that the curves became steeper (1) as  $a$  increases, *i. e.*, as the surface becomes finer, and (2) as the angle of incidence increases; these results are in general agreement with Gorton's observations. The actual results calculated from the formula are given below for the different surfaces used by him. The surfaces are denoted by *A, B, C, D*, where *A* refers to curves *I* and *II* in Fig. 6 (p. 73) of his paper, *B* refers to curve *III* in the same figure, and *C* and *D* refer respectively to the curves in Figs. 7 and 8. It will be seen from Table I. that for moderate incidences the agreement is fairly good.

TABLE I.

TABLE I.

Sur- face.	Angle of Inci- dence.	Relative Reflecting Power (as Percentages) for Wave-lengths (in $\mu$ ).											
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3
B	45°	Obsd.	—	15.9	33.8	50.3	62.7	71.5	77.7	81.7	—	—	—
		Calcd.	—	7.1	30.8	51.6	65.5	74.5	80.5	84.7	—	—	—
C	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	—
		Calcd.	0.0	0.0	4.1	16.6	31.6	45.0	55.6	63.8	70.1	75.0	—
	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.1	73.1	77.4	79.5	82.3
		Calcd.	0.0	1.3	14.7	34.0	50.2	61.9	70.3	76.4	80.8	84.2	86.7
D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
		Calcd.	—	4.0	24.0	44.8	59.8	70.0	76.9	—	—	—	—

But at oblique incidences, the simple single-constant formula fails to express the results quantitatively, as can be seen from the figures in Table II. below.

TABLE II.

Sur- face.	Angle of Inci- dence.	Percentage of Reflection for Wave-lengths (in $\mu$ ).											
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.
A	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2
		Calcd.	0.7	28.9	57.5	73.1	82.0	87.1	90.4	92.5	94.1	95.1	96.0
C	{ 72°  84°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—
		Calcd.	3.7	43.9	69.4	81.4	87.6	91.3	93.5	95.0	96.0	96.7	—
		Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1
		Calcd.	68.9	91.1	95.9	97.7	98.5	99.0	99.2	99.4	99.5	99.6	99.7
D	{ 70°  81°	Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—
		Calcd.	1.3	33.7	61.7	76.2	84.0	88.6	91.5	—	—	—	—
		Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—
		Calcd.	41.0	80.0	90.5	94.6	96.5	97.5	98.2	—	—	—	—

The constants for the different surfaces are as follows:<sup>1</sup>

Surface.	<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>
<i>a</i> 10 <sup>8</sup> ×	1.86	3.72	2.29	2.13
<i>d</i> μ	0.61	0.43	0.55	0.56

the height above or depth below the mean plane for which the proportional frequency of occurrence is one half being given by *d* in the second line.

The failure of the simple theory given above to account for the phenomena observed at very oblique incidences could have been anticipated; for, the assumption made that the elements of the rough surface are all equally illuminated by the incident radiation would obviously be wide of the mark at such incidences. The extent of the discrepancy evidently depends on the degree of rugosity of the surface. If this is very marked, and the radiation is of relatively short wave-length and is incident very obliquely, hardly anything more than the upper parts of the surface would be illuminated. These would act as diffracting sources of radiation, the permanent phase differences between them being, of course, much smaller than the phase differences between the radiations from the highest and deepest parts of the surface. On the other hand, with greater wave-lengths, even the deeper parts of the surface would contribute to the reflection, but the increase due to this may to some extent be set off by the larger phase differences that come into play.<sup>2</sup> In view of these complications, it seems very difficult to give even an approximate theoretical treatment for the case of very oblique incidences. An empirical formula may however be devised to fit the experimental results. This should obviously satisfy the following conditions indicated by theory. (a) For very large wave-lengths, the relative reflecting power should be unity. (b) For moderate obliquities and over a range of incidences depending on the rugosity of the surface, it should reduce to the simple single-constant formula (3) above. (c) For very oblique incidences, it should give results, which, for short wave-lengths, should be greater, but which increase with the wave-length less rapidly than as suggested by the single-constant formula (3). In devising such a formula, preference has naturally been given to terms of the exponential type suggested by the simple theory. The formula which has been found suitable is

$$I = e^{-a \tan^3 \theta} e^{-(b \cos^2 \theta)/\lambda^2} + (1 - e^{-a \tan^3 \theta}) e^{-(c \cos \theta + d \cos^2 \theta)/\lambda}.$$

<sup>1</sup> The surfaces *A* and *B* had been silvered, *C* had been platinized cathodically, but *D* was unsilvered.

<sup>2</sup> The expression  $2\pi \cos \theta \cdot 2\pi/\lambda$  for the phase difference between the radiations from the mean plane and a plane at a distance  $x$  from it will not, also, be valid in these circumstances.

This formula evidently satisfies the conditions stated above. For, when  $\lambda = \infty$ ,  $I$  is unity. When  $\theta$  is small, the factor  $e^{-a \tan^3 \theta}$  is practically unity, and the expression reduces to the formula (3). On the other hand, when  $\theta$  approaches  $\pi/2$ , this factor becomes negligible, and the expression reduces to the form

$$I = e^{-(c \cos \theta + d \cos^2 \theta)/\lambda}$$

which approaches unity more slowly than (3) as  $\lambda$  is increased. The actual results calculated from this empirical formula have been plotted out in Fig. 1 for the surface  $C$ . The numerical values have also been given in Table III. for detailed comparison with Gorton's observations. The constants used have the following values. It will be seen that of the four constants,  $a$  is the one most sensitive to alterations in the character of the surface.

Surface.	$A$ .	$C$ .	$D$ .
$a$ .....	0.24	0.079	0.0115
$b$ .....	42.4	34.5	32.9 ( $\times 10^{-8}$ )
$c$ .....	2.5	1.43	0.0 ( $\times 10^{-4}$ )
$d$ .....	17.8	15.75	7.89 ( $\times 10^{-4}$ )

TABLE III.

Sur- face.	Angle of Inci- dence.		Percentage of Reflection for Wave-lengths (in $\mu$ ).											
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	
A	23°	Obsd.	—	0.8	4.0	13.9	26.6	38.2	47.4	56.7	63.8	68.5	71.6	
		Calcd.	—	0.0	2.0	10.6	23.8	36.9	47.1	57.0	64.2	69.8	74.3	
	70°	Obsd.	4.0	25.3	39.8	48.1	55.3	60.2	63.7	66.9	70.1	72.6	75.2	
		Calcd.	5.3	23.0	37.5	48.0	55.6	61.3	65.7	69.2	72.1	74.5	76.5	
	24°	Obsd.	2.0	3.2	7.9	19.1	32.1	44.5	54.1	63.7	69.8	75.0	77.9	
		Calcd.	0.0	0.0	4.1	16.6	31.6	44.9	55.6	63.8	70.1	75.0	78.8	
C	45°	Obsd.	3.2	5.3	17.3	33.2	48.0	59.0	67.0	73.1	77.4	79.5	82.3	
		Calcd.	0.0	1.3	14.2	32.3	47.4	59.1	67.3	73.3	77.7	81.1	83.6	
	72°	Obsd.	17.0	36.9	51.0	62.3	69.3	74.4	77.9	80.9	83.1	84.7	—	
		Calcd.	14.5	36.1	55.5	64.9	71.0	75.3	78.4	80.9	82.9	84.5	—	
	78°	Obsd.	38.2	55.6	64.7	70.3	73.3	76.8	79.6	81.7	83.8	85.8	87.7	
		Calcd.	39.1	62.5	73.1	79.1	82.9	85.6	87.5	89.0	90.1	91.0	91.8	
	84°	Obsd.	73.3	85.7	90.4	93.3	95.1	95.9	96.5	97.0	97.5	97.8	98.1	
		Calcd.	73.3	85.6	90.2	92.5	94.0	95.0	95.6	96.2	96.6	97.0	97.2	
	D	54°	Obsd.	—	8.2	28.0	45.6	59.5	69.9	75.2	—	—	—	—
			Calcd.	—	4.5	24.0	44.5	59.5	69.6	76.3	—	—	—	—
70°		Obsd.	11.0	46.3	66.9	77.3	83.4	85.1	85.1	—	—	—	—	
		Calcd.	9.4	40.0	64.0	76.7	83.6	87.9	90.5	—	—	—	—	
75°		Obsd.	31.0	66.1	77.1	84.0	88.1	90.3	91.2	—	—	—	—	
		Calcd.	32.6	66.4	80.8	87.3	90.8	92.9	94.2	—	—	—	—	
81°		Obsd.	82.3	92.5	94.7	96.3	97.7	98.7	99.3	—	—	—	—	
		Calcd.	80.6	90.4	93.7	95.3	96.2	96.9	97.5	—	—	—	—	

The agreement between the calculated and the observed values in Table III. seems fairly good, except in the case of Surface C for an incidence of  $78^\circ$  for which the corresponding experimental curve given by Gorton appears to be rather irregular. The fact that the empirical formula suggested is found to give fair agreement with observations on three different surfaces at different angles of incidence and for different wave-lengths over a wide range makes it probable that it is of fairly general application to rough surfaces.

# AMPLIFICATION OF THE PHOTOELECTRIC CURRENT BY MEANS OF THE AUDION.

BY CARL ELI PIKE.

A METHOD has been outlined by Jakob Kunz<sup>1</sup> by means of which the photoelectric current may be amplified, thus making the photoelectric cell more useful as a photometer, especially in the region of ultra-violet light and also for technical purposes.

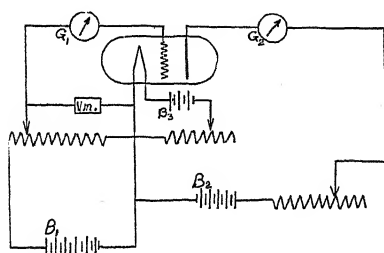


Fig. 1.

The amplification is produced by means of a vacuum tube with three electrodes, or the audion. In order to determine the best potentials to use in the primary and secondary circuits it is necessary to know the characteristic curves for the audions used. The characteristic curves of several audions have been determined by an arrangement of apparatus

shown in Fig. 1, which is self explanatory. If we plot the grid potentials as abscissæ and the plate current as ordinates, the curve obtained is called the characteristic.

Due to the fact that the plate current as well as the grid current was so sensitive to small changes in the temperature of the filament, it was necessary to keep the heating current very constant. Large storage cells, well insulated from the ground, were used for this purpose. A large resistance was placed in the external circuit, so that a small variation in the resistance of the filament would not affect the current appreciably. The characteristic curves of three types of audions are shown in Figs. 3, 4 and 5. Audion no. 1 is an oscillion made by the DeForest Radio Telephone and Telegraph Co. Audion no. 2 of Fig. 4 is a W-type;

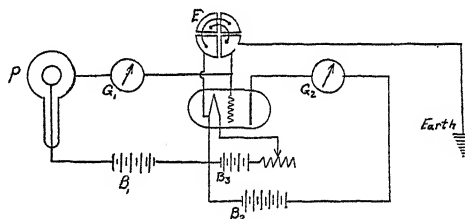


Fig. 2.

<sup>1</sup> PHYS. REV., Vol. X., No. 2, p. 205.

Audion no. 3 of Fig. 5 is a V-type instrument made by the Western Electric Co. It is noted that the plate current in the oscillion reaches its saturation value more abruptly than it does in either of the other two instruments. In the oscillion it is necessary to heat the filament to incandescence before the electrons are emitted, while in audion W and V the light from the filament was scarcely visible.

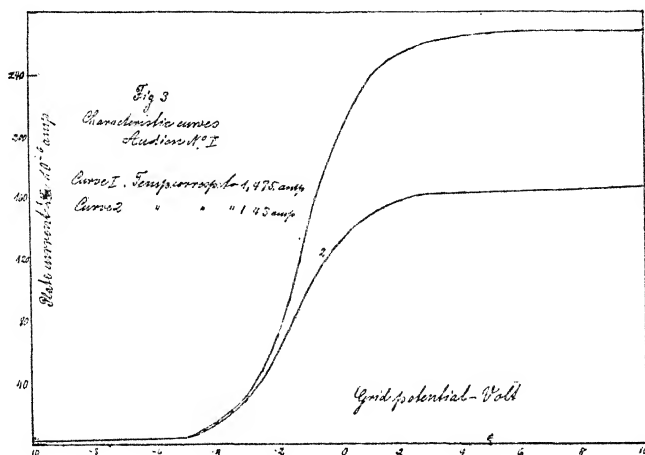


Fig. 3.

Audion no. 2, the W-type instrument, was used for the amplification of the photoelectric current, with an arrangement of apparatus shown in Fig. 2. Twenty-four volts were used in the secondary circuit and a hundred and twenty-five volts in the primary. The photoelectric cell used was a larger type of those made by Kunz in our laboratory. With 125 volts in the primary circuit, the drop of potential inside the audion was very small; measured with an electrometer it was found to be 0.56 volt for nearly the highest intensity of light incident on the photoelectric cell. Since the drop of potential between the grid and filament is very small in comparison to that across the terminals of the photoelectric cell, the photoelectric current is very nearly equal to what it would be if the audion were out of the circuit. The curve giving the relation between the intensity of light and the photoelectric current is shown in Fig. 6. It is unfortunately not a straight line. If this were a straight line and if the portion of the characteristic curve of the audion, used for the amplification, were straight, then we would expect a straight line relation between the intensity of light and the amplified current, and the amplification  $i_2/i_1$ , the ratio of the secondary to the primary current would be constant, represented by a straight line parallel to the hori-

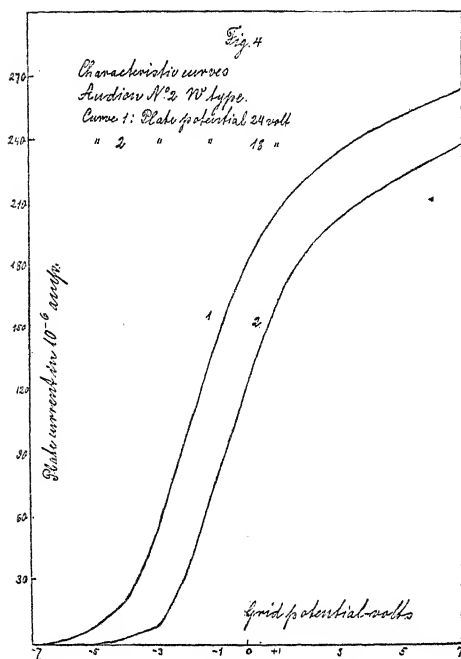


Fig. 4.

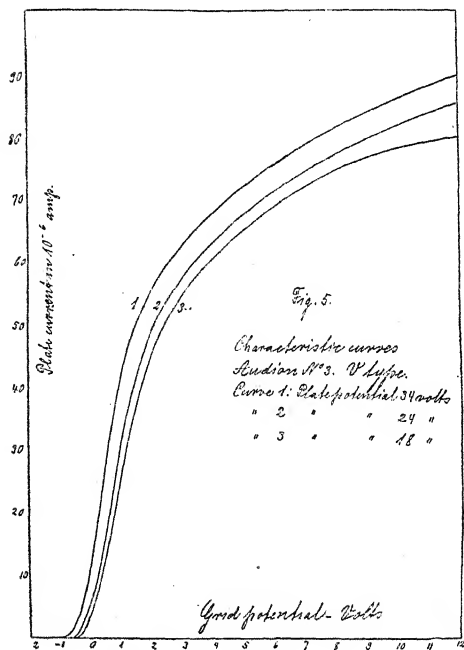


Fig. 5.



zontal axis of Fig. 7. Instead of this straight line, the curve of Fig. 7 has been obtained for intensities varying from 3 to 30 candle meters. For the highest intensity the amplification is about 1750, for the smallest

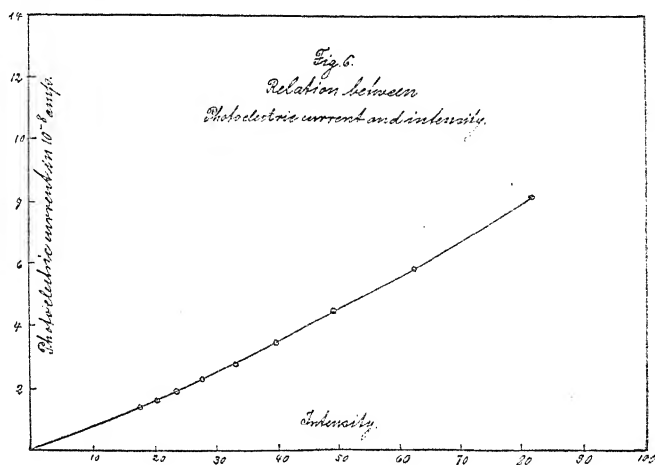


Fig. 6.

intensity it is over 5,000; above an amplification of 4,000 the points appear somewhat scattered around the curve, but this was only so because the primary current deflections of the Leeds and Northrup galvanometer,

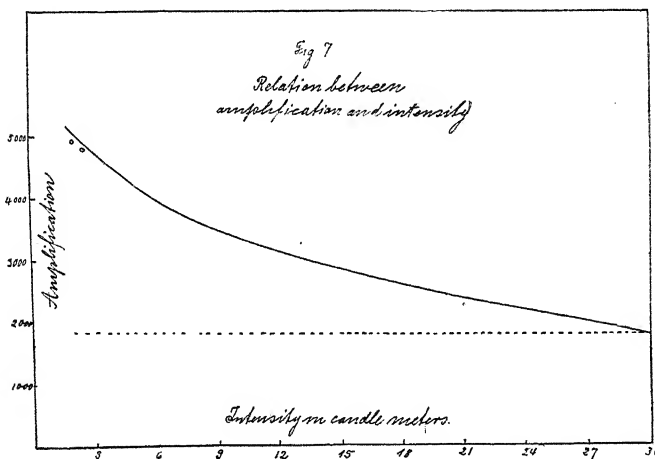


Fig. 7.

$G_1$  (with a figure of merit  $3.74 \cdot 10^{-9}$  for the scale distance used), were very small. If the primary or photoelectric current  $i$  is zero, there is already a large current through the secondary galvanometer with a

figure of merit  $2.9 \cdot 10^{-6}$ . It goes without saying that this "dark" current was subtracted from that current which was obtained in the galvanometer,  $G_2$ , when the photoelectric cell was under the action of light. The difference between the two deflections was proportional to the current  $i_2$ . The deflections of the galvanometers were very steady and could easily be repeated. Table I. gives the data that have been plotted in Fig. 7. A satisfactory theory of the audion, based upon the motion, accumulation and absorption of electrons has not yet been given. The current amplification can therefore not yet be determined theoretically. But we can find a simple expression for the amplification, namely,  $i_2/i_1$  in the following way, which involves only Ohm's law and the experimental relation between the plate current and the grid potential; as long as we restrict the amplification to the straight portion of the characteristic  $i_2 = Cp_1$ , we get the following equations.

$$i_1 = \frac{E_1}{R_0 + R_1} = \frac{p_1}{R_1},$$

$$i_2 = \frac{E_2}{R_2 + R_3} = \frac{p_2}{R_2} = Cp_1 = Ci_1R_1,$$

$$\frac{i_2}{i_1} = C \cdot R_1.$$

The amplification is therefore constant if  $C$  and  $R_1$  are constants, that is, if the straight part of the characteristic is used and if the resistance  $R_1$  between the filament and the

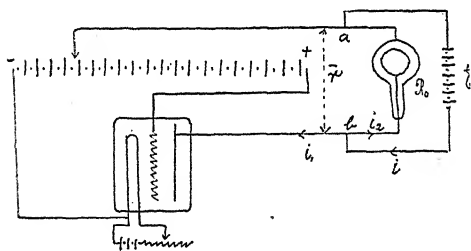


Fig. 8.

between the filament and the grid is constant. For large amplifications  $C$  and  $R_1$  have to be large.

A different principle has recently been indicated by A. W. Hull,<sup>1</sup> of the General Electric Company, for the amplification of small currents. By a proper choice

of the potentials, the electrons emitted from the incandescent filament pass through the grid and strike the plate where they are reflected. A system of this kind, shown in Fig. 8, presents a negative resistance  $\bar{r}$  between the points  $a$  and  $b$ . If we place a photo-electric cell with the positive resistance  $R_0$  in parallel with  $\bar{r}$ , then we get;

<sup>1</sup> P. I. R. E., February, 1918.

TABLE I.

Increase in $d_2$ .	$i_2$ Amperes.	$d_1$ .	$i_1$ Amperes.	Amplification.	Intensity in Candle Meters.
73.5	$213.0 \times 10^{-6}$	35.0	$131.0 \times 10^{-9}$	1600	33.0
64.8	188.0	23.5	87.7	2100	24.0
56.5	164.0	17.0	63.5	2600	18.5
49.0	142.0	12.9	48.2	2900	14.8
41.0	119.0	10.0	37.4	3270	12.0
35.5	101.5	8.1	30.2	3400	9.9
31.4	91.0	6.8	25.4	3570	8.3
28.0	81.2	5.7	21.3	3800	7.1
23.8	69.0	4.8	17.9	3840	6.1
22.5	65.2	4.1	15.3	4250	5.3
19.5	56.6	3.5	13.1	4320	4.7
17.5	50.7	3.1	11.6	4370	4.2
16.5	47.8	2.8	10.5	4570	3.7
15.1	43.7	2.5	9.4	4680	3.3
13.0	37.7	2.1	7.9	4800	3.0
12.0	34.8	1.9	7.1	4890	2.7
10.9	31.6	1.8	6.7	4720	2.5
11.5	33.3	1.7	6.4	5100	2.3
10.0	29.0	1.6	6.0	4840	2.1
10.0	29.0	1.5	5.6	5200	1.9

"W" type audion, no. 2. Lamp current, 3.75 amperes; candle power, 3.0. Heating current, 0.665 amperes.  $B^1$  equaled 125 volts.  $B^2$  equaled 25 volts. Figure of merit of  $G^1$   $3.74 \times 10^{-9}$ . Figure of merit of  $G^2$   $2.9 \times 10^{-6}$  when shunted with 1.7 ohm resistance. Ratio of figure of merit of  $G^2$  to that of  $G^1$  equaled 775.

$$i = i_1 + i_2 = E \left( \frac{R_0 + \bar{r}}{R\bar{r}} \right),$$

and

$$i_1 = \frac{E}{\bar{r}}, \quad i_2 = \frac{E}{R},$$

hence the amplification

$$\frac{i_2}{i} = \frac{\bar{r}}{R_0 + \bar{r}}$$

may be made very large by making  $r$  and  $R_0$  nearly equal in absolute values.

One application of the amplification of photoelectric currents in wireless telegraphy may be pointed out. J. Kunz and J. Kemp<sup>1</sup> have at first used the photoelectric cell as receiver in wireless telegraphy. Their method has been modified by H. Behnken<sup>2</sup> who showed that the photoelectric cell with a string electrometer forms a constant detector of high sensitiveness, especially useful for photographic registrations. With

<sup>1</sup> Jahrbuch d. drahtlosen Telegraphie, 6, 405, 1913.

<sup>2</sup> Verhdlg d. deutschen phys. Ges., 16, p. 668, 1914.

the amplification of the weak currents here involved it should be possible to increase the usefulness of the photoelectric detector. It was intended to continue this investigation with ultra-violet and interrupted light, with alternating currents, and with a differential galvanometer in the secondary circuit.

#### SUMMARY.

It has been shown that photoelectric currents can be amplified by means of the audion from 1,600 to 5,000 times. The weaker the light the smaller the primary photoelectric current and the larger the amplification. With different audions amplifications of 18,000 have been obtained.

In conclusion the writer wishes to express his appreciation to Jakob Kunz for suggesting the problem and directing the work.

LABORATORY OF PHYSICS,  
UNIVERSITY OF ILLINOIS,  
October, 1918.

## THE EMISSION AND ABSORPTION OF PHOTOELECTRONS BY PLATINUM AND SILVER.<sup>1</sup>

BY OTTO STUHLMAN, JR.

AS the result of light passing through very thin metal foil, photoelectrons will be emitted from both sides of the metal simultaneously. Since it will be necessary to distinguish between those electrons which are liberated from the side upon which the light is incident and those that come from the side from which the light emerges, we shall refer to them as "incidence" and "emergence" rays respectively.

It was hoped that some relations could be established between the atomic properties of metals and their photoelectric emissivity. Secondly to establish the magnitude of the coefficient of absorption of the electrons as a function of their speed and atomic properties of the metal in which the absorption was to take place. Finally to use the above information, if adequate, to throw some light on the structure of the atom.

All previous work by the writer and other investigators<sup>2</sup> who have been interested in the photoelectric properties of very thin metallic foil, or still thinner and transparent films of metal was undertaken with cathode deposited films supported by quartz.

The photoelectric currents and velocity investigations were, however, found to depend too much upon the previous history of the cathode and the potential gradient under which the cathodic deposit had been made to warrant any quantitative conclusions.

Thus in many cases, as in those investigations by the writer and K. T. Compton, both the magnitude of the current and the velocity of emission of the electrons could be altered at will, depending upon the impurities that are present through occlusion, in the cathode. The most serious difficulty, however, was found in mounting the very thin transparent films in such a way as to insure perfect electrical contact. These sources of error were so great that no quantitative conclusions could be drawn from the data at that time.

<sup>1</sup> Presented at the Amer. Phys. Soc. meeting, December 27, 1917.

<sup>2</sup> Stuhlman, *Phil. Mag.*, 20, p. 331, 1910; 22, p. 854, 1911; *Phys. Rev.*, 4, p. 195, 1914. Stuhlman and Compton, *Phys. Rev.*, 2, Sept., 1913. Kleeman, *Proc. Roy. Soc.*, 84, p. 92, 1910. Robinson, *Phil. Mag.*, 23, p. 542, 1912; p. 115, 1913; *V.*, 32, p. 421, 1916. Werner, *Ark. Mat. Ast. Fys.*, 8, No. 27, p. 1, 1913. Partzsh and Hallwachs, *Annalen* XLI., p. 247, 1913. Compton and Ross, *Phys. Rev.*, 9, p. 559, 1917.

In the present work it was found necessary to investigate metallic films of less than  $10^{-6}$  cm. in thickness with a degree of accuracy much beyond that heretofore attained in the above experiments. It was necessary to provide films of extreme purity and make electrical contact with them which would exclude the errors introduced through direct contact clamping between these very thin films and their supports leading to the electrometer. Heretofore successive thicknesses of films were produced either through successive additions of metal to an initial thin film, or successive plates were covered with various thicknesses and then separately investigated. This necessitated the investigation of many films of varying thickness deposited on the assumption that a linear relation existed between time of sputtering, and thickness of metal deposited. To attain this experimental linear relation by keeping the working conditions constant enough in the cathode-depositing-chamber through control of vacuum and cathode fall is itself quite a difficult undertaking.

#### TRANSPARENT METALLIC WEDGES WERE DEPOSITED.

In order to obviate the necessity of examining a large number of transparent films of successive increasing thickness, or of examining one film which had been built up of successive layers which had been photoelectrically fatigued through long use, the feasibility of substituting a transparent or semi-transparent metallic wedge was considered.

The deposition of such a wedge by an electrolytic method was however rejected. The difficulties of this method of deposition may be appreciated since we learn from Kundt's<sup>1</sup> well-known experiments on the optical constants of metals, that out of several thousand attempts only a score of wedges were produced which were sufficiently perfect to warrant investigation. A further objection however arises. A chemically deposited metal would contain impurities through occlusion of the electrolyte with the subsequent formation of polarized electrical double layers, which may become so prominent as to prevent all electronic emission.

A metallic wedge deposited by evaporation in vacuo from a fine wire, heated to luminescence, was eventually found to meet the above requirements and objections.

The method<sup>2</sup> essentially consists of a wire, heated to incandescence by means of an electric current, while the quartz plate upon which the wedge is to be deposited is placed below and to one side of it. If the whole is now placed in vacuo, the metal will vaporize and condense upon the quartz surface and surrounding objects.

<sup>1</sup> See Kayser *Handbuch der Spectroscopie*, IV. Band. p. 542.

<sup>2</sup> Stuhlman. Presented before the Amer. Phys. Soc., New York meeting, December, 1916. Also *Jour. Optical Soc. of Amer.*, V., 1, p. 78, 1917.

Under similar conditions Langmuir<sup>1</sup> has shown that the rate of evaporation  $m$  of an incandescent body is expressed by the relation

$$m = \sqrt{\frac{M}{2\pi RT}} \cdot p,$$

where  $M$  is the molecular weight of the vapor and  $p$  its vapor pressure. He has also shown that no reflection of molecules of the metal takes place at the surface upon which they initially strike, unless the surface is so hot as to cause reëvaporation. Under the above conditions we can therefore conclude that the total amount condensed per second is equal to the loss per second which the wire undergoes through evaporation.

Hence given a wire whose cross-sectional area is small compared with its distance above the plane, over which it is placed horizontally, it may be treated as a linear source which is sending out metallic vapor in all directions at right angles to its surface. Since no reflection takes place from the horizontal plane under the working conditions, then  $dm/2\pi$  can represent the amount of evaporated metal passing through unit angle per second and subsequently condensed from a length  $dl$  of wire. Let the wire be stretched parallel to and at a height  $h$  cm. above the horizontal plane in which the quartz plate is placed, upon which the deposition is to form. After  $n$  seconds let  $t_0$  represent the thickness of the metal condensed on the plane just below the wire. The flux per second of metal through an angle  $d\theta$  condensing just below the wire producing this thickness  $t_0$  is then

$$t_0 h d\theta = \frac{dm}{2\pi} d\theta.$$

The simultaneous condensation at any other point in the plane at a perpendicular distance  $r$  from the wire is given by

$$t_n ds = \frac{dm}{2\pi} d\theta.$$

Since

$$t_0 h d\theta = t_n ds = \frac{t_n r d\theta}{\cos \theta}$$

it follows that the thickness of deposit at any point in the plane is

$$t_n = \frac{t_0 h^2}{h^2 + x^2}, \quad (1)$$

where  $x$  represents the distance of the point  $n$  from the foot of the perpendicular  $h$  let fall from the wire to the plane. If we set  $h = 2a$  and  $t_n = y$  the equation reduces to the well-known curve

<sup>1</sup> PHYS. REV., V., 2, p. 340, 1913.

$$y = \frac{8a^3}{x^2 + 4a^2}, \quad (2)$$

the "Witch of Agnesi," so that a section through the solid deposit formed from an evaporating wire, at constant temperature, with vacuum at constant pressure can be represented by the above curve. The deposit just below the wire is however not a wedge. It has a pronounced curvature concave downward, followed by a curvature concave upward. Further along, that is for the values of  $x$  ten times larger than values of  $y$ , the curve approaches a straight line. It was in this region that the quartz plate was placed, which was to receive the metal, where the deposit would approach nearest to the desired form of a true wedge.

#### APPARATUS USED TO DEPOSIT WEDGES.

Fig. 1 shows a diagrammatic representation of the apparatus. Although originally<sup>1</sup> designed to make interferometer or opaque mirrors of uniform texture and optical purity, it was found to answer the requirements for the present purpose. The working parts are mounted

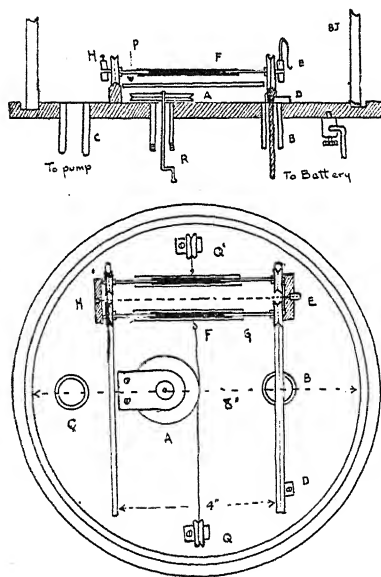


Fig. 1.

on a thick plate of rolled brass, grooved to hold the bell jar *BJ*. Hard sealing wax floated in this groove formed the seal for the vacuum which was maintained by means of a Gaede rotary mercury pump. The carriage *HE* supports the incandescent wire. At *H* the wire is attached to a screw at the top of the rod connecting the two wheels, from where it passes down and then through a large hole in the rod between the wheels, then across through a similar hole in the rod at *E*, thence to the spring brass hook, *E*. This spring hook supplies the necessary tension to the wire. Upon heating the wire, it expands and the slack is further taken up by the spring in the hook. For the heating current a 110-volt alternating current was used. The quartz plate was placed in the plane *P* below and to one side of the wire.

<sup>1</sup> Read as a paper before the Amer. Phys. Soc., New York, December, 1916. Jour. Optical Soc., V., 1, p. 78, 1917.



The heating-current enters the base plate at the binding post to the right. It then passes into the metallic track at *H* which is screwed to the base plate. From here it passes through the left side of the carriage, thence through the wire across to *E* into the right-hand track. This track is insulated from the base-plate by means of two fiber L-shaped supports. The current passes from this insulated track through the rod *B* back to the battery. The rod *B* passes through a hole in the base-plate and is sealed in place with hard red sealing wax, which serves to insulate it from the plate and at the same time makes the opening air tight.

To prevent the carriage supports from short circuiting the glowing wire, they are separated by hard fiber rods. This joint *F* was then covered with a tight-fitting glass tube extending one centimeter beyond each end of the fiber insulation. This prevented the fiber from being coated with a metallic deposit which would otherwise short circuit the glowing wire. In order to shift the carriage it was necessary to have a mechanical control outside of the apparatus. (A magnetic control would be simpler for the present purpose.) It was supplied by means of a small crank which passed through a ground glass stopper sealed in the tube at *R*. This crank turned the horizontal V-grooved wheel *A*, around which passed a belt to the small wheel *Q*, from where it passed to a hook on the carriage at *F*. The other end of the belt passed over a similar wheel at *Q'* and thence to a similar hook on the other side of the carriage. By turning the crank *R*, the carriage and wire could be rolled to any position above the plane *P* which contained the quartz plate upon which the metallic wedge is deposited.

In order to insure a very pure deposit it was found necessary to acid clean the wire thoroughly. It was then mounted and heated in vacuo, but not so high as to produce evaporation. This removed most of the occluded gases and such impurities which would otherwise have been condensed on the quartz plate. The wire is now set at the proper distance from the quartz plate and allowed to vaporize until the desired thickness of metal has been attained.

A metallic deposit condensed as outlined above, was first examined for change in thickness as the distance from the wire increased; with a view of ascertaining how near photometric measurements could determine changes in thickness. A glass microscope slide  $10 \times 1$  cm. was placed directly beneath and 1 cm. below the wire and the metal was deposited until the region just below the wire was opaque. The deposit was next examined by means of a microphotometer with unresolved light from a tungsten filament lamp. A beam  $1 \times 10$  mm. was directed normally on

the metallic side and after passing through the wedge was focussed on a linear thermopile in vacuo. The throw of a ballistic galvanometer in series was used to measure the current, which was assumed to indicate, in a rough way, the change in thickness of the wedge as successive millimeter portions were passed before the slit of the thermopile.<sup>1</sup> No quantitative conclusions were expected from these data due to the change in the optical constants with change in thickness of the film.

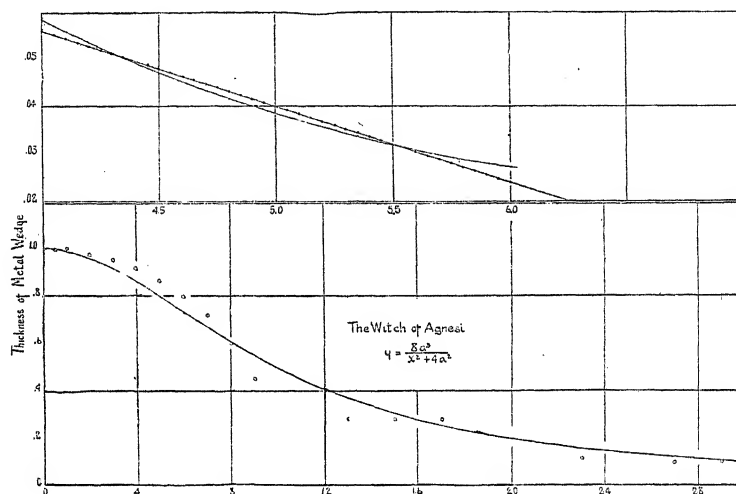


Fig. 2.

These more or less approximate results are shown in the lower curve of Fig. 2. The agreement between the observations as recorded and the theoretical equation  $y = 8a^3/(x^2 + 4a^2)$  plotted, as a full line, are as close as might be expected under the circumstances.

In order to determine how near to the wire the best approach to a true wedge could be expected, the following graphical method was resorted to. The theoretical curve was plotted on large scale coördinate paper and that part of the curve chosen which fitted nearest to a straight line within an average of five per cent. The upper graph in Fig. 2 shows the portion of the curve chosen. The solid line represents values computed from the theoretical equation, for the thick end of the wedge at a distance of four centimeters from the foot of the normal let fall from the wire to the plane. The broken graph is a straight line drawn through the average value of the points. Although the metal deposited at this distance from the wire has a slightly concave surface, yet it approaches a straight

<sup>1</sup> I am indebted to Dr. Ives, of the United Gas Improvement Co., Philadelphia, for the use of this apparatus, and to Dr. Eckhardt for assistance with these observations.

line wedge with not more than a maximum of five per cent. variation at any point. This five per cent. error, contributed by the straight line assumption, is within the error of observation contributed by the subsequent photoelectric current measurements.

The metallic wedges investigated in this paper were all deposited in this region.

#### THE WEDGES.

The metals investigated were platinum and silver. They were deposited from chemically pure wires .043 cm. in diameter. The wire was 12 cm. above the plane which contained a  $2.5 \times 1.0 \times 0.1$  cm. quartz plate placed four centimeters away from, with its narrow side parallel to, the incandescent wire.

The platinum wedges, when examined by reflected light, showed for the opaque deposit at the edge nearest to the glowing wire, the characteristic metallic lustre of the metal. Semi-opaque thicknesses transmitted red light and reflected blue. As the thickness decreased the reflected light changed from blue to brown, to yellow, to blue and at the very thinnest end to indigo. The colors of the transmitted light for the same thicknesses were gray brown, greenish blue and reddish brown.

Silver wedges which were opaque at one end and shaded to zero thickness at the other end in 4 cm. of length showed the following variation in color. Deep violet was transmitted through the nearly opaque thicker end, shading into violet, blue, light blue, deep blue, purple, red, brown and into yellow at its thinnest end. The corresponding points when viewed by reflected light showed at the opaque end the metallic luster of silver followed by deep green, green, green blue, red green, red, blue red, blue and a lighter blue at its thinnest end.

An absorption band might hence be expected to exist in the green part of the spectrum about  $\lambda$  4500. An absorption spectrum photograph obtained by means of a Hilger monochromatic illuminator through one of the silver wedges, showed a continuous gradually increasing absorption through the red and yellow, terminating at about  $\lambda$  5300, from here a weak broad green blue absorption band exists terminating at about  $\lambda$  4000 from where the absorption again drops off regularly into the violet but at a greater rate than the ascent in the red end.

While the thick and nearly opaque ends of some of the platinum wedges are transparent to red light (in fact the glowing filament of a tungsten lamp was visible through the thick end as a brilliant red image); the thick or nearly opaque end of the silver wedge showed the same filament as an indigo-blue image. Showing that the shorter wave-lengths penetrate further into a silver surface than into platinum. What evidence the

photoelectric observations throw on this topic will be discussed in a separate chapter.

#### THE ARRANGEMENT OF THE APPARATUS.

The metallic wedge ( $1 \times 2.5$  cm.) supported by the quartz plate is shown in Fig. 3 as *Q*. It was mounted with the thick end of the wedge

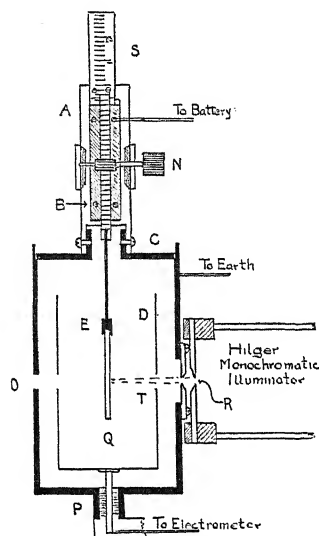


Fig. 3.

attached to the brass holder *E*. A counter-sunk screw pressed the metal-covered side of the quartz plate rigidly against the opposite facet of the clamp at *E*. This clamp was screwed to a rod ending in a rack *B*, which slid under V-shaped grooves in the fixture *A*. The pinion *N* engaging the rack, pushed the wedge into any desired position with respect to the beam of light *T* which was used to excite the photoelectric phenomena. The position with respect to the illumination was indicated on the steel millimeter scale *S* fastened to the fixture at *A*. This motion of the wedge allowed one to expose successively increasing or decreasing thickness of the metal to the beam of ultra-violet monochromatic illumination passing through a Hilger

Monochromatic Illuminator placed in front of the slit *R*. A 110-volt Cooper Hewitt quartz mercury arc running under constant power supplied the illumination. The slit at *R* was so adjusted that a clear cut image 8 mm. long and 0.23 mm. wide fell on the quartz plate at *T*. The degree of sharpness of the image and its position was at all times controlled by means of a fluorescent plate which could be substituted for *Q* and by means of a fluoroscope eye piece placed at *O*. The photoelectric chamber *D* is an oxidized copper cylinder covered with camphor soot whose photoelectric constants were well known, having been repeatedly used in former experiments. This cylinder *D* was mounted coaxially with the outer brass casing *P*. It was connected as shown, to an electrometer after passing through the hard sealing wax support at *P*. The electrometer showed a consistent sensibility of 1,220 divisions per volt which did not vary more than 1.3 per cent. during the course of the experiments.

The present method differs fundamentally from previous measurements on thin metal films in so far as photoelectric current can be mea-

sured from one film, possessing various thicknesses and having the same history of production and ageing. An additional improvement consists in being able to examine the very thinnest film without introducing special appliances or precautions for electrical contact. It also allows one to measure the thickness of the very thin films to a greater degree of accuracy through extrapolation from the thicker films, which in turn lend themselves more easily to accurate measurements.

The wedge was exposed at millimeter intervals through its entire length. The present paper deals with the photoelectric current obtained from such a wedge when the metal side faced away from the light. Under these conditions the illumination was allowed to fall under normal incidence on the quartz side of the plate, it then passed through the transition layer quartz-metal and liberated photoelectrons in the metal.<sup>4</sup> The electrons leaving the metal under these conditions are classified as "emergence." To measure the "incidence" effect all that was necessary was to rotate the superstructure in the groove *C* through  $180^\circ$  and repeat the measurements for thicknesses as relatively determined by the scale *S*.

#### FILM THICKNESS.

The determination of the thickness of the film contributed the largest error to the final results. Optical methods as developed by Wiener<sup>1</sup> were given up as too unreliable. A very promising method, though not used on the data presented below, and especially adaptable to silver films, is that in which the silver film is locally converted into silver iodide by means of the proximity of a minute piece of iodine. A convex lens placed upon the resulting hump allows one to use the Newtonian ring interference method for computing the thickness of the hump and from it a value of the thickness of the original silver.

Here either mechanical or electrical methods were resorted to. As in the case of platinum, when the specific resistance of the material is known in terms of thickness,<sup>2</sup> it was found feasible to determine the thickness at several points along the wedge by using the following indirect method.

A quartz plate, similar in size to the one upon which a wedge was deposited, was covered with a uniform film, of thickness comparable to the thick end of the wedges. This uniform film can be deposited in the apparatus discussed above by simply moving the carriage, supporting the incandescent wire, back and forth across the plate with uniform speed. After attaining its equilibrium value the film was examined for its spe-

<sup>1</sup> Wied. Annalen, V., 31, p. 629, 1887.

<sup>2</sup> Patterson, Phil. Mag., IV., p. 663, 1902. Longden, PHYS. REV., II., p. 49, 1900. I. Stone, PHYS. REV., VI., p. 1, 1898.

cific resistance. This resistance data could be used to compute its thickness. A photoelectric examination of this uniform surface made under similar conditions as existed when the wedge was investigated, determined the thickness of that part of the wedge which showed an equal photoelectric current.

When the thickness of the plane film was large enough, the above method was checked by direct weighing. The plane silver films could be changed to the iodide form, and then weighed, and from these results the thickness can be computed.<sup>1</sup>

#### EMERGENCE PHOTOELECTRIC CURRENT OBSERVATIONS.

The photoelectric currents for wave-lengths 2260, 2301, 2536 and 3131 were obtained for platinum and silver with variations in thickness of the metal exposed. Since the various spectral lines are not of equal

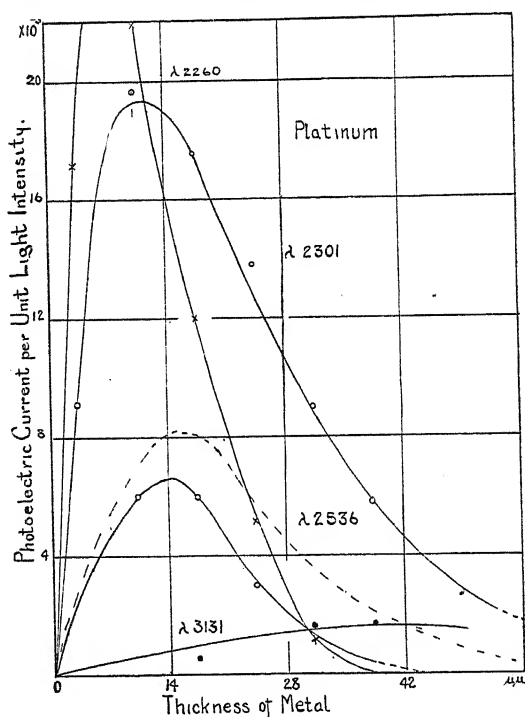


Fig. 4.

intensity the observations were reduced to represent current per unit incident light intensity, by dividing the number of electrons liberated per unit time by the energy of the light of the corresponding wave-length.

<sup>1</sup> Quinicke, Pogg. Ann., 129, p. 178, 186 .

The ordinates thus plotted in subsequent curves represent the relative number of electrons liberated in unit time by a unit amount of light-energy for the wave-length indicated.

In the case of platinum an attempt was made to get some results from  $\lambda 3131$  but with little success, since platinum is no longer photoelectric sensitive for this wave-length. These latter results are only introduced for qualitative comparison, and since no light filter was used, no doubt the observed current was not really due to the excitation from this wave length, but from a somewhat shorter wave-length.

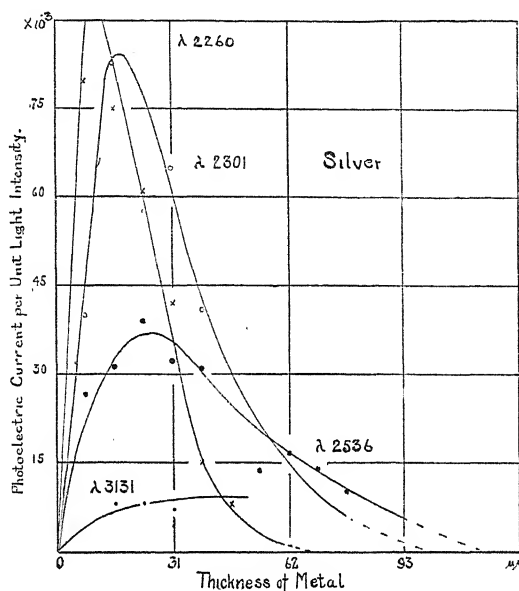


Fig. 5.

Figs. 4 and 5 show a typical set of observations for each of the metals. To avoid the error that might be introduced by photoelectric fatigue it was found necessary to prepare a separate wedge of metal for the observations obtained for each wave-length examined.

The photoelectric observations could be duplicated with a mean error of five per cent., but since the thickness determinations possessed an average error of ten per cent., no additional precautions were introduced into the photoelectric measurements to insure a greater accuracy. The constants computed from these curves show a degree of consistency in agreement with the above errors, which serve as a good check on the observational values.

In Fig. 4 the dotted line for  $\lambda 2536$  gives the unfatigued values of the

photoelectric current computed from the solid fatigued curve just below it. The ordinates of this curve were obtained by increasing the observed fatigued values by an amount experimentally determinable and proportional to the time elapsing between the removal of the wedge from its vacuum chamber and its subsequent photoelectric examination.

The curves for both metals have this in common; all make finite angles with the abscissa. There is no sign of inflexion on the ascent of the curve, so that the tangent to the curve at the origin is more inclined to the abscissa than the tangent at any other point. This means that for the very thinnest part of the wedge (atomic thickness) the photoelectric effect occurring there for any frequency of the exciting light, bears a definite ratio to the mass per square centimeter of the film. The slopes of the curves at the origin are relative measures of the energy absorbed by a film thickness comparable to the diameter of one molecule.

The curves can, for the sake of discussion, be divided into two parts. The initial rise which appears to be a type of saturation curve, and superimposed upon this a second phenomena of a more complex nature approximately represented by an exponential drop followed by a linear relation ending in another less rapid fall.

These two phenomena will be treated separately under the subject of scattering and absorption of photoelectrons.

The photoelectric current is excited by a monochromatic beam of light falling normally on the metal surface, and if no electronic absorption takes place the photoelectric current should be proportional to the light energy absorbed. If the optical constants of the metal do not change with change in film thickness for the wave-lengths used, it would follow that as the thickness of the metal increases linearly the photoelectric current should increase exponentially. This would continue until a thickness is attained beyond which the monochromatic light could no longer penetrate. This does not however hold true. When we measure the emergence effect, the curves seem to indicate that the light gives rise to electrons in amount apparently greater than the amount of light absorbed. It becomes therefore necessary to determine the cause of this additional electronic activity.

#### THEORY.

For the sake of simplicity let us consider only those electrons which leave the atom in the emergence direction, possessing energy proportional to the frequency of the exciting light but independent of the kind of atom from which they initially escape.

As the liberated electrons pass through successive layers of matter,



let us further assume for simplicity that they are absorbed according to an exponential law.

If then a beam of light of frequency  $\nu$  strikes a metal film, and if  $I_0$  is the intensity penetrating the surface normally, if the intensity has fallen to  $I$  after passing through a thickness  $dx$ , then

$$I = I_0 e^{-\epsilon x}.$$

If all of the absorbed energy is used in liberating photoelectrons in such a way that the energy absorbed is equal to the photoelectric energy liberated then

$$I_0 - I = I_0(1 - e^{-\epsilon x}), \quad (3)$$

where  $\epsilon$  is a constant, denoting the rate of appearance of electrons. It depends on the frequency of the light used and may be identified with the coefficient of absorption of the light. Since electronic density and light intensity are proportional under these circumstances,  $I_0$  can be considered a variable, subsequently to be absorbed exponentially in the distance  $t - x$ . Then

$$\frac{dI_0}{dx} = -\alpha I_0$$

where  $\alpha$  is the coefficient of absorption of the electrons passing through thickness  $t - x$ . The solution of this relation is given by

$$I_0 = I' e^{-\alpha(t-x)}.$$

Since all the electrons due to any frequency  $\nu$  of the light have the same initial velocity and hence may initially be considered a homogeneous beam coming from a layer of metal between  $x$  and  $x + dx$ , it follows if we assume that only those electrons are considered that move off in the emergence direction, that the intensity of the emergence radiation is proportional to

$$\frac{dN}{dx} = \epsilon I_0 e^{-\epsilon x} = \epsilon I' \int_0^t e^{-\epsilon x} e^{-\alpha(t-x)} dx$$

and

$$N = \frac{\epsilon I'}{\alpha - \epsilon} (e^{-\epsilon t} - e^{-\alpha t}). \quad (4)$$

It is seen that for small thicknesses the photoelectric current is proportional to the thickness or number of atoms per cubic centimeter. With increased thickness the curve passes through a maximum when

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon} \quad (5)$$

and apparently decreases exponentially with the intensity  $I' e^{-\epsilon t}$ .

In order to determine to what extent the above curves check this theory the discussion is for convenience divided into two parts, the initial absorption of light as a function of the thickness and the absorption of the photoelectrons as determined by that part of the curve showing the decrease of the photoelectric effect with its subsequent disappearance as the thickness increases.

#### ABSORPTION OF LIGHT WITH VARIATION IN THICKNESS.

If under the experimental conditions outlined above, the photoelectric current is represented for any thickness  $t$ , by the relation  $I_0(1 - e^{-\epsilon t})$  then this is equivalent to saying that the intensity of the electronic radiation from a unit area of the metal is always proportional to the number

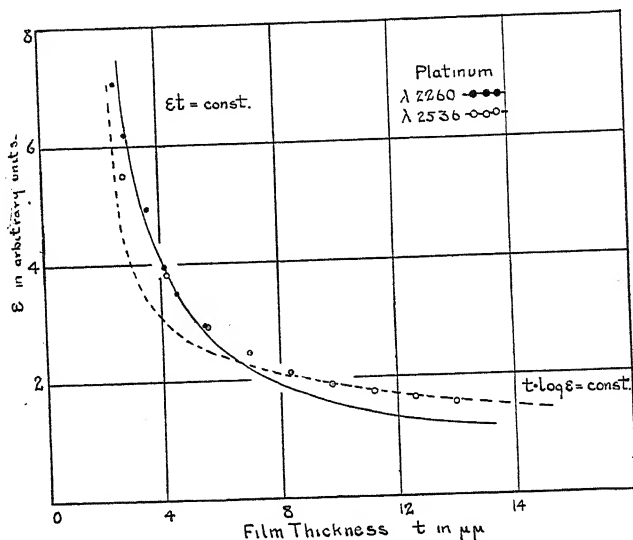


Fig. 6.

of atoms present in one cubic centimeter. The values of  $\epsilon$  therefore obtained from these curves would help to determine the validity of this relation. If the number of electrons liberated are proportional to the amount of light absorbed, and if the metal film is so thin that the absorption of the electrons as they pass through the successive layers of molecules can be neglected, then successive similar layers of the metal will absorb equal fractions of the amount of light energy they receive, provided no scattering takes place, and emit a quantity of electronic energy which is a measure of the light energy absorbed. Under these conditions we can measure the electronic energy and use it as a measure of the light energy absorbed. If then we let  $\epsilon$  represent this coefficient as measured

by the electronic results,  $I_0$  the intensity of the energy entering the slab of material,  $I$  the intensity leaving, then  $I = I_0 e^{-\epsilon t}$  where  $t$  is the thickness of the slab. This may be rewritten in the form

$$\epsilon = \frac{2.3}{t} \log \frac{I_0}{I}$$

If now  $\log I_0/I$  is plotted against  $t$ , the graph should be a straight line (for  $\epsilon$  constant) and  $\epsilon$  may be determined from the relation  $\epsilon = 2.3 \times$  slope.

The curves thus obtained were however not linear, they showed a continuous decrease in the coefficient  $\epsilon$  (Fig. 6) as the thickness increased. The coefficients were then computed from the photoelectric curves using the relation  $\epsilon = 0.693/D$  where  $e^{-0.693} = 0.5$  and  $D$  is the thickness which reduces the radiation to half value. Here  $1/\epsilon$  represents the distance to which the energy penetrates before its intensity is reduced  $1/e$  of its original value.

TABLE I.

Platinum, $\lambda 2301$ .			Silver, $\lambda 2301$ .		
$t$ .	$\epsilon$ .	$\epsilon t$ .	$t$ .	$\epsilon$ .	$\epsilon t$ .
1.40 $\mu\mu$	.990 $\mu\mu^{-1}$	1.39	1.70 $\mu\mu$	.393 $\mu\mu^{-1}$	.668
2.10	.660	1.39	2.54	.260	.660
2.70	.520	1.40	3.30	.206	.679
3.50	.396	1.38	4.25	.160	.680
4.20	.330	1.39	5.00	.135	.665
5.05	.283	1.43	5.77	.114	.657
5.90	.230	1.35	6.90	.100	.690

TABLE II.

Platinum,  $\lambda 2536$ .

$t$ .	$\epsilon$ .	$\epsilon t$ .	$t \log \epsilon$ .
2.8 $\mu\mu$	.550 $\mu\mu^{-1}$	.154	264
4.2	.381	.160	244
5.6	.291	.163	260
7.0	.247	.173	275
8.4	.211	.177	273
9.8	.190	.186	273
11.2	.176	.197	276
12.6	.165	.208	273
14.0	.157	.216	271

The above extended values give some idea of the general trend of the results. From the samples cited in Table I. it is seen how very close the product  $\epsilon t$  approaches a constant for small values of thickness. The

constancy of this product was found to hold, in the case of both platinum and silver for  $\lambda$  2260 and 2301, so long as the values thus obtained remained associated with very small values of  $t$ . When the thickness became larger  $\epsilon t$  no longer remained constant but increased with increasing values of  $t$ . A characteristic set of observations illustrating this condition is cited in Table II. If however, the relation  $t \log \epsilon$  is computed from the data it is found that this relation is constant for large values but not for small values of  $t$ . As an example the case of Pt  $\lambda$  2536 is cited, the change taking place at thickness  $7 \mu\mu$ .

One characteristic common to all these results is that  $\epsilon t$  seems constant under two circumstances, either for quite a range in values of  $t$  for large values of electronic velocities, excited by means of  $\lambda$  2260 or 2301, or for a very short range in the values of  $t$  (about  $7 \mu\mu$ ) when slower electronic velocities are used, as for example those excited by  $\lambda$  2536 or 3131. Although the curve for platinum  $\lambda$  3131 is shown in Fig. 4, yet the coefficients calculated from this curve were omitted owing to insufficient points on the curve to determine its initial slope. In addition the minimum wave-length sensibility for platinum is just inside the wave-length used so that the above curve must have been obtained through part of the shorter wave-lengths in the spectrum overlapping this region. The value for silver  $\lambda$  3131 has only been introduced to show the order of magnitude of the value  $\epsilon t$ .

TABLE III.

*Mean Values of  $\epsilon t = \text{const. for small values of } t$ .*

$\lambda$ .	Pt.	Ag.	Ratio.
2260	1.41	.681	2.07
2301	1.39	.671	2.07
2536	1.61	.983	1.64
3131		1.80	
Atomic weights	195.2	107.9	1.82

A summary of these results for values of  $t$  small, representing mean values similar to the extended data given above are shown in Table III. The last column gives the ratio of the values for this relation. It is of the same order of magnitude as the ratio of the atomic weights. We might conclude that the light energy absorbed per atom, is proportional to its atomic weight.

Before going into the possible origin of the above relations it may be of interest to examine the constancy of the relation  $\epsilon t$  further. The values of  $t$  were therefore plotted against values of  $1/\epsilon$  (Fig. 7). Under these conditions one would expect to get a straight line whose slope would

give a relative measure of the absorption of the light in passing through a molecular layer.

Since the amount of light energy absorbed is proportional to the photoelectric energy liberated it follows that the values of  $\epsilon$  determined for changes in thickness, are also measurements of the photoelectric characteristics. Under these conditions we may view the electron as liberated from its parent atom, passing through successive layers of molecules being deflected through collision with them but never entering the system of the atom to disappear through absorption.

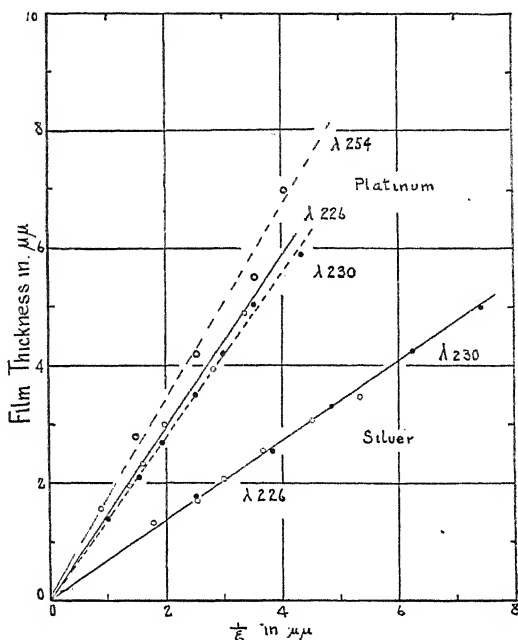


Fig. 7.

If therefore we consider two electrons, liberated with the same initial velocity, in the two metals, we will find after having passed through equal thicknesses of metal that the electron must have undergone a greater energy change per unit distance in passing through platinum than through silver. This ratio is, within experimental error, the same as the ratio of the densities of the two metals. This means that if equal masses are compared the amount of energy scattered is the same in the two cases. A similar relation exists for secondary radiation produced by Röntgen rays.<sup>1</sup> The above results are shown graphically in Fig. 7. In general they also seem to indicate that the greater the velocity of separation of

<sup>1</sup> Barkla, *Phil. Mag.*, 7, p. 543, 1904.

the electron from its parent atom, the less the scattering of its energy as it passes through neighboring atoms. The observations for platinum  $\lambda$  226 do not fall in line with this conclusion. This exception can however be attributed to experimental error. The results for Ag  $\lambda$  254 are here omitted so as to prevent too much crowding in the figure. The results for  $\lambda$  226 and 230 lie so close together that it is impossible to separate them in the graph, though the actual results from which the curve is compiled show that  $\lambda$  230 lies to the left of the results for the smaller wave-length.

If  $\epsilon$  represents the fraction of the total light energy lost through absorption and reappearing as electronic energy, in going through a sheet of metal of molecular thickness  $t$ ,  $I_0$  the intensity of the beam entering the sheet and  $I$  the intensity of the beam leaving it, then  $1/\epsilon$  the intensity coming out of the sheet is given by

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = kt, \quad (6)$$

where  $k$  is a constant. From which we get

$$I = I_0(1 - kt). \quad (7)$$

For larger values of  $t$  under the same circumstances it was found experimentally that

$$t \log \epsilon = \text{const.},$$

then

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = e^{-c/t}, \quad (8)$$

or

$$I = I_0(1 - e^{-c/t}). \quad (9)$$

These relations do not however conform to our simple theory advanced at the beginning of the paper which involved the assumption of an exponential absorption leading to equation (4).

#### THEORY OF SINGLE AND COMPOUND SCATTERING.

Rutherford<sup>1</sup> has advanced a theory of single scattering which leads to the conclusion that the proportion  $I/I_0$  of the rays which remain undeflected after passing through a thickness  $t$ , when  $t$  is small enough,<sup>2</sup> is given by the relation

$$I = I_0(1 - kt).$$

A relation deduced from the above experimental data for thicknesses less than  $7 \mu\mu$  (Fig. 6).

<sup>1</sup> Rutherford, Phil. Mag., V., 21, p. 684, 1911.

<sup>2</sup> Richardson, Electron Theory, p. 496.

For very small thicknesses we must therefore have electrons passing through the metal film, colliding according to Rutherford's hypothesis of "single scattering" and little or no energy is contributed to the atom through which the electrons passed. From this it is seen that the relation between  $I/I_0$  and  $t$  should be nearly linear in the initial stage, and this is found to be the case for all wave-lengths for both metals investigated.

If however the thickness penetrated by the electrons becomes greater a larger number of collisions necessarily results, and then the relation  $t \log \epsilon = \text{const}$  expresses the state of affairs. It seems as if the electron might be considered as at first moving in fairly straight lines with little deflection and little or no energy exchange, but as the penetration increases the deflexions become more numerous and violent resulting in the rate of expenditure of energy increasing so rapidly that the electron seems to come to a sudden stop.

This stage of the process seems to be equivalent to what Thomson<sup>1</sup> describes as "compound scattering." He showed that the probability of an average deflexion of an electron on passing through a metal sheet of thickness  $t$ , if less than a given angle, is equal to  $1 - e^{-c/t}$  where  $c$  is a constant for any particular substance, which under the present experimental circumstances means that the proportion  $I/I_0$  of the electrons undeflected is given by  $I = I_0(1 - e^{-c/t})$  a relation identical with the empirical equation (9).

The experimental arrangement and method therefore appears to fulfill the requirements for single scattering when the metal films are very thin (less than  $7 \mu\mu$ ), and compound scattering when the metallic film is thicker (Fig. 6).

In these experiments the angular opening  $\phi$  through which the electrons escape from the film is not varied, and if the same metal be examined so that the absorbing medium is also kept the same and the velocity of the electrons varied through change in wave-length of the illumination both Rutherford's and Thomson's theory leads to the relation,

$$\left[ \frac{mv^2}{t_0^{1/2}} \right] = \text{const.}, \quad (10)$$

where  $t_0$  is the value of the thickness required to cut down the radiation to half value.

This relation can be tested with film thicknesses in which single scattering takes place. In the experimental arrangement  $\phi$  is kept constant. Now the maximum energy of release of the electron is independent of the kind of atom and only depends on the frequency of the exciting light.

<sup>1</sup> Camb. Phil. Proc., 15, p. 465, 1910. See Richardson, Electron Theory, p. 493.

If therefore we consider values of  $1/\epsilon$  for equal values of  $t_0$  and  $\lambda$  in the two metals, we have a measure of the relative average energies with which the electrons appear after passing through equal thicknesses. This also gives us a comparison of the distances to which electrons penetrate so as to have their initial energies reduced to  $1/e$  of their original value. The results obtained from the data as shown in Fig. 7 indicate that equation (10) is valid within experimental error for film thickness less than  $10^{-7}$  cm.

An interesting and rather important change takes place after the electron penetrates further into the metal. In Fig. 6, we see an illustration of the thickness at which this occurs. We are here confronted with another illustration of the sudden change in the property of a metal when its thickness is less than  $10^{-7}$  cm. Similar changes occur in the optical and electrical properties of these metal films in this immediate vicinity.<sup>1</sup>

Unfortunately not enough data were available from individual results for  $\lambda$  2536, to show clearly that the smaller values of  $t$  followed the  $\epsilon t = \text{const.}$  relation (Fig. 6), so that the values for  $\lambda$  2260 were added to show how close the results fitted the theoretical curves, here shown as solid and broken lines. The latter results were therefore shifted so that the axes of the curve coincided with the axes of the curve for values  $\lambda$  2536. In this way the results show graphically just where the region of separation exists to which the above theories apply.

If we now consider thicknesses in which compound shattering exists we are limited on one side by the thickness comparable to  $7 \times 10^{-7}$  cm. and the maxima of the curves shown in Figs. 4 and 5. As seen from the sample data shown in Table II. the coefficient decreases less rapidly than the increase in thickness. This is to be expected, if the above theory can be applied; for the amount of scattering becomes more marked with decrease in speed and increase in thickness. This continues until a critical thickness is reached where the absorption of the number of electrons becomes predominant, finally leading to a decrease in the amount of emergence radiation.

The initial rise in the curves can therefore be divided into two distinct regions. For thicknesses less than  $10^{-7}$  cm. the velocities of the emergence electrons must be nearly equal or so nearly that very few if any electrons appear whose energy is less than that demanded by the Einstein relation. Here little if any absorption takes place so that the electron passes through subsequent layers with slight loss of energy. From here on however the number of electrons possessing less than this maximum energy decreases, they undergo frequent and violent collision

<sup>1</sup> Patterson, Phil. Mag., IV., p. 663, 1902. Minor, Ann. d. Phys., 10, p. 581, 1903.



with a rapid loss in energy until some electrons seem to come to a sudden stop. Further penetration is then accompanied by loss in numbers in addition to loss in energy.

#### ABSORPTION OF PHOTOELECTRONS.

Our theory as outlined at the beginning of the paper indicated a maximum for values of  $t$  given by the relation

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon}.$$

The values for  $t_m$  as picked off of the ionization curves are given in Table IV., Fig. 8. The values of  $t_m$  there indicated represent the distances to

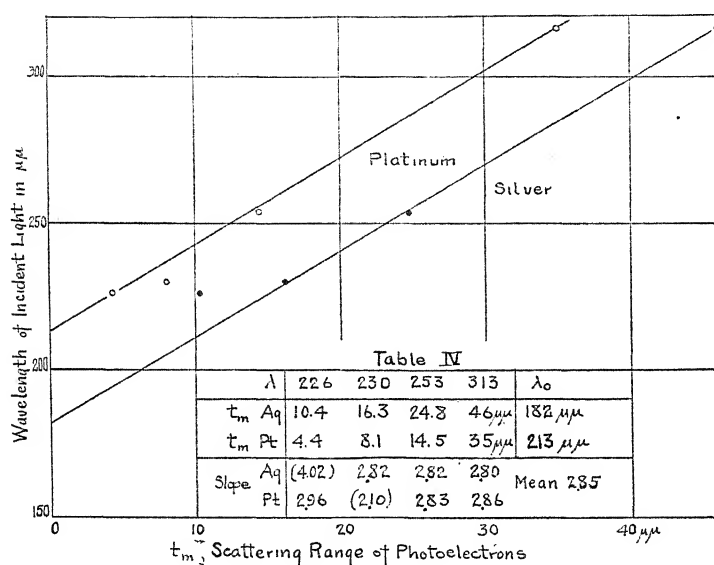


Fig. 8.

which an electron, starting from the lowest layer with initial energy  $\frac{1}{2}mv^2 = h\nu$ , can penetrate, before it disappears through absorption. The distance  $t_m$  is here called the scattering range. It does not measure the actual distance pursued by an electron in the form of the summation of all the zigzag paths, but the distance perpendicular to the surface of the metal. The values obtained in this way are plotted as a function of the wave-length of light causing the separation of the electron from its parent atom. The slope of the curve is the same for the two metals, the bracketed values in the table are however marked exceptions to this rule. The cause for this unusual difference cannot be accounted for through

experimental error. It seems likely however that the straight line relation is perhaps a first approximation to the truth and more numerous data determined for longer and shorter wave-lengths than here investigated will determine to a more exact degree the necessary relations.

The present curves however fit the relation  $\lambda - \lambda_0 = at_m$  to a marked degree. Here  $a$  is the slope of the curve and equal to 2.85,  $\lambda_0$  is the value of the minimum wave-length producing electrons possessing an effective

TABLE V.

$\lambda$ in $\mu\mu$ .	Velocity $\times 10^{-7}$ Cm. per Sec.				Mean Velocity.	$\lambda_0$ .
	226.	230.	253.	313.		
Silver . . . . .	14.5	14.0	13.3	13.5	$13.8 \times 10^7$	182 $\mu\mu$
Platinum . . . . .	14.1	13.8	13.2	13.4(?)	$13.6 \times 10^7$	213 $\mu\mu$
$v = \sqrt{\frac{2h\nu}{m}}$ . . . . .	14.05	13.95	13.25	11.92	$\times 10^7$ cm/sec	

scattering range equal to zero. This would mean that  $\lambda_0$  would be the minimum wave-length limit which could produce a photoelectric current under the above experimental conditions. Higher frequencies might liberate electrons, but their energies would be so great that they could penetrate the next atoms in their path and completely disappear in their system. These electrons would be absorbed so promptly that we could expect a metal to possess the property of selective absorption for electrons possessing these critical velocities. The minimum wave-lengths of the incident light energy producing electrons under these conditions are 213 and 182  $\mu\mu$  for platinum and silver respectively.

If however the above wave-length relation is reduced to its equivalent form we have

$$\frac{1}{v^2} - \frac{1}{v_0^2} = bt_m, \quad (11)$$

where  $b = 1.42m/hc$ , assuming the relation  $\frac{1}{2}mv^2 = h\nu$  to hold in the above case. We define  $m$  as the mass of the electron  $h$  as Planck's element of action and  $c$  as the velocity of light. Under these conditions  $v$  would be the velocity the electron must possess, to be able to penetrate a thickness  $t_m$ . Table V. shows the initial velocities the electrons should have, to penetrate thickness  $t_m$ , as computed from equation (11) and the velocities computed from the Einstein photoelectric equation. The contact difference of potential is here assumed equal to zero, since  $t_m$  is defined as the distance the electron travels in the metal without passing through the contact difference of potential layer. This use of the value

of  $t_m$  is however not rigidly true but owing to lack of further information it is offered as an approximate solution. We see in Table V. to what extent the values for the computed velocities check the experimental values. In all cases the velocity decreasing with increased wave-length of the exciting energy.

Now  $1/t_m$  may be defined as the stopping power. So that the stopping power of the metal increases as the energy of the electron increases, to eventually approach a limiting value. Comparing these values we note that the heavier the atom the greater the stopping power. If we therefore compare the distance traveled in platinum to that in silver for electrons of equal initial velocity we find that the distance traveled in silver is 1.17 as great as that in platinum when the energy in each is reduced to the same amount. Hence if we compare equal volumes of the material traversed we find that the ratio of the atomic volumes of silver to platinum is 1.13 or of the same magnitude as the relative distances traveled in the two metals. So that electrons after traversing equal masses have their initial energies reduced to the same extent.

#### ABSORPTION COEFFICIENTS.

After passing through a thickness  $t_m$  the photoelectric thickness curve falls off slowly at first, then nearly linear and finally more slowly again

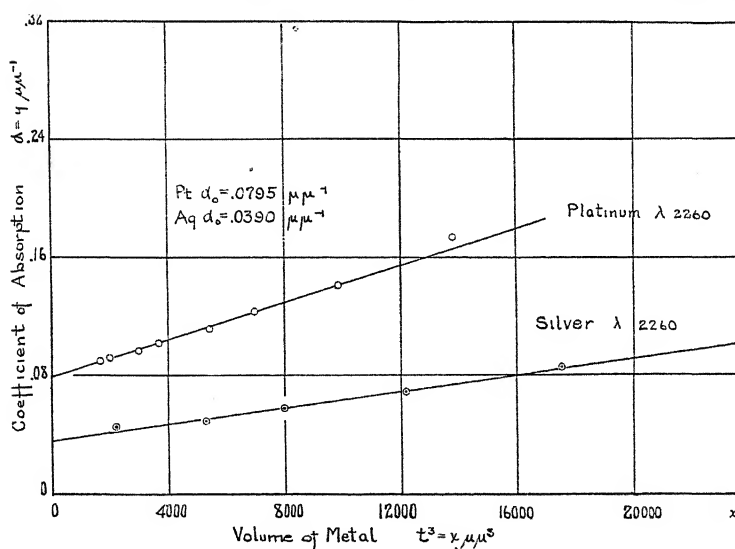


Fig. 9.

cutting the thickness axis at what appears to be a finite value. Unfortunately the latter part of the curve was very difficult to determine

owing to the very minute number of electrons coming through such thick metallic layers.

The initial drop of the curve seems to indicate that there is little absorption of electron numbers as yet and that scattering is still predominant. From here on however the curve takes a very definite form. The problem is a very complex one, the stream of photoelectrons which are traveling in any given direction must suffer loss in numbers both through true absorption and scattering to such a degree as to completely disappear from the original direction and in doing so must lose energy as well. What the relative importance of these actions is in the present case is difficult to tell. A partial solution is however attempted.

The coefficients of absorption  $\alpha$  were computed in the usual way for the absorption branch of the curve. These results were then plotted as ordinates and values of  $t$ , as abscissæ. The curve approximated nearest to a straight line when values of  $t^3$  were used as shown in Fig. 9. Thus the relation  $\alpha - \alpha_0 = mt^3$  would represent the relations to a comparative degree of accuracy limited by the experimental errors in the original observations. The values computed for  $\alpha$  and values for  $m$ , the slope of the curves, are given for comparison, to show to what degree of accuracy the results may be valid.

TABLE VI.

Platinum, $\lambda$ 226.			Silver, $\lambda$ 226.		
$\alpha$ .	$t$ .	$m$ .	$\alpha$ .	$t$ .	$m$ .
.167 $\mu\mu^{-1}\text{cm}^{-1}$	24.0 $\mu\mu$	63.0 $\mu\mu^{-3}\text{cm}^{-3}$	.099 $\mu\mu^{-1}\text{cm}^{-1}$	29.5 $\mu\mu$	23.4 $\mu\mu^{-3}\text{cm}^{-3}$
.141	21.5	62.8	.083	26.0	25.0
.123	19.2	62.3	.070	23.0	25.4
.112	17.5	61.6	.059	20.0	25.0
.102	15.5	61.9	.050	17.5	20.5
.0972	14.5	60.0	.043	12.8	20.0
.0917	12.6	61.2			
.0907	11.8	68.0			
	Mean	62.6 $\mu\mu^{-3}\text{cm}^{-3}$		Mean	23.2 $\mu\mu^{-3}\text{cm}^{-3}$

The coefficients of absorption thus obtained are not constant, but increase very rapidly with increasing values of thickness traversed. The intercepts on the  $y$  axis are however of interest. They would represent the absorption coefficients for values of thickness commensurable to a molecular layer. These values of  $\alpha_0$  for platinum and silver were found to be .0795 and .0390  $\mu\mu^{-1}$  respectively for photoelectrons emitted with a velocity of  $13 \times 10^7$  cm. per sec. from their parent atoms. If we divide

the above values by the density of the respective metals to which they apply we arrive at the following interesting relation

$$\left(\frac{\alpha_0}{\rho}\right)_{\text{Pt}} = 37,000 \text{ cm}^2\text{gm}^{-1},$$

$$\left(\frac{\alpha_0}{\rho}\right)_{\text{Ag}} = 37,100 \text{ cm}^2\text{gm}^{-1},$$

showing the mass absorption to be constant for  $\lambda 226$ . This is the "density law" of absorption at first suggested by Lenard.<sup>1</sup> Crowther,<sup>2</sup> experimenting with  $\beta$  rays from uranium showed that the "density law" did not hold for these larger velocities. Lenard experimenting with heterogeneous cathode rays arrived at the following values for the metals gold and silver here given for comparison.

$$\frac{\alpha_0}{\rho} = 2880 \text{ cm}^2\text{gm}^{-1},$$

$$\frac{\alpha_0}{\rho} = 3070 \text{ cm}^2\text{gm}^{-1}.$$

Compton and Ross working with sputtered platinum obtained from photoelectric measurements the average value  $\alpha/\rho = 28,900$ . While Partzsch and Hallwachs (*l. c.*), using some data supplied by Rubens and Ladenburg, obtained the value  $\alpha/\rho = 47,400$  for gold in the form of a leaf of about 100  $\mu\mu$  thickness.

The exigencies of the times have interrupted the completion of this subject at this time, it is hoped however that in the near future further data can be published which will support the above conclusions. The experimental part of this paper was performed at the University of Pennsylvania and I take great pleasure in acknowledging the ever generous coöperation of Prof. Goodspeed in furnishing the funds for the necessary apparatus.

<sup>1</sup> Lenard, *l. c.*

<sup>2</sup> Crowther, *Phil. Mag.*, 12, p. 379, 1906.

THE PHYSICS LABORATORY,  
STATE UNIVERSITY OF IOWA,  
IOWA CITY, IOWA.

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WALLACE CLEMENT SABINE.

WALLACE CLEMENT SABINE, Hollis Professor of Mathematics and Natural Philosophy at Harvard University and formerly dean of the Harvard Graduate Schools of Applied Science, Vice-President of the American Physical Society and recent member of the editorial board of the *PHYSICAL REVIEW*, died at Boston on the tenth day of January, 1919, in his fifty-first year.

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PROCEEDINGS  
OF THE  
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE PITTSBURGH MEETING, December 27-29, 1917.

THE ninety-second meeting of the American Physical Society (the nineteenth Annual meeting) was held in the Applied Science Building of the Carnegie Institute of Technology, December 27 to 29, 1917. This was a joint meeting with Section B (Physics) of the American Association for the Advancement of Science. Two sessions were held on each of the three days of the meeting, Thursday, Friday, and Saturday.

On the first two days the following program of research papers was presented. This program was in charge of the officers of the Physical Society.

The Optical Properties of Rubidium. J. B. NATHANSON.

A Preliminary Study of the Luminescence of the Uranyl Salts Under Cathode Ray Excitation. FRANCES G. WICK and LOUISE S. MCDOWELL.

Note on a Phosphorescent Calcite. E. L. NICHOLS and H. L. HOWES.

The Visibility of Radiation in the Blue End of the Visible Spectrum. L. W. HARTMAN.

An Improved Form of Mercury Vapor Air Pump. CHAS. T. KNIPP.

Heat Conductivity of Cerium. C. N. WENRICH and G. G. BECKNELL.

Temperature and Heat of Fusion. J. E. SIEBEL. (By title.)

Report on the Construction of Certain Mathematical Tables. C. E. VAN ORSTRAND.

Mobilities of Ions in Vapors. KIA-LOK YEN.

The Size and Shape of the Electron. ARTHUR H. COMPTON.

The Coefficient of Emission and Absorption of Photo-electrons from Platinum and Silver. OTTO STUHLMANN, JR.

Ionization and Excitation of Radiation by Electron Impact in Nitrogen. BERGEN DAVIS and F. S. GOUCHER.

Energy in Continuous X-Ray Spectra. C. T. ULREY.

An Experimental Investigation of the Characteristic X-Ray Emission from Molybdenum and Palladium. BENJAMIN A. WOOTEN.

Characteristic X-Ray Emission as a Function of the Applied Voltage. BERGEN DAVIS.

A Standard of Sound. (Demonstration.) CHAS. T. KNIPP.

The Air Damped Vibrating System: Theoretical Calibration of the Condenser Transmitter. I. B. CRANDALL.

A Photographic Method of Measuring the Velocity of Sound Waves near the Source of Sound. ARTHUR L. FOLEY. (By title.)

The Effect of Intensity and Distance on the Velocity of Sound. ARTHUR L. FOLEY. (By title.)

The Influence of the Pressure and Time Employed in Condensing a Dental Amalgam upon its Crushing Strength, at Temperatures Between 10° and 100° C. ARTHUR W. GRAY and PARIS T. CARLISLE.

Absorption in Paraffined Paper Condensers. L. PYLE.

An Interesting Case of Resonance in an Alternating Current Circuit. H. K. DODGE.

On Electromagnetic Induction and Relative Motion, II. S. J. BARNETT.

Eddy-current and Hysteresis Losses in Iron at High Frequencies. C. NUSBAUM.

Some Energy Transformations with Oscillatory Currents. E. F. NORTHROP.

The Effects produced upon Audion Characteristic Curves by Different Kinds of Signals (Buzzer, Electron Relay and 60-cycle A.C.) A. D. COLE.

Influence of a Series Spark on the Direct Current Corona. S. F. CROOKER. (By title.)

On the Effect of a Magnetic Field upon Cathode Rays. L. T. MORE and LOWELL M. ALEXANDER. (By title.)

The Physical Characteristics of X-Ray Fluorescent Intensifying Screens. MILLARD B. HODGSON.

Barometric Ripples. W. J. HUMPHREYS.

The Ultra-Violet and Visible Absorption Spectra of Phenolphthaleins. W. E. HOWE and K. S. GIBSON.

The Ultra-violet and Visible Absorption Spectra of Orcinolphthaleins. R. C. GIBBS, H. E. HOWE and E. P. T. TYNDALL.

The Ultra-Violet Absorption Spectra of Acetone. E. P. T. TYNDALL.

Thermal Conductivity of Metals. EDWIN H. HALL. (By title.)

A Mercury Manometer of High Sensibility. J. E. SHRADER.

A Simple Gauge for Very Low Pressures. J. E. SHRADER.

Resonance and Ionization Potentials for Electrons in Magnesium Vapor. PAUL D. FOOTE and F. L. MOHLER.

The Spectral Photoelectric Sensitivity of Molybdenite. W. W. COBLENTZ and M. B. LONG.

The program on Saturday was in charge of Section B, A.A.A.S. In the forenoon there was a symposium on "The Relationship of Physics to the War." Among the speakers were Lieut. G. P. Thomson, R.F.C. (son of Sir Joseph J. Thomson), Lieut. Giorgio Abeti of the Italian Mission to the United States, Dr. A. L. Day, Director of the Geophysical Laboratory, Washington, who gave an interesting account of the present optical-glass situation in this country and Major C. E. Mendenhall, who discussed the recent war activities of the National Research Council.

In the afternoon Professor Henry A. Bumstead delivered an address as



retiring Vice-president of Section B on "Present Tendencies in Theoretical Physics" (published in *Science*, January 18, 1918).

At a short business session of the Physical Society the result of the mail ballot for officers of the society for 1918 was announced. They are as follows: H. A. Bumstead, president; J. S. Ames, vice-president; D. C. Miller, secretary, and G. B. Pegram, treasurer. A. L. Day and G. F. Hull are the new members of the Council. (Former elected members holding over are C. E. Mendenhall, G. W. Stewart, I. Langmuir, G. B. Pegram, G. O. Squier and H. A. Wilson.) G. K. Burgess, A. C. Lunn and A. D. Cole are the newly elected members of the Editorial Board of the *PHYSICAL REVIEW*. (Former members of the board holding over are F. Bedell, Managing Editor, and E. P. Lewis, W. C. Sabine, A. Trowbridge, N. E. Dorsey, Wm. Duane and O. M. Stewart.)

The reports of the treasurer and managing editor were presented, accepted, and ordered printed. (These printed reports were afterward mailed to all the membership.)

The visiting members of the Society were much indebted to Dean F. L. Bishop of the University of Pittsburgh and to Prof. H. S. Hower, of the Carnegie Institute of Technology, and to other resident members for many courtesies extended to them.

ALFRED D. COLE,  
*Secretary for 1917.*

#### MINUTES OF THE BALTIMORE MEETING, DECEMBER 27 AND 28, 1918.

THE ninety-fifth meeting (the twentieth annual meeting) of the American Physical Society was held at Johns Hopkins University, in Baltimore, Maryland, on December 27 and 28, 1918, in affiliation with Section B—Physics of the American Association for the Advancement of Science. Professor Bumstead is now serving as Scientific Attaché to the American Embassy in London and his resignation as president of the society was accepted by the Council on November 30, 1918. The vice-president, J. S. Ames, thus became acting president, and he presided at the several sessions of the society and the council. The maximum attendance at the technical meetings was about one hundred, while eighty-eight members and visitors were present at the time of the business session.

On the afternoon of December 27 there were two sessions under the auspices of Section B, the presiding officer being the vice-president and chairman of the section, Major G. F. Hull. At two o'clock, p.m., the retiring vice-president and chairman W. J. Humphreys, gave an address on "Some Recent Contributions to the Physics of the Air." At five o'clock, p.m., Dr. George E. Hale gave an address before the entire Association on "The National Research Council."

The annual business meeting was held at eleven o'clock, a.m., on December 28, 1918. The revised form of the Constitution and By-Laws was unanimously adopted by letter ballot. The amendments do not alter the intent or purpose

of the constitution in the old form, except in one respect; the managing editor is made a member, *ex-officio*, of the council. The amended constitution will be published in the next printed list of members.

The following officers were elected for the year 1919: For President, J. S. Ames; for Vice-President, W. C. Sabine; for Secretary, D. C. Miller; for Treasurer, G. B. Pegram; for Members of the Council (four-year term), G. K. Burgess, J. C. McLennan; for Member of the Council (one-year, unexpired term), Max Mason; for members of the Board of Editors of the *PHYSICAL REVIEW*, Henry Crew, L. V. King, H. S. Uhler.

Colonel Millikan explained the purposes of the Smith-Howard Bill now before Congress, authorizing Federal coöperation with the States for the promotion of engineering and industrial research. After general discussion it was unanimously voted that the American Physical Society favors federal aid and coöperation with the several States in support of research in science and engineering and in industrial research. The society favors the creation of Boards of eminent scientists and engineers within each state for the administration of the funds appropriated for all research within the state.

The reports of the treasurer and of the managing editor of the *PHYSICAL REVIEW* were presented at the meeting of the council. It was ordered that, after auditing, these reports be printed and sent to all members.

The secretary reported that within the year sixty-two persons had been elected to associate membership and eight elected to regular membership. Eighteen associate members had been transferred to regular membership. There was one resignation from membership. Two members had died within the year, C. C. Trowbridge, and Ernest Weibel, the latter having been killed in action in France. The total membership now is 935, there being 402 regular members.

As a matter of record it is noted that, because of conditions growing out of the war, three meetings, which, following custom, would have been held in the year 1918, were cancelled. These meetings were the usual Washington meeting of April, a special joint meeting with the American Institute of Electrical Engineers which had been arranged for October 11 and 12, and the Chicago meeting of November 30. A special meeting of the council was held in Baltimore on November 30, 1918, to transact necessary business.

At a meeting of the Council held in Baltimore on November 30, 1918, the following elections were made: *elected to regular membership*, Shoji Nishikawa; *elected to associate membership*, W. F. Angell, H. C. Arnold, J. W. Cook, F. A. Ferguson, O. F. Gish, Peter Hidnert, Samuel C. Hoffman, Frank C. Hoyt, I. L. Jones, Lewis V. Judson, Herbert Kahler, Charles B. Kazda, Otto Koppius, M. B. Long, John P. Minton, Newell C. Page, Herbert H. Palmer, Arthur R. Payne, C. E. Pierce, Anthony S. Santos, C. D. Shallenberger, Joseph Valasek, Hugo P. Wahlin, Dorothy W. Weeks, W. Wubbe.

At a meeting of the Council held in Baltimore on December 27, 1918, the following elections were made: *elected to regular membership*, M. D. Hershey,

J. C. Hunsaker, Vladimir Karapetoff, Paul D. Merica, John M. Miller, H. B. Phillips; *elected to associate membership*, R. C. Colwell, Wm. B. Creagmile, G. A. Dasney, A. E. F. Fant, E. O. Hoffman, J. J. Hopfield, F. L. Hunt, George Lewis, H. J. Lockwood, C. W. D. Parsons, D. E. Sharp, Margery Simpson, Marguerite D. Tschaler, I. H. Van Horn, F. C. Wentz, C. F. Whitemore, D. T. Wilber; *transferred from associate to regular membership*, E. C. Crittenden, A. T. Jones, E. F. Kingsbury, Louise S. McDowell, Wm. F. Meggers, A. Norman Shaw, H. B. Williams.

At the morning sessions of December 27 and 28, 1918, thirty papers were presented as follows, four being read by title:

The Unique System of Units. W. W. STRONG.

A Simple Stretched Wire Dilatometer. ARTHUR W. GRAY.

Monochromatic and Neutral Tint Screens in Optical Pyrometry. W. E. FORSYTHE.

The Temperature, Pressure, and Density of the Atmosphere in the Region of Northern France. W. J. HUMPHREYS.

Refinements in Spherometry. G. W. MOFFITT.

A New Type of Hot Wire Anemometer. T. S. TAYLOR.

The Linear Thermal Expansion of Glass at High Temperatures. C. G. PETERS.

Some Characteristics of Glasses in the Annealing Range. A. Q. TOOL and J. VALASEK.

Striae in Optical Glass. L. E. DODD and A. R. PAYNE.

Preliminary Determination of the Thermal Expansion of Molybdenum. LLOYD W. SCHAD and PETER HIDNERT.

On the Characteristics of Electrically Operated Tuning Forks. H. M. DADOURIAN. (Read by title.)

Ionization and Resonance Potentials for Electrons in Vapors of Arsenic, Rubidium, and Caesium. PAUL D. FOOTE, O. ROGNLEY and F. L. MOHLER.

Absorption Coefficient of the Penetrating Radiation. OLIVER H. GISH.

Photoelectric Sensitivity vs. Current Rectification in Molybdenite. W. W. COBLENTZ and LOUISE S. MCDOWELL.

A Device for the Automatic Registration of the  $\alpha$ - and  $\beta$ -Particles and  $\gamma$ -Ray Pulses. ALOIS F. KOVARIK.

Note on the Distribution of Energy in the Visible Spectrum of a Cylindrical Acetylene Flame. EDW. P. HYDE, W. E. FORSYTHE and F. E. CADY.

Preliminary Note on the Luminescence of the Rare Earths. E. L. NICHOLS, D. T. WILBER and F. G. WICK.

On the Critical Absorption Frequencies of Chemical Elements of High Atomic Numbers. WILLIAM DUANE and TAKIO SHIMIZU.

Some Interesting Results of Eclipse Magnetic Observations. L. A. BAUER.

The Minimum Temperature at the Base of the Stratosphere. W. J. HUMPHREYS. (Read by title.)

Why Clouds Never Form in the Stratosphere. W. J. HUMPHREYS. (Read by title.)

- Speeds in Signaling by the Use of Light. W. E. FORSYTHE.  
Thermal Conductivity of Various Materials. T. S. TAYLOR.  
Further Observations on the Production of Metallic Spectra by Cathode Luminescence. EDNA CARTER and ARTHUR S. KING.  
Effect of Crystal Structure Upon Photoelectric Sensitivity. W. W. COBLENTZ. (Read by title.)  
A Mechanically Blown Wind Instrument. A. G. WEBSTER.  
The Dynamics of the Rifle Fired at the Shoulder. A. G. WEBSTER.  
Interior Ballistics, by a New Gun Indicator. A. G. WEBSTER.  
Residual Gases in Highly Exhausted Glass Bulbs. J. F. SHRADER.  
Silvering Quartz Fibers by Cathodic Sputtering. J. F. SHRADER.

DAYTON C. MILLER,  
*Secretary.*

### THE SPECTRAL PHOTOELECTRIC SENSITIVITY OF MOLYBDENITE.<sup>1</sup>

BY W. W. COBLENTZ AND M. B. LONG.

THE samples of molybdenite  $\text{MoS}_2$ , examined were thin folia, 0.01 to 0.05 mm. in thickness, soldered to copper electrodes.

In this examination, which is of a preliminary nature, the sample of molybdenite was exposed to radiations of equal energy (measured with a thermopile) of different narrow spectral regions, isolated by means of screens. These bands of spectral radiation, having their maximum intensity at wave-lengths  $\lambda_m = 0.5, 0.55, 0.6, 0.75, 0.9, 2.4, 4.4$  and  $8.5 \mu$  respectively, were obtained by transmission of the radiations from a Nernst glower through colored glasses, by the emission from the Bunsen gas flame and by utilizing residual radiations reflected from quartz.

The results obtained indicate that molybdenite, like selenium and stibnite, has its maximum photoelectric sensitivity just beyond the visual red,  $0.85 \mu$ ; but it differs from these substances in that it is quite sensitive to infra-red rays. For example, at  $2.0 \mu$  the photoelectric sensitivity is perhaps from one third to one half that of the maximum, and at  $4.4 \mu$  (Bunsen flame which has weak emission bands at  $1.8$  and  $2.7 \mu$ ) it is about one tenth the maximum.

The temperature coefficient of resistance of molybdenite is negative; and hence a rise in temperature produces a change in resistance which is in the same direction as that caused by the photoelectric effect. It will therefore be necessary to determine the magnitude of these two effects in order to establish a true photoelectric effect for wave-lengths greater than  $4.4 \mu$ . These various factors including reflection, etc., will be considered in the complete paper.

WASHINGTON, D. C.

December 22, 1917.

<sup>1</sup> Abstract of a paper presented at the Pittsburgh meeting of the American Physical Society, December 27-29, 1917.

THE UNIQUE SYSTEM OF UNITS.<sup>1</sup>

BY W. W. STRONG.

SOME thirty years ago it would have almost been universally admitted by physicists and chemists that all masses were integral multiples of the masses of the atoms constituting the given mass. The masses of the atoms were universal invariants and an atomic system of units of this kind has been proposed. The effort of Prout was that of making a unique and universal atomic unit of mass that could be used wherever matter is present.

Today it is almost universally assumed that the charge of the electron is a unit of electrical charge in much the same way as the atomic unit of mass in the system of Prout. The variable mass of the electron as its velocity changes indicates that if there is an invariable element of mass it is much smaller than Prout imagined and that it cannot be larger than that quantum of mass that appears when the electron is accelerated. Let us assume that there are such invariant quanta of electrical charge and mass. If some five such invariant quantities as we may consider as applying to electrical charge, mass quanta and radiant energy (with a supposed constancy of the velocity of light) were consistent with phenomena we would be approaching a unique set of units. Let us then state some of the properties of such an ideal system of units.

1. There are in nature certain invariant atomic quantities that serve as the basis for a unique and a universal system of units that is absolute in reality. At the present time the electron (unit) is usually made to play such a rôle.

2. The science of measurement is not then based upon a complicated and an artificial process of creating and comparing standards and in defining terms such as the coincidence of two events or of comparing two lengths in relative motion but upon a unique set of natural and invariant units that are universally available.

3. If there is an ether it appears logical to assume that this set of unique units should be provided for us by the structure of the ether or at least as many of them as are necessary to fully describe all phenomena relating to the ether, the electric, magnetic and gravitational fields. Derived units such as  $T$  and  $L$  should be based on an ether structure since these are used in treating the propagation of light.

4. The unique set of units are made so by all phenomena being consistent with the assumption of their being unique and hence their use is supported by all scientific knowledge. If all electrons are alike there is no necessity of defining equal charges of electricity as is done in connection with the derivation of the electrostatic system of units. If there is an elementary quantum of mass there is no necessity to consider the matter of relative motion in the definition of mass. How can a consistent system of units be obtained by assuming the Newtonian mechanics in the definition of our units and the de-

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

scription of phenomena and then trying to build a relativity theory on another system of mechanics and at the same time using the old system of units?

5. The derived units should be uniquely determined by the fundamental or natural unique units.

6. It hardly seems possible to develop a consistent system of physics without at first determining a system of units that is universally applicable to all phenomena to be investigated.

S. I. E. M. Co.,  
MECHANICSBURG, PA.

#### A SIMPLE STRETCHED WIRE DILATOMETER.<sup>1</sup>

BY ARTHUR W. GRAY.

AT the request of Dr. Arthur L. Day, Optical Glass Committee, War Industries Board, the writer undertook to determine the thermal expansivity of some optical glass which had been produced under the direction of the former for naval searchlight mirrors. The decision as to whether or not the glass was suitable rested exclusively upon its thermal expansivity, a knowledge of which was desired up to about 200° C. With the dilatometer which had to be hastily constructed for the purpose, expansions were, however, measured at various temperatures up to 300° C.

The apparatus, which uses the stretched wire method of measuring linear displacements,<sup>2</sup> could easily be adapted for commercial testing. It differs from the precision apparatus developed by the writer at the Bureau of Standards, mainly in the substitution of an ordinary Brown and Sharpe micrometer screw for the expensive microscope comparator and reference bar.

Extending vertically downwards from the cover of the temperature bath are two tubes, the axes of which are separated by approximately the length of the specimen under investigation. At the lower end of each tube are two pointed co-axial screws for clamping the specimen from opposite sides. One of the tubes is rigidly attached to the cover, the other is hinged so as to permit free expansion of the specimen. A fine wire, of annealed tungsten, attached to the lower end of each tube extends vertically upwards through the tube and the cover for several decimeters above the latter, where it passes over a pulley and supports a weight which keeps it tight and straight. The wire bears firmly against the rounded end of the specimen and likewise against a round

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

<sup>2</sup> Expansion measurements by this method were reported to the Physical Society at the Washington meeting in December, 1911, and at various times since then. The principle was also described in a communication to the Washington Academy of Sciences, in which the two arrangements that form the basis of the Bureau of Standards' present expansivity equipment were illustrated diagrammatically. (A. W. Gray, "New Methods for Displacement Measurements and Temperature Uniformity Applied to the Determination of Linear Expansivity," *Journ. Wash. Acad. Sc.*, 2, 248-258, 1912.) One arrangement is used with electric furnaces, the other with liquid baths.

pin situated a decimeter or so below the pulley. By means of a micrometer screw and suitable guides this pin is movable horizontally in the plane of the two wires, so that their upper ends can be brought closer together or separated a measured distance. Just above the cover of the temperature bath is placed a horizontal bar of low and known thermal expansivity, protected as far as possible from temperature changes accompanying warming and cooling of the bath below.

After the specimen is placed in the temperature bath, the upper end of each stretched wire is moved until the wire just touches the end of the bar above the cover. Contact is indicated electrically by means of an inexpensive reflecting galvanometer in series with a dry cell and a suitable resistance. As the specimen under investigation expands, it moves the wires away from the contact bar. The magnitude of the expansion is determined from the movements of the micrometer screws required to restore contact.

Only one graduated screw is actually used. An ordinary screw of moderately fine pitch moves a horizontal bar to which are rigidly attached both the pin that communicates the motion to the left-hand wire *A* and the nut of the graduated screw which moves the right-hand wire *B*. In this way a single micrometer screw is made to measure the total displacement of both wires.

The accuracy of the apparatus just described is limited mainly by the accuracy with which contact can be detected between the stretched wires and the bar above the temperature bath. The effect of errors in the measuring screw varies inversely as the ratio of the distances from the contact bar to the pins for moving the wires and to the specimen. In the apparatus actually used these pins were 600 mm. above and the specimen 150 mm. below the bar, so that the micrometer screw measured four times the expansion of the specimen. The results showed that with proper use deviations of individual screw readings from the average should not exceed 5 microns. The average deviation of 107 individual readings from the average reading for each temperature was less than 2 microns, corresponding to an average deviation of 0.5 micron in determining the expansion of the specimen.

The apparatus actually constructed as outlined above illustrates merely one particular arrangement for applying the general principles of the method. It was hastily designed and built to meet an emergency. Various details could obviously be modified. For example, two independent graduated screws for moving the stretched wires *A* and *B* would be a convenience; any sensitive detector of contact is usable; tungsten wires are not necessary, etc. Moreover, micrometer screws can be arranged to replace microscopes for use with either an air-bath (as in an electric furnace) or a liquid bath; so that the range of application is not limited to the interval between room temperature and 300° C.

PHYSICAL RESEARCH LABORATORY,  
THE L. D. CAULK COMPANY,  
MILFORD, DELAWARE,  
Nov. 29, 1918.



MONOCHROMATIC AND NEUTRAL TINT SCREENS IN OPTICAL PYROMETRY.<sup>1</sup>

By W. E. FORSYTHE.

IT is very difficult to find a monochromatic or a neutral tint glass screen for optical pyrometry. Very fortunately absolutely monochromatic or absolutely neutral tint screens are not necessary for such work. For the so-called monochromatic screen what is wanted, for the most part, is a screen that is sufficiently monochromatic to enable comparisons in brightness to be made between the comparison source, or pyrometer filament for the Morse pyrometer, and the source that is being investigated.

When it is necessary to use glass absorbing screens to reduce the apparent brightness of the source studied, the main requirement is to have a screen that is nearly enough neutral tint to permit comparisons in brightness to be made. The degree to which it is necessary for the absorbing screen to have a spectral transmission independent of the wave-length, depends upon the so-called monochromatic glass used. It is quite evident that if the colored glass used is absolutely monochromatic, any absorbing glass would answer.

With two pieces (thickness about 6 mm.) of Jena red glass no. 4512 in the eyepiece of the pyrometer the total transmission, which amounted to about 1.5 per cent., of a piece of noviweld absorbing glass was measured for a particular temperature by two observers whose visibility curves are quite different and the two results agreed to within a fraction of a per cent. If a red glass is used, by total transmission is meant the ratio of the brightness observed through the red glass and the black glass to the brightness observed through the red glass alone. If the entire visible spectrum is used, it is generally very hard to make such measurements owing to the color differences involved with even the best absorbing glasses. If, however, a good red glass is used in the eyepiece, transmission measurements can easily be made.

The transmission of the absorbing glass when used with a red glass can be calculated for any black body distribution by the following formula taken from Preston's theory of light:

$$(1) \quad T_B = \frac{\int J_\lambda V_\lambda T'_R T'_B d\lambda}{\int J_\lambda V_\lambda T'_R d\lambda}.$$

$J_\lambda d\lambda$  = black body energy for interval  $\lambda$  to  $\lambda + d\lambda$ ,  $V_\lambda$  = visibility,  $T'_R$  and  $T'_B$  = spectral transmission of red and absorbing glasses respectively. An absorbing glass that is not neutral tint is sometimes used to cut down the apparent brightness of a source when measuring the temperature with an optical pyrometer having a red glass in the eyepiece. It has been stated<sup>2</sup> that if an absorbing glass which is not neutral tint is used in connection with the red glass, the effective wave-length to be used is different from the effective wave-length that is to be used with a sector. In what follows it is shown that

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

<sup>2</sup> Foote, Bull. Bureau of Standards, Vol. 12, p. 491.



such is not the case but that the same effective wave-length is to be used in each case. Suppose that a sector with a transmission  $T_s$  were found such that the brightness observed through the sector would equal that observed through the black glass using the same red glass in both cases, that is

$$\int J_\lambda V_\lambda T'_B T'_R d\lambda = \int J_\lambda V_\lambda T'_s T'_R d\lambda = T_s \int J_\lambda V_\lambda T'_R d\lambda.$$

The question to be considered is what effective wave-length is to be used in calculating the temperature of the source whose brightness is thus measured. When the brightness is measured using the rotating sector the temperature  $T_2$  is calculated from  $T_1$  the temperature corresponding to the pyrometer reading if no sector is used by means of the formula

$$(2) \quad 1/T_2 - 1/T_1 = \frac{\lambda_e \log T_s}{c_2 \log e},$$

when  $T_s$  = transmission of sector  $\lambda_e$  the ordinary effective wave-length between  $T_1$  and  $T_2$ . The effective wave-length has been defined as the wave-length such that

$$\left[ \frac{J(\lambda, T_1)}{J(\lambda, T_2)} \right]_{\lambda_e} = \frac{\int J(\lambda, T_1) V_\lambda T'_R d\lambda}{\int J(\lambda, T_2) V_\lambda T'_R d\lambda}.$$

$T_s$  = transmission of sector. Suppose that using the black glass screen before the pyrometer lamp the same current  $A$  through the pyrometer filament is required to cause an apparent brightness match between the same source at temperature  $T_2$  and the pyrometer filament. This means that the sector and glass have the same transmission. The temperature  $T_2$  can be calculated by an equation similar to equation (2).

$$(3) \quad 1/T_2 - 1/T_1 = \frac{\lambda'_e \log T_B}{c_2 \log e}.$$

$T_B$  = transmission of black glass screen and  $\lambda'_e$  an effective wave-length. As  $T_1$  and  $T_2$  are the same in each case

$$\lambda_e \log T_s = \lambda'_e \log T_B,$$

or

$$\lambda_e = \lambda'_e,$$

that is, as the transmission of the black glass given by equation (1) is the same as would be obtained by comparing the transmission of the glass with a sector the same wave-length is to be used in each case.

NELA RESEARCH LABORATORY,  
NATIONAL LAMP WORKS OF GENERAL ELECTRIC COMPANY,  
NELA PARK, CLEVELAND, OHIO,  
December, 1918.

THE TEMPERATURE, PRESSURE AND DENSITY OF THE ATMOSPHERE IN THE  
REGION OF NORTHERN FRANCE.<sup>1</sup>

BY W. J. HUMPHREYS.

THE records of 416 sounding balloon flights made at Trappes, Uccle, Strassburg and Munich during the years 1900-1912 have been grouped in such manner as to give the average summer and winter distribution of temperature, humidity, pressure and density from the surface up to 20 kilometers above sea level. The temperatures have further been grouped according to weather conditions.

All this data is given in brief tables and shown in graphs.

A NEW TYPE OF HOT WIRE ANEMOMETER.<sup>1</sup>

BY T. S. TAYLOR.

IN all hot wire anemometers thus far used, the instruments have been so designed as to measure the average velocity over the area of the pipe or for a length of 10 cm. or greater at different points in the plane of cross section. None of these types is suitable for the measurement of velocity in small tubes. It was for the purpose of making these latter measurements, that the present instrument was constructed. It consists of a platinum heating wire 0.007 inch in diameter and one half to one inch long, stretched across a suitable framework, say of glass, and having a copper-constantan thermocouple made of .002 inch wire attached at its mid point. The instrument can be used satisfactorily by measuring the current that is necessary to maintain the temperature, as determined by the thermocouple, say 300° C., above the temperature of the gas in which it is placed. This method facilitates the measurements greatly, and avoids the possibility of accidentally burning out the wire. As for the other types of anemometers, it is necessary to calibrate it under the conditions under which it is to be used. This can be done by revolving it at various speeds in the gas whose velocity is to be measured at various temperatures. To do the latter it is necessary to have the revolving part of the calibration device contained in an oven whose temperature can be regulated quite accurately.

By using this method it is possible to measure quite accurately the velocity of gases at various temperatures, and has the great advantage of really measuring the velocity of gas at a point.

A second type, constructed after the usual one, has also been used in exactly the same manner as the above. It determines the average velocity over a length of from one to three millimeters. It is constructed by having two small .002-inch platinum wires attached about 2 millimeters apart near the middle of the .007-inch platinum heating wire. In calibrating and using it, the cur-

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

rent is measured that is required to maintain a certain potential drop, say 60 millivolts between the two potential leads. A good portable potentiometer such as Leeds and Northrup's potentiometer indicator is suitable both for the temperature measurements in the first type and the potential drop in the second type. The current can be measured quite satisfactorily by use of a good ammeter of proper range. To be sure more accurate results can be obtained when the measurements of both current and potential or thermal electromotive force are made by means of a precision potentiometer.

A decided advantage of the thermocouple type is that it enables one to determine the temperature of the gas directly, immediately before or after taking velocity measurements. This can be done also for the second type by the insertion of a separate thermocouple near the resistance wire or from the temperature coefficient of the platinum wire.

December 3, 1918.

#### THE LINEAR THERMAL EXPANSION OF GLASS AT HIGH TEMPERATURES.<sup>1</sup>

BY C. G. PETERS.

THE thermal expansions of different kinds of optical glass, chemical glass-ware, and ordinary glass tubing, were measured by the Fizeau method. Rings cut from the samples were placed between two fused quartz interferometer plates and heated in an electric furnace from 20° to 650° C. Curves with degrees Centigrade plotted against increase in length show that the rate of expansion of each sample was nearly constant until the annealing temperature was reached. This annealing temperature varied with different glasses from 450° to 550° C. After the annealing temperature was passed the glass seemed to be in a slightly plastic condition and the rate of expansion increased by four to six times. This rapid expansion continued for about 100° when a softening point was reached. From the curves the temperature where careful annealing is required can be accurately located.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.

#### SOME CHARACTERISTICS OF GLASSES IN THE ANNEALING RANGE.<sup>1</sup>

BY A. Q. TOOL AND J. VALASEK.

A STUDY has been made of the behavior of a number of glasses near their softening temperatures in order to obtain data that would serve to indicate the proper annealing process, as well as the maximum temperatures at which the various glasses should be annealed.

The "relaxation times" for the stresses were determined by two methods, the first being by the bending of a glass strip as described by Twyman and the

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

second by observing the rate at which the double refracting power of a sample of strained glass disappeared. This relaxation time  $T$ , defined by the equation

$$\frac{dF}{dt} = E \frac{ds}{dt} - \frac{F}{T}$$

due to Maxwell, decreases with increasing temperature in such a way that

$$T_{\theta} = T_0 e^{(\theta_0 - \theta)/k}.$$

The magnitude of the constant  $k$  was found to depend on the nature of the glass. For optical glasses it was found to range from 10 to 13 for temperatures expressed in degrees Centigrade.

In the temperature range where the glass softens to such a degree that  $T$  becomes only a few tenths of an hour, it was found that heating curves, obtained by the use of a differential couple, showed a relative cooling of the glasses with respect to the neutral body. Cooling curves show a relative heating.

Measurements by C. G. Peters, of this bureau, have shown that in this same range there is an abnormal increase in the expansion coefficient. Consequently it is to be suspected that the softening of the glass, the increased expansion and this apparent cooling effect are closely related.

HIGH TEMPERATURE MEASUREMENT SECTION,  
BUREAU OF STANDARDS,  
WASHINGTON, D. C.

#### PRELIMINARY DETERMINATION OF THE THERMAL EXPANSION OF MOLYBDENUM.<sup>1</sup>

BY LLOYD W. SCHAD AND PETER HIDNERT.

THE thermal expansion of an exceptionally pure specimen of molybdenum was determined from  $-142^{\circ}$  to  $+305^{\circ}$  C.

A short description of the apparatus and of the method used in obtaining high and low temperatures is given.

The results are shown in the form of tables, from which were computed by the method of least squares, the two following empirical equations which satisfy the observations:

$$L_t = L_0 (1 + 5.15t \times 10^{-6} + 0.00570t^2 \times 10^{-6})$$

and

$$L_t = L_0 (1 + 5.01t \times 10^{-6} + 0.00138t^2 \times 10^{-6}),$$

where  $L_t$  is the length of the specimen at any temperature  $t$  within the proper range; in the first case  $19^{\circ}$  to  $-142^{\circ}$  C. and in the second case  $19^{\circ}$  to  $+305^{\circ}$  C. The probable error of the length computed from these equations is less than  $3 \times 10^{-6}$  per unit length.

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918. Complete paper may be found in the Scientific Papers of the Bureau of Standards, No. 332.

SPEEDS IN SIGNALING BY THE USE OF LIGHT.<sup>1</sup>

BY W. E. FORSYTHE.

IN connection with some other work it was necessary to have some data concerning the maximum speed of light signaling as limited by the inertia of the eye and to compare this with the speeds attainable with various types of lamps.

A preliminary investigation was carried out to determine the relative lengths of dot, space and dash necessary for the greatest speed. Three observers made readings for this test and all agreed very well with the following ratio for the maximum speed:

dot: dash: space:: 1: 4: 3.

To test out the speed for a sharp cut off of the signals, using this ratio for dot, dash and space, a sector was made with which the following signals could be shown ( space, of course, follows each dot or dash):

- 1—dot dash dot dash
- 2—dash dot dot dash
- 3—dash dot dot
- 4—dash dot dash dot
- 5—dash dash dot dot
- 6—dot dash dash dot
- 7—dot dash dash

This sector was mounted so that the observers saw the light source through a small opening. The observer was shown a series of twelve signals for each of a series of speeds beginning at a speed so low that there was no trouble in reading all the signals. This was repeated for higher and higher speeds until the observer was unable to read the signals.

To test out the speed where the heating and cooling of the different lamps was a factor, a commutator was constructed that would turn the lamps on and then off so that for any particular speed the voltage was applied to the lamp for the dot and dash the same length of time that the flashes of light were visible with the sector. Three observers made comparative readings on the signals as given by the sector and the commutator using a tungsten lamp with a ribbon filament mounted in argon. The results for each observer were about the same for either method.

As almost the same results were obtained from the lamp as from the sector this lamp, as a standard, was compared with a vacuum ribbon lamp and a gas-filled wire lamp. The lamps were so mounted as to have the same intensity in the direction of the observer stationed at a distance of 100 or 170 yards. Three observers took part in these tests and the results without exception were in favor of the gas-filled ribbon lamp.

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

As all the above tests were carried out at a comparatively short distance it was thought well to make a test at a much larger distance. To do this the gas-filled ribbon lamp was mounted in a signaling unit which gave about a  $5^\circ$  spread and two observers made readings on signals sent as above at a distance of about 2,700 yards, both in the daytime and also at night. Assuming a breakdown at the speed at which two of the twelve signals shown were missed, one observer was able to read the signals on a clear day when the four-part signals were shown at such a speed that the time from the beginning to the end of the signal was about 1.6 seconds. This observer made about the same record at night. The other observer was able to read the signals on a clear day when the time from the beginning to the end of the signal was 2.2 seconds and at night his record was the same as that of the first observer.

A word might be said about how the signals appeared to the observer at the breakdown point. Each observer was instructed to record what he saw, not by code letter, but by actually writing down the dots and dashes. A study of these records shows that on clear days the breakdown seemed to be due to the observer's inability to see initial dots, finally seeing only the dashes. At night at the distance at which observations were made, the light was very intense and at the breakdown point the observers reported that they did not see the spaces between the light flashes. The light just seemed to flicker.

NELA RESEARCH LABORATORY,  
NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,  
NELA PARK, CLEVELAND, OHIO,  
December, 1918.

#### THERMAL CONDUCTIVITY OF VARIOUS MATERIALS.<sup>1</sup>

By T. S. TAYLOR.

THE present investigation was undertaken to determine the thermal conductivity of various materials used in certain manufacturing processes. It was found that the direct method was by far the most satisfactory. This consisted of measuring the quantity of heat that passed through a given thickness of the material between two equi-temperature surfaces and the temperature drop through the sample. The hot equi-temperature surface or source consisted of a soapstone heater about 9 inches in diameter and  $1/2$  inch thick. It was composed of two discs each having a heating element wound in a spiral groove cut in its surface. The two discs were cemented together with faces having heating wire adjacent. Potential leads were brought out so that the potential drop could be determined for a determined area (about 4 sq. in.) surrounding the center of the heater. The cool equi-temperature surfaces were reservoirs through which water was kept circulating. The heat generated in the heater passed laterally through the samples to the cold reservoirs. The temperature drop through the samples was measured by means of small

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

copper constant thermocouples inserted on each side of the sample. Extra turns of wire being wound around the outer edge of the heater made it possible to have the surface of the heater a very constant temperature source. Consequently from the amount of heat developed per unit area of the heater, the thickness of the sample and the temperature drop through the sample the thermal conductivity of the material is readily calculated.

By this method the conductivity of a large number of materials has been determined. The following list contains some of the results.

Material.	Thick- ness, Inches.	Direction of Heat Flow.	Temp.	Cal./cm. °C./sec.	Temp. Coef.
Hard rubber . . . . .	0.380	Transversely.	25-50	0.000380	.0015
White fibr. . . . .	0.383	"	20-80	0.000695	.0012
<i>Woods:</i>					
White pine . . . . .	0.519	Across grain.	20-120	0.000255	.0015
" " . . . . .	0.732	Along grain.	30-80	0.000613	.0020
Maple . . . . .	0.508	Across grain.	20-80	0.000434	.0006
" " . . . . .	0.785	Along grain.	20-80	0.001037	.0008
White oak . . . . .	0.516	Across grain.	20-80	0.000455	.0018
" " . . . . .	0.754	Along grain.	40-70	0.000944	.0006
Plate Glass . . . . .	0.252	Transversely.	20-100	0.00195	.0018
" " . . . . .	0.289	"	20-120	0.00201	.0012
Soapstone . . . . .	0.715	"	70-130	0.00800	
Sheet steel .0172 inch . . . . .	0.416	Transversely.	20-80	0.00158	.0025
Same painted with asphalt . . . . .	0.425	"	20-80	0.00625	.0009
Wool felt dark gray . . . . .	0.98	"	40-100	0.000175	.0076
Solid graphite . . . . .	1.04	"	50-130	0.110	.0012
<i>Powdered graphite:</i>					
I. through 20- on to 40-mesh . . . . .	0.476	"	40-100	0.00320	.0048
II. through 40-mesh screen . . . . .	0.476	"	40-100	0.00100	.0040
III. through 100 . . . . .	0.476	"	40-110	0.000482	.0034
<i>Lamp Black:</i>					
Eagle brand, Germantown . . . . .	0.476	"	40-150	0.000166	.0006
Sil-O-Cel Brick . . . . .	0.977	"	30-150	0.000262	.0015

The values as recorded are given in each case for the average temperature over which the observations extended. Thus for white pine the average temperature extended from 20° C. to 120° C. The conductivity as recorded is therefore for 70° C. The temperature coefficient of thermal conductivity was determined from the observations over the range of temperature in question.

One fact that is noticeable in particular is the amount the transverse thermal conductivity of iron sheets can be increased by painting the component sheets when building up the sample.

FURTHER OBSERVATIONS ON THE PRODUCTION OF METALLIC SPECTRA BY CATHODE LUMINESCENCE.<sup>1</sup>

BY EDNA CARTER AND ARTHUR S. KING.

IN continuation of previous experiments, the spectra of titanium, manganese, iron, magnesium, calcium and cadmium have been examined. The metal was vaporized by the impact of cathode rays in a high vacuum and the spectrum of the vapor in the cathode stream observed. The later apparatus was constructed largely of quartz in order to withstand the considerable heating of certain parts.

The experiments have shown the possibility of obtaining by this method fairly rich spectra even for metals of high melting point. Thus for titanium about 140 lines were identified, and 150 for manganese. In the detailed publication a comparison will be made with the spectra of other sources. In general, the majority of lines in the luminescence spectrum are those easily excited in the arc and furnace, but peculiarities appear which mark this as a source having special characteristics, and the action appears to be materially different for different vapors. For titanium the spectrum closely resembles that of the furnace, the enhanced lines being absent, while with cadmium, magnesium, and calcium there is a marked tendency to show enhanced lines. A relatively high intensity of lines in the ultra-violet was noted.

In the regular arrangement, the vapor whose spectrum was photographed was out of the path of the current, the anode being at one side of the chamber. When the anticathode was used as the anode, a much more intense spectrum was obtained, but for the elements tested, no decided difference in relative intensity of lines was observed. A discharge of this kind gave about 120 lines in the iron spectrum, the relative intensities among these lines showing many features of interest. Certain groups are strongly emitted, some of these being characteristic of low temperature conditions, while others require in other sources an intense excitation. In further work, the discharge conditions governing the appearance of certain classes of lines will be more fully tested.

VASSAR COLLEGE,  
MOUNT WILSON SOLAR OBSERVATORY,  
December, 1918.

RESIDUAL GASES IN HIGHLY EXHAUSTED GLASS BULBS.<sup>1</sup>

BY J. E. SHRADER.

THE investigation of high vacua has been extended by making observations on the effect of the sealing off of highly exhausted glass bulbs, and noting the subsequent change in pressure with time over an extended period.

In this work the mercury diffusion pump and the Knudsen Absolute gauge were employed. The glass ware was cleaned with potassium bichromate

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.



solution followed by thorough rinsing with hot water to remove soluble salts. Heat treatment of the bulbs and gauges in an electric oven at  $500^{\circ}\text{C}$ . for two hours sufficed to remove gases and vapors from the glass so that pressures of the order of  $5 \times 10^{-8}$  mm. Hg. could be obtained.

On sealing off the bulbs and gauges by melting the glass at a constriction there is a comparatively large evolution of gases and vapors which can be partly removed by pumping during the time of sealing off, but the pressure in the bulbs is always four or five times as large as the pressure before sealing off. For several days after sealing off, there is a comparatively rapid increase in pressure, after which the change in pressure is slower and is proportional to the time.

After the exhausted bulbs had stood for about two months they were subjected to further heat treatment at successively higher temperatures and the change in pressure was noted after each heat treatment. Reheating at temperatures below  $250^{\circ}\text{C}$ . resulted in only slight increase in pressure while higher temperatures produced increasingly larger changes in pressure.

#### A DEVICE FOR THE AUTOMATIC REGISTRATION OF THE $\alpha$ - AND $\beta$ -PARTICLES AND $\gamma$ -RAY PULSES.<sup>1</sup>

BY ALOIS F. KOVARIK.

THE detection of individual  $\alpha$ - or  $\beta$ -particles has been in all cases dependent on the visual or audible method. These methods are quite trying on the nerves, because close attention is necessary on account of the probability law of emission and long periods of counting are essential because the method is a statistical one. The method described in this paper makes it possible to get a record of individual particles on a chronograph paper, permitting a study of it at convenience, and thus enhancing the accuracy of the statistical method. The method consists in amplifying the current to the needle-point electrode in the counting chamber by means of a three electrode vacuum tube amplifier and using this current to operate a very sensitive relay which in turn operates a local battery circuit which operates the electromagnet on a chronograph. The writer wishes to express his sincere thanks to the Western Electric Co. for the fine spirit of coöperation in making him a loan of both the sensitive relay and the amplifier.

The accuracy of the method was tested (1) for the  $\alpha$ -particles, by checking the count by the scintillation method as in previous work, (2) for the  $\beta$ -particles, by determining their number by a measurement of the charge carried by the  $\beta$ -particles and (3) by getting a record of a large number of  $\beta$ -particles emitted over a long time and finding the number emitted in a given interval of time and checking up, or rather, in the case of the  $\beta$ -particles, proving the application of the law of probability to the emission of the  $\beta$ -particles. To

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

the writer's knowledge, this is the first automatic record of any random distribution which could be readily applied to test the probability law.

A record of the  $\gamma$ -rays is also obtained. The writer is not ready, at present to say that it is the  $\gamma$ -ray pulses which are registered although evidence seems to point that way. The law of inverse square distance (absorption in air for penetrating  $\gamma$ -rays neglected) is verified.

SHEFFIELD SCIENTIFIC SCHOOL,  
YALE UNIVERSITY,  
NEW HAVEN, CONN.,  
November 23, 1918.

#### PHOTOELECTRIC SENSITIVITY *vs.* CURRENT RECTIFICATION IN MOLYBDENITE.<sup>1</sup>

BY W. W. COBLENTZ AND LOUISE S. McDOWELL.

IN the course of an investigation of the spectral photoelectric sensitivity of molybdenite<sup>2</sup> it was found that samples which are sensitive photoelectrically have a considerably higher electrical resistance (100 to 1,000 times greater) than samples which are not sensitive to light. Moreover, the conductivity of the insensitive samples was found to be quite independent of the direction in which the current passed (lengthwise) through the crystal.

On the other hand, samples of molybdenite which are sensitive photoelectrically were found to possess a much higher conductivity when the electric current (from a 4-volt battery) was passed in one direction than when it was passed in the opposite direction through the crystal. This difference in current leakage ("dark current") as dependent upon the direction of the current through the crystal, varied from 10 to 30 per cent., for different crystals. The photoelectric current was therefore proportionately increased by connecting the crystal into the battery circuit in the proper direction to obtain the maximum conductivity.

It was, therefore, of interest to determine whether any relation exists between photoelectric sensitivity and the rectifying action which occurs when the crystal is placed in a high frequency oscillating circuit.

For this purpose crystals (size 10 by 4 by 0.1 mm.) were selected which showed one or two small photoelectrically sensitive spots, but which were quite insensitive throughout the remainder (of the surface) of the crystal.

The rectification tests were made by means of an ordinary tuned buzzer circuit. The telephone and detector, in series, were placed in parallel with the condenser of the secondary circuit. The coupling was adjusted to produce strong oscillations in the secondary circuit. The molybdenite crystal was explored for rectification by touching the surfaces by means of a fine metal point.

The results of these tests show that the low-resistance, photoelectrically-

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 27, 1918.

<sup>2</sup> *PHYS. REV.*, 11, p. 497, 1918.

insensitive samples of molybdenite are far more efficient rectifiers than the high-resistance, light-sensitive specimens. One such specimen, which showed no photoelectric sensitiveness whatever, as determined by a sensitive galvanometer, produced decided rectification over almost its entire surface. In fact it was difficult to find a spot which produced no rectification. On the other hand, the samples of molybdenite, having a high resistance exhibited the rectifying action only in spots, which usually did not coincide with the photoelectrically sensitive spots. These radio-electrically sensitive spots were widely scattered over the surface, but sometimes comparatively large areas were found which showed no electrical rectification, as indicated by the telephone. Only the most sensitive spots of the high resistance material produced a rectification approaching that of the specimens having a low resistance.

In two instances, especially electrically sensitive spots were found upon the light-sensitive areas, but other, equally sensitive spots occurred in the areas which were photoelectrically insensitive.

The conclusions to be drawn are therefore that: (1) the low-resistance, photoelectrically-insensitive molybdenite exhibits much greater rectifying action than the high-resistance, photoelectrically-sensitive specimens; (2) there is no apparent relation between the electrical rectification and the photoelectric sensitivity observed in molybdenite.

BUREAU OF STANDARDS,  
November 9, 1918.

#### ABSORPTION COEFFICIENT OF THE PENETRATING RADIATION.<sup>1</sup>

BY OLIVER H. GISH.

IT is stated several times in the literature that the "penetrating radiation of the atmosphere" has much greater penetrating power than the hardest  $\gamma$ -rays but no attempts to measure this directly have been made previously.

The following table gives values obtained indirectly from Kohlhörster's observations.

*Absorption Coefficient in Air (Computed).*

Altitude, <i>m.</i>	Absorption Coef. ( $\mu$ ) (Computed) ( $\text{Cm.}^{-1}$ ).	Absorption Coef. of $\gamma$ -Rays in Air Divided by $\mu$ .	Observer.
— — — —	$1.10 \times 10^{-5}$	4.8	Kohlhörster
111- 686	1.53 "	3.4	Gish
3861-9071	0.772 "	6.8	"
— — — —	0.746 "	7.0	E. v. Schweidler

Using a Wulf type electroscope in the form of a vertical cylinder and adding concentric cylinders of lead at intervals the following results were obtained.

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

*Absorption Coefficient in Lead (Measured).*

Altitude, m.	Absorption Coef. ( $\mu$ ) (Measured) (Cm. <sup>-1</sup> ).	Absorption Coef. $\gamma$ -Rays in Lead + $\mu$ .	Place.
359	1.22	0.41	Lincoln, Nebr.
1857	0.82	0.61	Colorado Spr's, Colo.
4308	0.45	1.11	Pike's Peak

The values in the first table are less than those found for gamma-rays ( $6.0 \times 10^{-5}$ ) while those in the second table are (except for the last) greater than the coefficient of absorption of gamma-rays in lead (0.50). It should however be said that in the latter case the values obtained are too large due to the method of computing them. Both tables are however significant in that they show a decrease in the absorption coefficient with altitude. In view of this it seems probable that the "upper component" of the penetrating radiation is more penetrating than gamma-rays.

WESTINGHOUSE RESEARCH LABORATORY,  
EAST PITTSBURGH, PA.

#### IONIZATION AND RESONANCE POTENTIALS FOR ELECTRONS IN VAPORS OF ARSENIC, RUBIDIUM, AND CÆSIUM.

BY PAUL D. FOOTE, O. ROGNLEY AND F. L. MOHLER.

THE study of electron currents in metallic vapors has been extended to arsenic, rubidium and cæsium. The resonance potential or potential of inelastic impact and ionization potential were measured by the method used in previous work.

The results with arsenic gave the resonance potential at 4.7 volts and ionization at 11.5 volts. As these values are close to the corresponding potentials for mercury of 4.9 volts and 10.3 volts the experiment was repeated with all possible precautions with similar results. A wehnelt discharge showed no visible arc with a current of 30 milliamperes but when some mercury was added to the arsenic the arc was visible with a current of .02 milliampere. In the case of metallic vapors so far studied the quantum relation  $Ve = h\nu$  shows these potentials to be related to the frequencies of the first line and limit of a prominent series in the spectrum of the metal. No such relation can be found in the case of arsenic but little is known of series relations in arsenic.

The potentials found may predict a series with its first line and limit near  $\lambda = 2,620$  and  $\lambda = 1,070$  respectively.

In rubidium vapor the resonance potential is 1.6 volts and the ionization potential 4.1 volts, in cæsium 1.48 volts and 3.9 volts respectively. Tate and Foote<sup>2</sup> have shown that in the case of sodium and potassium these potentials are determined from the quantum relation  $Ve = h\nu$  by the frequencies of the first line and limit of the principal series.

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

<sup>2</sup> Phil. Mag., 36, p. 75, 1918.

These series in the case of rubidium and caesium are widely separated doublets. Foote and Mohler<sup>1</sup> found in the case of thallium that the short wave-length line of the doublet determined the value of the resonance potential. The accompanying table shows that the series relations of rubidium and caesium are analogous to sodium and potassium. The agreement is well within experimental error if we take as in the case of thallium the short wave-length line of the doublet.

*Resonance and Ionization Potentials for the Alkali Metals.*

Metal.	$\lambda(\text{\AA})$ ( $\nu = 1.5 \times 10^8 - 2 \times 10^8$ ).	Resonance Potential Volts.		$\lambda(\text{\AA})$ ( $\nu = 1.5 \times 10^8$ ).	Ionization Potential Volts.	
		Computed.	Observed.		Computed.	Observed.
Lithium.....	6707.85	1.839	—	2299.67	5.363	—
Sodium.....	{ 5895.94 5889.97 }	{ 2.092 2.094 }	2.12	2412.63	5.112	5.13
Potassium.....	{ 7699.01 7664.94 }	{ 1.602 1.609 }	1.55	2856.69	4.318	4.1
Rubidium.....	{ 7947.64 7800.29 }	{ 1.552 1.581 }	1.6	2968.40	4.155	4.1
Caesium.....	{ 8943.46 8521.12 }	{ 1.379 1.447 }	1.48	3184.28	3.873	3.9

BUREAU OF STANDARDS,  
WASHINGTON, D. C.  
December 9, 1918.

#### NOTE ON THE DISTRIBUTION OF ENERGY IN THE VISIBLE SPECTRUM OF A CYLINDRICAL ACETYLENE FLAME.<sup>2</sup>

By EDW. P. HYDE, W. E. FORSYTHE AND F. E. CADY.

KNOWLEDGE of the distribution of energy in the visible spectrum of an acetylene flame has become important within the last few years through the use of this flame, in cylindrical form, in investigations of the visibility of radiation. It can be shown by computation that the data on acetylene published by Coblenz form a curve in the visible spectrum which will not agree with that of a black body at any temperature to better than 7 or 8 per cent. As this would mean that no color match could be obtained and as previous experience of the authors had led to the conclusion that the energy curve of acetylene differed in shape from that of a black body only in the extreme red, a short investigation was undertaken to verify this conclusion.

Tungsten lamps whose current color-temperature relation was carefully determined in this laboratory, were sent to the Eastman Kodak Company and to the Bureau of Standards with the request that they be compared with the

<sup>1</sup> PHYS. REV., 11, p. 487, 1918.

<sup>2</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

acetylene flame and the current for color match be found. The results gave an average value of  $2360^{\circ} \text{K} \pm 10^{\circ} \text{K}$ ., and neither laboratory reported any difficulty in obtaining a match in color. However the Bureau of Standards reported a difference amounting to about  $75^{\circ} \text{K}$ . between the flame as given by the Eastman standard burner and that given by the "Crescent Aero" burner, the latter being higher.

The spectral distribution of the flame was measured by means of a spectrophotometer and a spectral-pyrometer and the results gave a curve agreeing within the limits of error with that of a black body at  $2360^{\circ} \text{K}$ . In the extreme red, beyond  $0.70 \mu$  there was indication of a higher emissivity for the acetylene. A photographic method gave results corroborating those just mentioned.

A test of the sensibility of the color-match method to show differences in the spectral energy curve, showed that if two spectral curves matched at  $0.5 \mu$  and  $0.7 \mu$  and differed by as little as 4 per cent. in the middle of the spectrum, the two light sources could not be made to match in color.

In conclusion it is recommended that the relative emission intensities of a cylindrical acetylene flame, at least for that type represented specifically by the Eastman standard burner and for the wave-length interval from  $0.4 \mu$  to  $0.7 \mu$  should be taken as identical with those of a black body at  $2360^{\circ} \text{K}$ .

NELA RESEARCH LABORATORY,

NATIONAL LAMP WORKS OF GENERAL ELECTRIC CO.,

NELA PARK, CLEVELAND, OHIO,

December, 1918.

#### PRELIMINARY NOTE ON THE LUMINESCENCE OF THE RARE EARTHS.<sup>1</sup>

BY E. L. NICHOLS, D. T. WILBER AND F. G. WICK.

THE rare earths in solid solution in  $\text{CaO}$  and certain other media exhibit fluorescence and absorption of special interest on account of the narrow bands in their spectra.

By experiments now in progress we have established the following points:

1. All bands visible when luminescence is excited by the iron spark coincide precisely with the bands of kathodo-luminescence. It is probable therefore that, as in the case of the uranyl salts, willemite, etc., the location of bands is independent of the mode of excitation.
2. When excited at the temperature of liquid air, these spectra undergo a marked narrowing of the bands and, particularly in the blue and violet, broad hazy bands are resolved into numerous line like components.
3. At low temperatures the bands usually ascribed to erbium in the blue appear to be greatly enhanced whereas the bands due to samarium, chiefly in the red, are weakened. In preparations containing both of these elements, the change in the color of fluorescence thus produced is very striking.
4. Since bands due to several elements are present in all preparations thus

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

far made, the search for series is greatly complicated. There is evidence however of the occurrence of constant frequency intervals, as in the case of all other fluorescent substances thus far examined.

ON THE CRITICAL ABSORPTION FREQUENCIES OF CHEMICAL ELEMENTS OF HIGH ATOMIC NUMBERS.<sup>1</sup>

BY WILLIAM DUANE AND TAKEO SHIMIZU.

IN several papers<sup>2</sup> presented to the American Physical Society accounts have been given of measurements of the critical X-ray absorption frequencies of some of the chemical elements. In making these measurements an X-ray spectrometer with a calcite crystal has been employed. Before striking the crystal the X-rays passed through two narrow slits and after reflection through a third slit in front of the ionization chamber, which was wide enough to admit the entire reflected beam of X-rays into the chamber. This arrangement makes a large correction for the absorption of the X-rays by the crystal unnecessary. The critical absorption wave-lengths were obtained by drawing curves representing the ionization currents as functions of the angle made by the crystal planes with the X-ray beams (the grazing angles of incidence). If a thin sheet of a chemical element (or of one of its compounds) is placed in the path of the X-rays, a marked break in the curve appears at a certain point. The ionization current on the high frequency side of this break is much smaller than on the low frequency side, indicating a substantial increase in the absorption of the shorter X-rays by the chemical element. The formula

$$\lambda = 6.056 \sin \theta \times 10^{-8} \text{ cm.} \quad (1)$$

gives the wave-length  $\lambda$ , in which the double grazing angle is obtained by measuring from the center of the sharp drop in the ionization curve on one side of the zero line of the spectrometer to that on the other side.

The papers above referred to contain the results of experiments on chemical elements up to and including cerium, with an atomic number of 58. The present paper deals with an extension of the measurements to chemical elements of atomic numbers higher than 58. We used the same apparatus as in the previous researches with the single exception that the current exciting the X-ray tube came from a high tension transformer (with a system of kenotrons and condensers, etc., to give approximately a constant difference of potential), instead of from the high tension storage battery previously employed. With this apparatus we have measured the critical absorption frequencies for a num-

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society December 28, 1918.

<sup>2</sup> The Critical Absorption of Some of the Elements for High Frequency X-rays, F. C. Blake and William Duane, *PHYS. REV.*, December, 1917. On the Relation between the K X-ray series and the Atomic Numbers of the Chemical Elements, William Duane and Kang-Fuh Hu, *PHYS. REV.*, June, 1918. On the Critical Absorption and Characteristic Emission X-ray Frequencies, William Duane and Kang-Fuh Hu, *PHYS. REV.*, June, 1918.



ber of chemical elements up to mercury (atomic number 80). The lantern slides shown indicate that the square roots of these frequencies plotted against the atomic numbers depart from the straight line law to a still greater extent than the measurements previously recorded for chemical elements of lower atomic numbers.

In one of the papers referred to it was shown that the velocity  $v$  of the electron required to produce the  $K$  characteristic X-rays, calculated from the simple formula

$$h\nu = \frac{1}{2} \frac{mv^2}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad (2)$$

is a linear function of the atomic number  $N$ , it being given by the equation

$$v = 0.00678c(N - 3/2) \quad (3)$$

to within about 1/5 per cent. up to the atomic number 58. As indicated by the lantern slide shown the departure from this law for chemical elements up to mercury ( $N = 80$ ) amounts to only 1 per cent.

We have plotted on the same diagram the results of the interesting experiments recently published by de Broglie. He used a photographic method for measuring the critical absorption wave-lengths. The correction for the absorption by the crystal in this method does not appear to be quite clear, especially as the correction must depend upon the finite size of the crystal. As a matter of fact the velocities we have calculated from his data by equation (2) fall about as far above the straight line represented by equation (3) as our values do below it.

In dealing with these chemical elements of high atomic numbers we must remember that the velocities of the electrons required to produce their characteristic X-rays exceed half the velocity of light, and that several formulas have been proposed for the electron's kinetic energy. The velocities calculated by these formulas differ from each other (in the case of mercury for instance) by several per cent. We postpone a discussion of these points, however, until the critical frequencies for radium, thorium and uranium have been obtained.

#### SOME INTERESTING RESULTS OF ECLIPSE MAGNETIC OBSERVATIONS.<sup>1</sup>

BY L. A. BAUER.

BECAUSE of the bearing of a definite detection of magnetic effects during solar eclipses upon theories of magnetic variations, the Department of Terrestrial Magnetism of the Carnegie Institution of Washington made special arrangements for magnetic and allied observations during the solar eclipse of June 8, 1918. In the program of work, there participated magnetic observa-

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.



ories and universities distributed over the entire zone of visibility, from China and Japan to the Atlantic seaboard. There were thus obtained magnetic data from about twenty-five stations.

The results gained confirm the general conclusions reached from similar observations made in accordance with the author's scheme of observation during various eclipses since 1900. It may now be regarded as definitely proved that during a solar eclipse the earth's magnetic field passes through a minute oscillation, the period of which approximates that of the eclipse and the range of which, as compared with that of the solar diurnal magnetic variation, is approximately proportional to the amount of sunlight and other solar radiations cut off from the earth by the moon.

The character of the magnetic effect during the eclipse is in general the reverse of that which occurs during the sunlit hours of the day. Its main characteristic features occur from station to station not according to absolute time, nor to local time but according to eclipse time, as of course should be the case if the effect is attributable to the eclipse.

The range of the effect on the magnetic declination is about 0.1 of that of the solar-diurnal variation, or about one minute of arc for our latitudes. The maximum effect on the strength of the earth's magnetic field is about 0.03 per cent., or equivalent to the effect found by the author for a ten per cent. change in the solar radiation as revealed by changes in the solar-constant values observed at Mt. Wilson by the Smithsonian Institution.

The mathematical analysis of the eclipse magnetic variation may throw light upon the causes of other variations of the earth's magnetic field, notably of those attributed to the sun and moon. Fuller publication will be found in the *Journal of Terrestrial Magnetism and Atmospheric Electricity*, Vols. 23 and 24.

#### WHY CLOUDS NEVER FORM IN THE STRATOSPHERE.<sup>1</sup>

BY W. J. HUMPHREYS.

IT is obvious from the occurrence of cirrus clouds at the base of the stratosphere that at this level the atmosphere frequently is fully saturated. We should, therefore, expect the stratosphere, even if wholly free from vertical convection, to become humid through long continued diffusion. Nevertheless clouds do not form in it even under the influence of rapid drops in temperature of 10° C. or more. It must, therefore, have a humidity far below saturation. And to this condition there are at least two contributing causes.

(a) In the absence of vertical convection the vapor pressure must rapidly decrease, under the influence of gravity, with elevation, just as does the pressure of the atmosphere as a whole. Hence, at anything like constant temperature, the stratosphere, even if saturated at the base, must rapidly become very dry with increase of elevation. But this is not sufficient to account for the total absence of clouds at these levels.

<sup>1</sup> Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

(b) The lower portion of the stratosphere normally has a minimum temperature during the passage of each anticyclone which obviously prevents water vapor from diffusing upward to a greater density at any level than that of saturation at this low temperature. The little vapor that does diffuse a short ways upward during the few days prevalence of any milder temperatures, such as accompany cyclonic conditions, is precipitated on the passage—never long delayed—of the next anticyclone. In this way the stratosphere is kept exceedingly dry, and clear skies instead of hazy made the rule for anticyclonic weather.

#### THE MINIMUM TEMPERATURE AT THE BASE OF THE STRATOSPHERE.<sup>1</sup>

BY W. J. HUMPHREYS.

NUMEROUS records obtained by sounding balloons have shown conclusively that, on the average, the temperature at the base of the stratosphere is lower than at any other level, above or below.

As this is not in accord with a state of thermal equilibrium one naturally looks for the cause in some type of forced convection.

An analysis of the records shows that this minimum temperature is especially pronounced in anticyclonic regions, or where the barometric pressure is high. But in these regions the chief flow of the atmosphere is towards lower latitudes. Hence it lags, or loses some of its eastward velocity, and thereby interferes with the flow of the prevailing winds from the west and causes them to rise, as over a mountain barrier, to unwonted heights, and thus to cool to abnormally low temperatures.

The base of the stratosphere recovers somewhat from its minimum temperature after the passage of an anticyclone, but only to be again dynamically cooled below the point of radiation equilibrium by the next anticyclone, and so on indefinitely.

On the average, therefore, the lower portion of the stratosphere has a mechanically enforced minimum temperature more or less below the point of radiation equilibrium.

#### STRIÆ IN OPTICAL GLASS.<sup>1</sup>

BY L. E. DODD AND A. R. PAYNE.

VARIOUS types of striæ in optical glass, classified on the basis of visible appearance, are described, and photographs shown. Certain of these types are then discussed from the standpoint of their nature and probable origin. A relationship between gas bubbles in the glass and striæ is pointed out. The application of pressure to the molten glass is advocated as a preventive, not only of bubbles, but also of those striæ associated with bubbles. Another probable source of the more numerous striæ is given as the separation

<sup>1</sup> Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

of lead or its oxide, in the lead glasses. Striae having such origin, however, are more localized, near the walls and bottom of the pot, and may be reduced to a minimum in number by proper heat treatment and stirring. It is thought that striae in the more central part of the glass mass in the pot, which are regarded as being mostly associated with bubbles, may be largely prevented by application of pressure, and that thereby the per cent. of good glass may be considerably raised from the present amount of 20 or 25 per cent. of the total melt. The experimental application of pressure is proposed as a special research, primarily of commercial value, but indirectly of scientific value by its possibility of opening the way to the manufacture of the larger lenses free from defect.

BUREAU OF STANDARDS,  
PITTSBURGH BRANCH.

#### EFFECT OF CRYSTAL STRUCTURE UPON PHOTOELECTRIC SENSITIVITY.<sup>1</sup>

By W. W. COBLENTZ.

IN a previous investigation<sup>2</sup> it was observed that plates of silver sulphide which had been prepared in the laboratory and subjected to hammering, were rather insensitive photoelectrically. On the other hand, the natural mineral acanthite,  $\text{Ag}_2\text{S}$  is sensitive photoelectrically. The sample tested was a spindle-shaped rod about 8 mm. long, irregular in outline, and consisting principally of three crystals. Wishing to produce a wider receiving surface for spectral photoelectric examination, the central crystal was flattened by hammering. It was then found that this flattened crystal was quite insensitive photoelectrically, whereas, the rest of the material remained sensitive. Evidently the change in internal configuration of the crystal had some effect upon the photoelectric sensitivity. It is, therefore, proposed to observe the spectral photoelectrical sensitivity of the sample before and after mechanical manipulation.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.,  
December 27, 1918.

#### SILVERING QUARTZ FIBERS BY CATHODIC SPUTTERING.<sup>1</sup>

By J. E. SHRADER.

THE device by Williams<sup>3</sup> for silvering quartz fibers has been so modified that the measurement of the resistance of the fibers can be measured in situ.

Sputtering of fibers has been done with D.C. from generators, rectified A.C. from a transformer, and A.C. from a transformer rectified only by the sputter-

<sup>1</sup> Abstract of a paper presented (by title) at the Baltimore meeting of the American Physical Society, December 27, 1918.

<sup>2</sup> Bull. Bur. Standards, 14, p. 591, 1918.

<sup>3</sup> PHYS. REV., 4, pp. 517, December, 1914.

ing device itself. The sputtering was done in hydrogen at different voltages and current densities, while the resistance was measured for varying lengths of time of sputtering.

After silvering the resistance of the fiber has been observed to change with time and this change has been measured over a short period of time.

#### ON THE CHARACTERISTICS OF ELECTRICALLY OPERATED TUNING FORKS.<sup>1</sup>

By H. M. DADOURIAN.

THE communication presents the results of a series of experiments performed to determine the conditions affecting the frequency of electrical tuning forks. Detailed descriptions of the experiments will be published in an early issue of THE PHYSICAL REVIEW.

PALMER PHYSICAL LABORATORY,  
PRINCETON UNIVERSITY,  
December 10, 1918.

#### A MECHANICALLY BLOWN WIND INSTRUMENT.<sup>1</sup>

By A. G. WEBSTER.

THE principle employed is that of the squeaky faucet or fluttering safety-valve. A spring of variable tension holds the valve in place and the proper pressure causes a puff of air, which generates a sound wave in the horn, which on reflexion arrives at the valve in the proper phase to maintain vibration. The theory, combined with the author's theory of acoustical impedance, leads to a new boundary problem in partial differential equations.

#### THE DYNAMICS OF THE RIFLE FIRED AT THE SHOULDER.<sup>1</sup>

By A. G. WEBSTER.

A RECORD is made on a smoked glass attached to a rifle in the act of being fired. From this, the acceleration of the gun may be calculated and thus the elastic properties of the shoulder determined. Experiments are made with various rifles, including the Browning Machine Gun.

#### INTERIOR BALLISTICS, BY A NEW GUN INDICATOR.<sup>1</sup>

By A. G. WEBSTER.

A N indicator is described in which the stiffness of the spring is so great and the damping so perfect that oscillations due to the apparatus are eliminated. Results obtained regarding the burning of the powder are described.

<sup>1</sup> Abstract of a paper presented at the Baltimore meeting of the American Physical Society, December 28, 1918.

## THE PHYSICAL REVIEW.

### THE STRUCTURE OF RADIOACTIVE ELEMENTS.

BY INGO W. D. HACKH.

THE progress of knowledge in chemical and physical sciences has come mainly through the interpretation of experimental facts. Many phenomena of modern physical research can only be explained by theories of the structure of atoms. Thus a branch of "sub-atomic" physics and chemistry has been created. The majority of papers dealing with the structure of the atom deal with the elements of lighter atomic weight, while the elements of highest atomic weight which exhibit the phenomena of radioactivity seem to be neglected. Nevertheless these later elements offer a more substantial basis for theories concerning the constitution of atoms, because, by their disintegration some of the building stones of the atoms are revealed.

In further developing the periodic system of the chemical elements<sup>1</sup> the following scheme was devised for the series of the radioactive elements, by which the change in their relative electromotive force and valency during the different stages of disintegration is brought in harmony with the total charge of the respective substance. This scheme not only offers proof of the completeness of the uranium-radium series, but also points to a theory of "metastasic" electrons and incidentally establishes temporarily structure formula and logical symbols for the radioactive substances.

According to the current interpretation of disintegration the radioactive elements exhibiting beta-radiation, discharge an electron, and become relatively more negative by an increase in valency and corresponding change of their position in the periodic system.

Thus, *e. g.*, tetravalent uranium  $\alpha'$ , an isotope of thorium in group 4, loses an electron and becomes pentavalent uranium  $\alpha''$  or brevium<sup>2</sup> occupying a position in group 5, which is thus more electro-negative. This very short lived substance loses another electron and is transformed

<sup>1</sup> J. Am. Chem. Soc., 40, 1023, 1918. Am. J. of Science, 46, p. 481, 1918.

<sup>2</sup> Fajans, Phys. Z., 14, 877, 1913. See footnote, Ber., 46, 3492, 1913.

into the more stable hexavalent uranium 2 of group 6. Therefore with each loss of an electron (or negative charge) the valency has increased and the relative electromotive force of the substance has become more electro-negative. According to the conception of the conservation of energy the *total* negative charge of uranium 2 must be less than the total negative charge of uranium  $x''$  or uranium  $x'$ , although the electromotive force of uranium 2 is more electro-negative than its two predecessors.

TABLE I.

*Structure of the Members of the Uranium-Radium Series. (Fig. 1.)*

	Atomic Number, AN.		Helium Nuclei, He.	Electrons.			Structure Formula.
				Unstable Shell, E <sub>i</sub> .	Valency Shell, E <sub>v</sub> .	Total, E <sub>m</sub> .	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
A	92	Uranium 1..	$8 + x$	$8 + y$	6	$14 + y$	$u\text{-He}_3E_{8+6}$
B	90	Uranium X <sup>1</sup> .	$7 + x$	$10 + y$	4	$14 + y$	$u\text{-He}_7E_{10+4}$
C	91	Brevium....	$7 + x$	$8 + y$	5	$13 + y$	$u\text{-He}_7E_{8+5}$
D	92	Uranium 2..	$7 + x$	$6 + y$	6	$12 + y$	$u\text{-He}_7E_{6+6}$
E	90	Ionium....	$6 + x$	$8 + y$	4	$12 + y$	$u\text{-He}_6E_{8+4}$
F	88	Radium....	$5 + x$	$10 + y$	2	$12 + y$	$u\text{-He}_6E_{10+2}$
G	86	Niton.....	$4 + x$	$12 + y$	0	$12 + y$	$u\text{-He}_4E_{12+0}$
H	84	Radium A...	$3 + x$	$6 + y$	6	$12 + y$	$u\text{-He}_3E_{6+6}$
I	82	Radium B...	$2 + x$	$8 + y$	4	$12 + y$	$u\text{-He}_2E_{8+4}$
J	83	Radium C...	$2 + x$	$6 + y$	5	$11 + y$	$u\text{-He}_2E_{6+5}$
J <sup>1</sup>	81	Radium C <sup>11</sup> .	$1 + x$	$8 + y$	3	$11 + y$	$u\text{-He}_1E_{8+3}$
K	84	Radium C <sup>1</sup> .	$2 + x$	$4 + y$	6	$10 + y$	$u\text{-He}_2E_{4+6}$
L	82	Radium D...	$1 + x$	$6 + y$	4	$10 + y$	$u\text{-He}_1E_{6+4}$
M	83	Radium E...	$1 + x$	$4 + y$	5	$9 + y$	$u\text{-He}_1E_{4+5}$
N	84	Radium F...	$1 + x$	$2 + y$	6	$8 + y$	$u\text{-He}_1E_{2+6}$
O	82	Radio-lead..	$x$	$4 + y$	4	$8 + y$	$u\text{-E}_{4+4}$

Assuming that the disintegration manifesting itself as alpha-radiation is the result of a *single* atom emitting a *single* helium-nucleus (which later becomes a helium-atom) and beta-radiation the result of a *single* atom discharging a *single* electron, it becomes evident that each step in disintegration must have been accompanied simultaneously by an intra-atomic shift of electrons between the outer valency ring and the interior part of the atom. This change in the position of "metastasic" electrons can occur according to the two types of disintegration, in two directions, namely; (1) in  $\alpha$ -radiation two valency electrons move from the outer shell to the unstable inner ring and take the place of the emitted helium nucleus, decreasing the valency thereby by two and the relative electromotive force of the resulting substance will be less negative; (2) in  $\beta$ -

radiation there is in addition to the emission of an electron the transfer of another electron from the unstable inner ring to the outer valency ring, thus increasing the valency and making the electromotive force more negative.

There are in the uranium-radium series 16 members, eight of which exhibit alpha-radiations, six show beta-radiations and two final end products exhibit apparently no radiation. Thus the difference from uranium I to radio-lead consists of eight helium-nuclei and six electrons. This difference between the structure of U and Pb has been used in the calculation of the number of helium-nuclei and electrons for the different members of the series as shown in Table I. Beginning with uranium I there are in the unstable ring eight helium-nuclei and eight electrons, while the valency ring contains six valency electrons. If now one helium-nucleus is thrown off, two of the valency electrons take temporarily the place of the helium-nucleus, the unstable ring of the new meta-bolon or atom consists of seven helium-nuclei and ten electrons, while four valency electrons are left in the outer ring (uranium  $x'$ ). This new system of seven He and ten E is very unstable and in the next step of disintegration one electron is thrown off entirely, while one electron

TABLE II.

*Structure of the Members of the Thorium Series.*

Atomic Number, AN.		Helium Nuclei, He.	Electrons.			Structure Formula.
			Unstable Shell, $E_i$ .	Valency Shell, $E_v$ .	Total, $E_m$ .	
(2)	(3)	(4)	(5)	(6)	(7)	(8)
90	Thorium . . . . .	$6 + x^1$	$8 + y^1$	4	$12 + y^1$	t.He <sub>6</sub> E <sub>8+4</sub>
88	Mesothorium 1 . . .	$5 + x^1$	$10 + y^1$	2	$12 + y^1$	t.He <sub>5</sub> E <sub>10+2</sub>
89	Mesothorium 2 . . .	$5 + x^1$	$8 + y^1$	3	$11 + y^1$	t.He <sub>5</sub> E <sub>8+3</sub>
90	Radio-thorium . . .	$5 + x^1$	$6 + y^1$	4	$10 + y^1$	t.He <sub>5</sub> E <sub>6+4</sub>
88	Thorium X . . . . .	$4 + x^1$	$8 + y^1$	2	$10 + y^1$	t.He <sub>4</sub> E <sub>8+2</sub>
86	Thorium-emanation	$3 + x^1$	$10 + y^1$	0	$10 + y^1$	t.He <sub>3</sub> E <sub>10+0</sub>
84	Thorium A . . . . .	$2 + x^1$	$4 + y^1$	6	$10 + y^1$	t.He <sub>2</sub> E <sub>4+6</sub>
82	Thorium B . . . . .	$1 + x^1$	$6 + y^1$	4	$10 + y^1$	t.He <sub>1</sub> E <sub>6+4</sub>
83	Thorium C . . . . .	$1 + x^1$	$4 + y^1$	5	$9 + y^1$	t.He <sub>1</sub> E <sub>4+5</sub>
81	Thorium D . . . . .	$x^1$	$6 + y^1$	3	$9 + y^1$	t.E <sub>6+3</sub>
84	Thorium C <sup>1</sup> . . . . .	$1 + x^1$	$2 + y^1$	6	$8 + y^1$	t.He <sub>1</sub> E <sub>2+6</sub>
82	Thorium-lead . . . .	$x^1$	$4 + y^1$	4	$8 + y^1$	t.E <sub>4+4</sub>

shifts to the outer ring, leaving in the unstable ring the seven helium nuclei and eight electrons (Ux'' or brevium), and increasing thereby the valency electrons from four to five.

In Fig. 1 this idea is graphically illustrated and the number of helium



nuclei and electrons of each member is shown. From these data are gotten the temporary structure formula shown in column 8 of Table I. In a similar way the results for the members of the thorium series are given in Table II. The sole difference between the two types of structure formula is in the value  $u$  and  $t$ . The symbol  $u$  indicates "lead from uranium," that is a nucleus with atomic weight 206.4 containing  $x$  helium nuclei and  $y$  electrons, while  $t$  indicates "lead from thorium" with atomic weight 208.2, which consists of  $x'$  helium nuclei and  $y'$

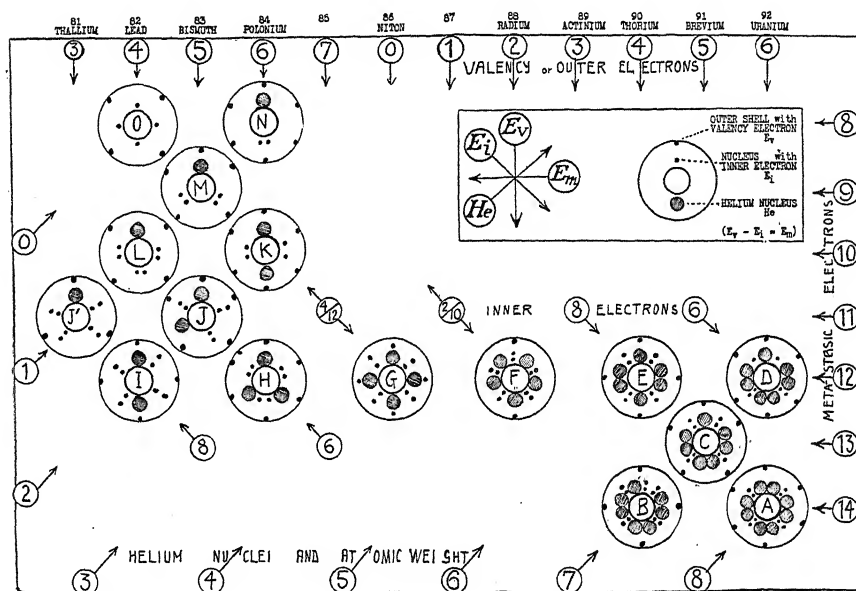


Fig. 1.

Diagram indicating the structure of the members of the uranium-radium series. The diagram is thought to be attached to the periodic system and the position of the radioactive substance in this scheme determines its structure. In any of the four axis the number of helium nuclei and electrons is the same for different members, thus the members which are located on the same southwest to northeast axis have the same atomic weight and the same number of helium nuclei (He), corresponding to the data of column 4 in Table I.; northwest to southeast axis have the same number of inner electrons ( $E_i$ ), column 5 of Table I.; north to south axis belong to the same group, have the same valency ( $E_v$ ) and are the isotopes of the element shown on the top of the axis (column 6 of Table I.); west to east axis have the same number of metastasis electrons ( $E_m$ ), column 7 of Table I.

electrons. Whether or not there is a difference in the values  $x$  and  $x'$ , and  $y$  and  $y'$ , can not be definitely decided at present, for the experimental difference existing between  $t$  and  $u$  may be due to a different arrangement of the same number of helium nuclei ( $x = x'$ ) and electrons ( $y = y'$ ).

For the present the structure formula offer certain advantages prefer-



The completeness of both series from uranium or thorium to lead, be-

### *The Periodic System and the Radioactive Isotopes.*

	4	5A	6A	7A	0	1A	2A	3A	4							
	(non-metals)				(inert gases)	(light-metals)										
	$\neq 0$	-	--	---	$\neq \infty$	+++	++	+	$\neq 0$							
Vb	82.Pb	83.Bi	84.Po	85.—	86.Nt	87.—	88.Ra	89.Ac	90.Th	VI						
IVb	50.Sn	51.Sb	52.Te	53.I	54.Xe	55.Cs	56.Ba	57.La	58.Ce	Va						
IIIb	32.Ge	33.As	34.Se	35.Br	36.Kr	37.Rb	38.Sr	39.Y	40.Zr	IVa						
IIb	14.Si	15.P	16.S	17.Cl	18.Ar	19.K	20.Ca	21.Sc	22.Ti	IIIa						
Ib	6.C	7.N	8.O	9.F	10.Ne	11.Na	12.Mg	13.Al	14.Si	IIa						
0	1.H				2.He	3.Li	4.Be	5.B	6.C	Ia						
	(heavy metals)															
III'	22.Ti	23.V	24.Cr	25.Mn	26.Fe	27.Co	28.Ni	29.Cu	30.Zn	31.Ga	32.Ge	III'				
IV'	40.Zr	41.Cb	42.Mo	43.—	44.Ru	45.Rh	46.Pd	47.Ag	48.Cd	49.In	50.Sn	IV'				
V''	58.Ce	Pr	Nd	—	Sa	Eu	Gd	Tb	Dy	Ho	Er	Tm	Db	Yb	72.Lu	V''
V'	72.Lu	73.Ta	74.W	75.—	76.Os	77.Ir	78.Pt	79.Au	80.Hg	81.Tl	82.Pb	V'				
	(radioactive isotopes)															
(Tl)	(Pb)	(Bi)	(Po)	(—)	(Nt)	(—)	(Ra)	(Ac)	(Th)	(Bv)	(U)					
PbRa	←	RaF														
		RaE	↗													
⎧ PbTh	↗	⎧ ThC'		⎧ ThEm	←	⎧ ThX	←	⎧ RaTh								
⎧ PbAc	←	⎧ AcC'		⎧ AcEm	←	⎧ AcX	←	⎧ RaAc								
⎧ RaD		⎧ RaC'		⎧ RaEm	←	⎧ Ra	←	⎧ Io								
								↗ MsT''								
⎧ ThD	←	⎧ ThC						Ac	←		← U <sub>2</sub>					
⎧ AcD		⎧ AcC									↗ Uz					
⎧ RaC''		⎧ RaC									Ux''					
								↗ MsT'								
⎧ ThB	↗	⎧ ThA							Th	↗						
⎧ AcB	←	⎧ AcA							Uy	←	U <sub>1</sub>					
⎧ RaB		⎧ RaA							Ux'							
+	±0			—	±∞	+			±0		—					

comes evident by the periodic system because all places are occupied according to the rule that the position is changed two places to the left by alpha-radiation and one place to the right by beta-radiation. The deviation of the actinium-series is still doubtful and the structure formula of its members have therefore not been calculated.

The relationship of the radioactive elements to the other elements is shown in Table III. In this table the radioactive elements are appended to the new periodic table, the isotopes being placed vertically under each other. It also shows that the disintegration of all three series runs parallel, beginning with RaTh, RaAc and Io.

#### CONCLUSIONS.

1. The phenomena of disintegration enables the establishment of a partial atom-structure formula for each radioactive element. These atom-structure formulæ indicate the atomic weight, the valency or place in the periodic system and the respective position in the uranium-radium or thorium-series.

2. A theory of metastasic electrons has been developed by which (a) alpha-radiation or the emission of a helium-nucleus causes the change of position of two electrons from the valency ring to the interior unstable ring, while (b) beta-radiation or the emission of an electron causes the shift of another electron from the inner or unstable ring to the outer or valency ring.

3. This theory supports the view that the seat of disintegration is located in the next inner shell, and not in the outer or valency shell, and is caused by the shifting of two electrons.

4. The periodic table offers indications that the uranium-radium series and the thorium series down to lead are completely known and that no more members of these two series can be expected.

ON THE VIBRATIONS OF ELASTIC SHELLS PARTLY  
FILLED WITH LIQUID.

BY SUDHANSUKUMAR BANERJI.

## I. INTRODUCTION.

THE problem considered in this paper is chiefly of acoustical interest in relation to the theory of "musical glasses." This class of instrument consists of a series of thin-walled elastic shells whose gravest modes of vibration are tuned to form a musical scale by partially filling them with a liquid and are excited either by striking or by tangential friction on the rims. The principal features of interest requiring elucidation are (1) the dependence of the pitch of the vibration upon the quantity of liquid contained in the vessel and (2) the mode of vibration of the liquid itself. These features are discussed in this paper for the three cases in which the elastic shell is respectively (1) a hemispherical shell, (2) a cylindrical vessel with a flat bottom and (3) a conical cup, these forms approximating more or less closely to those used in practice. The analytical expressions show that the motion of the liquid is very marked near the margin of the vessel and is almost imperceptible near the center and also at some depth inside the liquid. This feature becomes more and more marked in the case of the higher modes of vibration of the vessel. Numerical results have also been obtained and tabulated showing the theoretical relation between the quantity of liquid contained in the vessel and the vibration frequency. These show that the rapidity with which the frequency falls on addition of liquid is greatest when the vessel is nearly full, this being specially noticeable in the case of the higher modes of vibration.

The general principle of the analytical method used is similar to that adopted by Lord Rayleigh<sup>1</sup> in treating the two-dimensional case of a long cylinder completely filled with liquid which was studied by Auerbach.<sup>2</sup> This case has also been recently discussed by Nikoloi.<sup>3</sup> The lowering of

<sup>1</sup> Lord Rayleigh, *Phil. Mag.*, XV., pp. 385-389 (1883). *Scientific Papers*, Vol. 2, pp. 208-211.

<sup>2</sup> Auerbach, *Wied. Ann.*, 3, p. 157, 1878, and also *Wied. Ann.*, 17, p. 964, 1882. Reference may also be made to the papers by Montigny, *Bull. del' Acad. de Belg.*, [2], 50, 159, 1880, and by Koláček, *Wied. Ann.*, 7, 23, 1879, and also *Sitz. math. naturw. d. Wien*, 87, Abath. 2, 1883.

<sup>3</sup> Nikoloi, *Journ. Russk. Fisik Chimicesk*, 41, 5, pp. 214-227, 1909.

the pitch produced by the liquid is of course due to the added inertia exactly as in the related case of the vibrations of a bar or a string immersed in a liquid which have been studied by Northway,<sup>1</sup> Mackenzie and Kalähue.<sup>2</sup>

Musical glasses are sometimes excited by rotating them about a fixed vertical axis, the tangential friction being produced by a rubber kept in a fixed position. No attempt is made in this paper to consider this somewhat complicated case,<sup>3</sup> which I hope to be able to deal with on a future occasion.

## 2. HEMISPHERICAL CUPS.

The force which a thin sheet of matter subjected to stress opposes to extension is very great in comparison with that which it opposes to bending. From this Lord Rayleigh concluded that the middle surface of a vibrating shell remains unstretched and proposed a theory<sup>4</sup> of flexural vibrations of curved plates and shells in accordance with this condition. As the direct application of the Kirchhoff-Gehring method led to equations of motion and boundary conditions which were difficult to reconcile with Lord Rayleigh's theory, his theory gave rise to much discussion. Later investigations have, however, shown that any extension that may occur must be limited to a region of infinitely small area near the edge of the shell and that the greater part of the shell vibrates according to Lord Rayleigh's type.

Let the radius of the hemisphere be equal to  $a$ . Let a point whose natural coördinates are  $a, \theta, \phi$  be displaced to the position  $a + u, \theta + v, \phi + w$ , where  $u, v, w$  are to be treated as small.

From the condition of inextension

$$(\delta s)^2 = a^2(\delta\theta)^2 + a^2 \sin^2\theta(\delta\phi)^2 \\ = (a + u)^2(\delta\theta + \delta v)^2 + (\delta u)^2 + (a + u)^2 \sin^2(\theta + v) (\delta\phi + \delta w)^2, \quad (1)$$

Lord Rayleigh obtains the three differential equations

$$\frac{\partial v}{\partial \theta} + \frac{u}{a} = 0,$$

<sup>1</sup> Northway and Mackenzie, *PHYS. REV.*, 13, pp. 145-164, 1901.

<sup>2</sup> Kalähne, *Ann. d. Physik*, 46, 1, pp. 1-38, 1914.

<sup>3</sup> Reference may be made in this connection to papers by Prof. Love on "The Free and Forced Vibrations of an Elastic Spherical Shell Containing a Given Mass of Liquid," *Proc. Lond. Math. Soc.*, Vol. XIX., where he has studied the case of a rotating spherical shell completely full of liquid, and by Prof. Bryan on "The Beats in the Vibrations of a Revolving Cylinder or Bell," *Proc. Comb. Phil. Soc.*, Vol. VII., 1892.

<sup>4</sup> Lord Rayleigh, *Proc. Lond. Math. Soc.*, Vol. XIII., p. 4, 1881. See also *Proc. Roy. Soc.*, Vol. 45, pp. 45 and 443, 1881, *Theory of Sound*, Vol. I., Chap. XV, and Love's *Elasticity*, Chap. XXXIII.

$$\frac{\partial v}{\partial \phi} + \sin^2 \theta \frac{\partial w}{\partial \theta} = 0,$$

$$\frac{u}{a} + \cot \theta \cdot v + \frac{\partial w}{\partial \phi} = 0,$$

which can be integrated in the forms

$$\frac{u}{a} = \frac{\sin}{\cos} m\phi [A_m(m + \cos \theta) \tan^m \frac{1}{2}\theta - B_m(m - \cos \theta) \cot^m \frac{1}{2}\theta],$$

$$\frac{v}{\sin \theta} = -\frac{\sin}{\cos} m\phi [A_m \tan^m \frac{1}{2}\theta - B_m \cot^m \frac{1}{2}\theta],$$

$$w = \frac{\cos}{\sin} m\phi [A_m \tan^m \frac{1}{2}\theta + B_m \cot^m \frac{1}{2}\theta],$$

$A_m$  and  $B_m$  being arbitrary constants. These equations determine the character of the displacement of a point in the middle surface.

Since the pole  $\theta = 0$  is included the constant  $B_m$  must be considered to vanish and the type of vibrations in a principal mode is expressed by the equations

$$\begin{aligned} u &= A_m \alpha (m + \cos \theta) \tan^m \frac{1}{2}\theta \sin m\phi, \\ v &= -A_m \sin \theta \tan^m \frac{1}{2}\theta \sin m\phi, \\ w &= A_m \tan^m \frac{1}{2}\theta \cos m\phi, \end{aligned} \quad (4)$$

in which  $A_m$  is proportional to a simple harmonic function of the time.

The potential energy of bending of the vibrating shell is given by

$$\begin{aligned} V &= \frac{8}{3} \pi \mu \frac{\tau^3}{a^2} m^2 (m^2 - 1)^2 A_m^2 \int_0^{\pi/2} \tan^{2m} \frac{\theta}{2} \frac{d\theta}{\sin^3 \theta} \\ &= \frac{2}{3} \pi \mu \frac{\tau^3}{a^2} (m^3 - m) (2m^2 - 1) A_m^2, \end{aligned} \quad (5)$$

where  $\tau$  = thickness of the shell and  $\mu$  = rigidity.

The kinetic energy  $T$  is given by the expression

$$\begin{aligned} T &= \frac{1}{2} \pi \sigma \alpha^4 \tau \left( \frac{dA_m}{dt} \right)^2 \int_0^{\pi/2} \sin \theta \{ 2 \sin^2 \theta + (\cos \theta + m)^2 \} \tan^{2m} \frac{1}{2}\theta d\theta \\ &= \frac{1}{2} \pi \sigma \alpha^4 \tau \left( \frac{dA_m}{dt} \right)^2 \int_1^2 \frac{(2-x)^m}{x^m} [(m-1)^2 + 2(m+1)x - x^2] dx \\ &= \frac{1}{2} \pi \sigma \alpha^4 \tau f(m) \left( \frac{dA_m}{dt} \right)^2, \end{aligned} \quad (6)$$

where  $\sigma$  represents the density of the shell, and

$$f(m) = \int_1^2 \frac{(2-x)^m}{x^m} [(m-1)^2 + 2(m+1)x - x^2] dx, \quad (7)$$

which can be evaluated for any integral value of  $m$ .

Since the types of vibrations of the shell are entirely determined by the geometry of the middle surface of the shell, it is obvious that the types can under no circumstances be altered by the presence of the liquid in the shell. The liquid gives rise to a surface traction and affects only the arbitrary constant  $A_m$ , that is to say, the amplitude and the frequency of vibration of the shell.

The motion of the liquid will depend upon a velocity potential which satisfies the equation

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial \Phi}{\partial \theta} + \frac{\operatorname{cosec}^2 \theta}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} = 0. \quad (8)$$

A solution of this differential equation which will correspond to the type of vibration of the shell can be obtained by assuming  $\Phi$  to be of the form

$$\Phi = \left( C_m r + \frac{D_m}{r^2} \right) \sin m\phi \cdot \Delta_\theta,$$

where  $\Delta_\theta$  is a function of  $\theta$  only. Substituting in the differential equation we find that  $\Delta_\theta$  satisfies the equation

$$\frac{d^2 \Delta_\theta}{d\theta^2} + \cot \theta \frac{d\Delta_\theta}{d\theta} + (2 - m^2 \operatorname{cosec}^2 \theta) \Delta_\theta = 0.$$

The general solution of this differential equation is

$$\Delta_\theta = E_m \tan^m \frac{1}{2} \theta (m + \cos \theta) + F_m \cot^m \frac{1}{2} \theta (m - \cos \theta).$$

Neglecting solutions of the type  $\cot^m \frac{1}{2} \theta (m - \cos \theta)$ , we see that  $\Phi$  is of the form

$$\Phi = \left( C_m r + \frac{D_m}{r^2} \right) \sin m\phi \tan^m \frac{1}{2} \theta (m + \cos \theta), \quad (9)$$

where  $C_m$  and  $D_m$  are two arbitrary constants. Let us first take

$$\Phi = C_m \frac{r}{a} \tan^m \frac{1}{2} \theta (m + \cos \theta) \sin m\phi \cos pt. \quad (10)$$

The relation between  $C_m$  and  $A_m$  of (4) is readily found by equating the value of  $\partial \Phi / \partial r$ , when  $r = a$ , to  $\partial u / \partial t$ , both of which represent the normal velocity at the circumference. We get

$$C_m \cos pt = a^2 \frac{dA_m}{dt}. \quad (11)$$

The expression (10) determines the principal mode of vibration of the liquid. The simple character of the fluid motion as determined by this expression will however be a little disturbed on account of the existence of a free surface and we shall have to add a small correction to this

expression. The condition to be satisfied at the free surface is

$$\frac{\partial^2 \Phi}{\partial t^2} + g \frac{\partial \Phi}{\partial z} = 0, \text{ when } z = h,$$

where  $h$  denotes the depth of the liquid surface below the center of the hemisphere. We shall neglect the force of gravity, inasmuch as the period of free waves of length comparable with the diameter of the shell is much greater than that of the actual motion. The condition to be satisfied at the free surface then becomes simply

$$\Phi = 0, \text{ when } z = h.$$

Hence we must have

$$\Phi = C_m \frac{r}{a} \tan^m \frac{1}{2} \theta (m + \cos \theta) \sin m\phi \cos pt + f(r, \theta, \phi) \cos pt, \quad (12)$$

where  $f(r, \theta, \phi)$  is a solution of  $\Delta^2 \Phi = 0$  and is such that its differential coefficient with respect to  $r$  vanishes on the spherical boundary and it has the value

$$- C_m \frac{h \sec \theta}{a} \tan^m \frac{1}{2} \theta (m + \cos \theta) \sin m\phi \quad (13)$$

on the free surface.

In the particular case, when the shell is completely full of liquid, the differential coefficient of  $f(r, \theta, \phi)$  with respect to  $r$  vanishes on the spherical surface and  $f(r, \theta, \phi)$  has the value

$$- m C_m \frac{r}{a} \sin m\phi \quad (14)$$

on the surface defined by  $\theta = \pi/2$ .

For the determination of the function  $f(r, \theta, \phi)$ , spherical harmonics of the complex degree  $-\frac{1}{2} + p\sqrt{-1}$  are extremely suitable. The properties of these harmonics and their applications to some physical problems have been investigated by Hobson.<sup>1</sup> Solutions of Laplace's equation of the form

$$\frac{1}{\sqrt{r}} \sin(p \log Ar) \frac{\sin}{\cos} m\phi K_p^m(\cos \theta),$$

where  $K_p^m(\cos \theta)$  is a harmonic of degree  $-\frac{1}{2} + p\sqrt{-1}$  and order  $m$ , and is defined by the hypergeometric series

$$K_p^m(\cos \theta) = F(m + \frac{1}{2} + pi, m + \frac{1}{2} - pi, m + 1, \sin^2 \frac{1}{2} \theta),$$

are suitable for our present purpose. These solutions are finite and continuous for all points in the space inside the hemispherical shell (except

<sup>1</sup> Hobson, "On a Class of Spherical Harmonics of Complex Degree with Application to Physical Problems," Trans. Camb. Phil. Soc., Vol. 14, pp. 212-236, 1889.

infinitely near the origin which may be supposed excluded by surrounding it by an infinitely small sphere).

Let us assume

$$f(r, \theta, \phi) = \sum_p B_p \sqrt{\frac{a}{r}} \sin \left( p \log \frac{r}{h} \right) \frac{K_p^m(\cos \theta)}{K_p^m(\cos \alpha)} \sin m\phi, \quad (15)$$

where  $h/a = \cos \alpha$ . Then

$$\frac{\partial}{\partial r} f(r, \theta, \phi) = 0,$$

when  $r = a$ , if the  $p$ 's are the roots of the equation

$$\tan(p \log a/h) - 2p = 0,$$

and the summation in the above series extends over all the roots of this equation.

The values of the constant  $B_p$  have to be obtained from the equation

$$- Cm \frac{\cos \alpha}{\cos \theta} \tan^m \frac{1}{2} \theta (m + \cos \theta) = \sum_p B_p \sqrt{\frac{a}{h}} \frac{\sin[p \log(\sec \theta)]}{\sqrt{\sec \theta}} \frac{K_p^m(\cos \theta)}{K_p^m(\cos \alpha)},$$

which must be satisfied for all values of  $\theta$  between the limits  $0 < \theta < \alpha$ . Approximate values of the constants  $B_p$ 's can be easily obtained from this equation. In the particular case when the shell is completely full of liquid, the values of the constants  $B_p$ 's can be obtained in a very simple form. Since the origin is a singular point, we exclude the point by surrounding it with a small sphere of radius  $\epsilon$ , and assume that  $f(r, \theta, \phi)$  vanishes on the surface of this sphere. Since in this case  $\alpha = \pi/2$ , we can assume

$$f(r, \theta, \phi) = \sum_p B_p \sqrt{\frac{a}{r}} \sin \left( p \log \frac{r}{\epsilon} \right) \frac{K_p^m(\cos \theta)}{K_p^m(0)} \sin m\phi,$$

where

$$K_p^m(0) = \frac{\sqrt{\pi} \Pi(m)}{\Pi(\frac{1}{2}m - \frac{1}{4} + \frac{1}{2}pi) \Pi(\frac{1}{2}m - \frac{1}{4} - \frac{1}{2}pi)}$$

$$= \frac{\sqrt{\pi} \Pi(m)}{\left\{ \frac{(2m-1)^2 + p^2}{2^2} \right\} \left\{ \frac{(2m-3)^2 + p^2}{2^2} \right\} \dots}$$

and the summation extends for all values of  $p$  which are the roots of the equation

$$\frac{d}{da} \left[ \frac{\sin[p \log(a/\epsilon)]}{\sqrt{a}} \right] = 0,$$

that is to say, the equation

$$\tan \left( p \log \frac{a}{\epsilon} \right) - 2p = 0. \quad (16)$$



The constants  $B_p$ 's have to be determined by the condition that  $f(r, \theta, \phi)$  must have the value  $-mC_m(r/a) \sin m\phi$  on the free surface which is given by  $\theta = \pi/2$ . Hence  $B_p$ 's are given by

$$-mC_m \frac{r}{a} = \sum_p B_p \sqrt{\frac{a}{r}} \sin \left( p \log \frac{r}{\epsilon} \right).$$

Putting  $r = \epsilon \cdot e^\lambda$ , it is easy to see that

$$\begin{aligned} B_p &= - \frac{2(p^2 + \frac{1}{4}) \cdot mC_m}{p^2 \log(a/\epsilon) + \frac{1}{2}[\frac{1}{2} \log(a/\epsilon) - 1]} \left( \frac{\epsilon}{a} \right)^{3/2} \int_0^{\log a/\epsilon} e^{3/2\lambda} \sin p\lambda d\lambda \\ &= - \frac{8(4p^2 + 1) \cdot mC_m}{4p^2 \log(a/\epsilon) + [\log(a/\epsilon) - 2]} \left[ \frac{\sin[p \log(a/\epsilon)] + p(\epsilon/a)^{3/2}}{9 + 4p^2} \right]. \quad (17) \end{aligned}$$

To obtain an idea of the magnitude of the constant  $B_p$ , we shall obtain its value when  $a/\epsilon$  is a very large quantity. It is easy to see by the method of successive approximation that the roots of the equation (16) are given by

$$\begin{aligned} p \log \frac{a}{\epsilon} &= X - \frac{1}{2X} \log \frac{a}{\epsilon} - \left( \log \frac{a}{\epsilon} \right)^2 \left[ \frac{1}{4} - \frac{1}{2^{\frac{1}{2}}} \log \frac{a}{\epsilon} \right] \frac{1}{X^3} \\ &\quad - \left( \log \frac{a}{\epsilon} \right)^3 \left[ \frac{1}{4} - \frac{1}{1^{\frac{1}{2}}} \log \frac{a}{\epsilon} + \frac{1}{1^{\frac{1}{2}}} \left( \log \frac{a}{\epsilon} \right)^2 \right] \frac{1}{X^5} \\ &\quad - \text{etc.}, \end{aligned}$$

where  $X = (s + \frac{1}{2})\pi$ ,  $s$  being any integer. Now, if we take  $a/\epsilon = 10^5$ , the roots are successively the following:

$$p_1 = .321, p_2 = .628, p_3 = .960, p_4 = 1.205, \text{ etc.}$$

Hence we easily find that the constants  $B_{p1}, B_{p2}, B_{p3}$ , etc., have approximately the values  $B_{p1} = -.09 C_m, B_{p2} = -.08 C_m, B_{p3} = -.06 C_m$ , etc., from which we infer that the surface correction  $f(r, \theta, \phi)$  is a small one. The principal mode of vibration of the liquid is therefore expressed by

$$\Phi = C_m \frac{r}{a} \tan^m \frac{1}{2}\theta (m + \cos \theta) \sin m\phi \cos pt. \quad (18)$$

If  $q$  represent the velocity of the liquid as given by this expression, we have

$$\begin{aligned} q^2 &= C_m^2 \frac{1}{a^2} \tan^{2m-2} \frac{1}{2}\theta [(m + \cos \theta)^2 (\sin^2 m\phi \tan^2 \frac{1}{2}\theta + \frac{1}{4} m^2 \cos^2 m\phi \sec^2 \frac{1}{2}\theta) \\ &\quad + \{\frac{1}{2}m(m + \cos \theta) \sec^2 \frac{1}{2}\theta - \tan \frac{1}{2}\theta \sin \theta\}^2 \sin^2 m\phi] \cos^2 pt. \quad (19) \end{aligned}$$

Since  $q$  is independent of  $r$ , the velocity of the liquid at any point in a given radius vector is constant. We see that the velocity varies as  $\tan^{m-1} \frac{1}{2}\theta$ . Hence if we move along any given meridian, the velocity increases from a zero value at the pole at first very slowly then rather abruptly to a

large value at the surface, the abruptness of rise being greater the larger the quantity  $m$ , that is to say, the higher the mode of vibration of the liquid. Since the velocity of the liquid is constant along any given radius vector, we see that if we consider the motion of the liquid on the surface of a cone of semi-vertical angle  $\theta$ , and trace the motion of the liquid as a whole as  $\theta$  increases, the velocity remains small as  $\theta$  increases and assumes a large value only at or near the surface. It is obvious therefore that in every case when the cup is not quite filled to the brim, the velocity of the liquid has a very large value near the margin of the vessel and is almost imperceptible near the center and at some depth in the liquid. In the particular case, when the shell is almost filled to the brim, the velocity of the liquid as given by this expression at a point which is near the center and also near the free surface is not small. But in this case the free surface correction  $f(r, \theta, \phi)$  to the expression for the velocity potential becomes of some importance and has a sign opposite to it. Consequently the velocity of the liquid near the center always remains very small. These indications of theory are all confirmed by experiment.

To calculate the kinetic energy of the liquid, we have to integrate  $\Phi \times \partial\Phi/\partial n$  over the whole boundary of the fluid. At the free surface  $\Phi = 0$ . We have therefore only to consider the spherical surface.

Therefore

$$\begin{aligned}
 T &= \frac{1}{2}\rho \iint \Phi \frac{\partial\Phi}{\partial n} dS \\
 &= \frac{1}{2}\alpha\rho \cos^2 pt \int_0^{2\pi} \int_0^a [C_m \sin m\phi \tan^m \frac{1}{2}\theta (m + \cos \theta) + f(r, \theta, \phi)] \\
 &\quad \times C_m \sin m\phi \tan^m \frac{1}{2}\theta (m + \cos \theta) \sin \theta d\theta d\phi \\
 &= \frac{\pi}{2} a\rho \cos^2 pt C_m^2 \int_0^a \tan^{2m} \frac{1}{2}\theta (m + \cos \theta)^2 \sin \theta d\theta \\
 &\quad + \frac{\pi}{2} a\rho \cos^2 pt C_m \sum_p \beta_p \frac{\sin [p \log (a/h)]}{K_p^m (\cos \alpha)} \\
 &\quad \times \int_0^a \tan^m \frac{1}{2}\theta (m + \cos \theta) K_p^m (\cos \theta) \sin \theta d\theta,
 \end{aligned} \tag{20}$$

$\rho$  being the density of the liquid.

Since the liquid is supposed to be incompressible, the potential energy is zero.

The sum of the kinetic and potential energies of the solid and liquid together must be independent of the time. Thus we get

$$\begin{aligned}
 &\left[ a^5 \rho \int_0^a \tan^{2m} \frac{1}{2}\theta (m + \cos \theta)^2 \sin \theta d\theta + \alpha^5 \rho K + \alpha^4 \tau \sigma f(m) \right] \frac{d^2 A_m}{dt^2} \\
 &\quad + \frac{4}{3} \mu \frac{\tau^3}{\alpha^2} (m^3 - m)(2m^2 - 1) A_m = 0,
 \end{aligned}$$

where

$$K = \sum_p \frac{B_p \sin [p \log (a/h)]}{C_m K_p^m (\cos \alpha)} \int_0^a \tan^m \frac{1}{2} \theta (m + \cos \theta) K_p^m (\cos \theta) \sin \theta d\theta$$

$$= \sum_p \frac{B_p 4 \sin^2 \alpha}{C_m 9 + 4p^2} \cdot \frac{\sin [p \log (a/h)]}{K_p^m (\cos \alpha)} \left[ K_p^m (\cos \alpha) \frac{d}{d \cos \alpha} \{ \tan^m \frac{1}{2} \alpha (m + \cos \alpha) \} \right.$$

$$\left. - \tan^m \frac{1}{2} \alpha (m + \cos \alpha) \frac{d}{d \cos \alpha} K_p^m (\cos \alpha) \right].$$

If we put

$$F(\alpha, m) = \int_0^a \tan^{2m} \frac{1}{2} \theta (m + \cos \theta)^2 \sin \theta d\theta$$

$$= \int_{1+\cos \alpha}^2 \left( \frac{2-x}{x} \right)^m (m-1+x)^2 dx,$$

and if  $A_m$  varies as  $\cos (pt + \epsilon)$ , we get

$$[a\rho F(\alpha, m) + \tau\sigma f(m) + \alpha\rho K]p^2 = \frac{4}{3} \frac{\mu}{\alpha^3} \left( \frac{\tau}{\alpha} \right)^3 (m^3 - m)(2m^2 - 1). \quad (2I)$$

This equation gives the frequency of vibration of the shell with different quantities of liquid.

The fall of pitch for the three gravest tones given by  $m = 2$ ,  $m = 3$  and  $m = 4$  for a brass hemispherical shell 10 cm. in radius, 2 mm. in thickness and of density 8.6 with different quantities of liquid are shown in Table I. In Fig. 1, the frequencies have been plotted against the quantity of water in the vessel for these three modes of vibrations.

TABLE I.

a.	Quantity of Water in the Shell.	m = 2.			m = 3.			m = 4.		
		F(a, m).	f(m).	$\times \text{Const.}$	F(a, m).	f(m).	$\times \text{Const.}$	F(a, m).	f(m).	$\times \text{Const.}$
90°	$\pi a^3 \times .667$	1.114	1.53	1.80	1.580	1.88	4.63	2.030	2.296	8.76
80°	$\pi a^3 \times .494$	.570	"	2.24	.641	"	6.50	.479	"	14.51
70°	$\pi a^3 \times .338$	.291	"	2.75	.125	"	9.53	.097	"	19.42
60°	$\pi a^3 \times .208$	.123	"	3.29	.032	"	10.57	.009	"	21.45
50°	$\pi a^3 \times .133$	.058	"	3.61	.008	"	10.95	.002	"	21.64
40°	$\pi a^3 \times .034$	.026	"	3.81	.003	"	11.14	.000	"	21.68
30°	$\pi a^3 \times .014$	.015	"	3.88	.001	"	11.22	.000	"	21.69
20°	$\pi a^3 \times .003$	.013	"	3.90	.000	"	11.23	.000	"	21.69
10°	$\pi a^3 \times .001$	.011	"	3.91	.000	"	11.23	.000	"	21.69
0°	0	0	"	3.93	0	"	11.24	0	"	21.69

The frequencies of a brass hemispherical shell of about the same radius and thickness loaded with different quantities of water have also been

determined experimentally by a photographic method. The results showed a general agreement with the calculated values. As a shell of uniform thickness and of uniform elastic properties throughout could

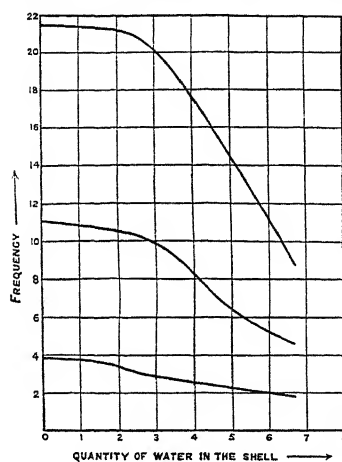


Fig. 1.

not be procured, and the one that was used was very much deficient in these respects, the slight discrepancy that was noticed between the calculated and the observed values of the frequency, was probably due to these defects.

### 3. CYLINDRICAL CUPS.

The problem of the flexural vibrations of a cylindrical shell is considered in Lord Rayleigh's Theory of Sound, Vol. I., §235C. If the displacements at any point  $a, \theta, z$  of the cylinder be  $\delta r, a\delta\theta, \delta z$ , then

$$\begin{aligned}\delta r &= -n(A_n a + B_n z) \sin n\theta, \\ a\delta\theta &= (A_n a + B_n z) \cos n\theta, \\ \delta z &= -n^{-1}B_n a \sin n\theta.\end{aligned}\quad (22)$$

Supposing now that the cup has been formed by an inextensible disk being attached to the cylinder at  $z = 0$ , the displacements  $\delta r, a\delta\theta$  must vanish for that value of  $z$ . Hence  $A_n = 0$ , and

$$\delta r = -nB_n z \sin n\theta, \quad a\delta\theta = B_n z \cos n\theta, \quad \delta z = -n^{-1}B_n a \sin n\theta, \quad (23)$$

the constant  $B_n$  is proportional to a simple harmonic function of the time, say,  $\cos pt$ .

Since the displacements  $\delta r$  and  $a\delta\theta$  are proportional to  $z$  and the displacement  $\delta z$  is independent of  $z$ , it is obvious, that when  $z$  is large the displacements  $\delta r$  and  $a\delta\theta$  are also very large compared to  $\delta z$ , that is to say, near the free end of the shell, the displacement  $\delta z$  is negligible compared to  $\delta r$  or  $a\delta\theta$ . But at the bottom of the shell, the displacements  $\delta r$  and  $a\delta\theta$  vanishes and  $\delta z$  remains constant. We conclude, therefore, from the law of continuity, that the disk at the bottom of the shell must have a small normal vibration. If  $w$  denote the normal displacement of the disk, it is well known that  $w$  satisfies the differential equation

$$\frac{\partial^2 w}{\partial t^2} + C^4 \nabla^4 w = 0, \quad (24)$$

where

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + \frac{1}{r^2} \frac{d^2}{d\theta^2} \text{ and } C^4 = \frac{E\tau^{12}}{3\sigma(1-\epsilon^2)},$$

$E$  being Young's modulus,  $\sigma$  the density,  $\tau^1$  the thickness and  $\epsilon$  the Poisson's ratio.

If  $w \propto \cos(pt + e)$ , then the equation becomes

$$\nabla^4 w - v^4 w = 0,$$

where  $v^4 = p^2/c^4$ . A solution of this differential equation is known to be

$$w = C_n J_n(vr) \sin n\theta.$$

Hence we shall take

$$w = C_n J_n(vr) \sin n\theta \cos pt. \quad (25)$$

The value of the constant  $C_n$  can be obtained from the condition that  $w$  and  $\delta z$  must be continuous at the boundary.

This gives

$$C_n \cos pt = -\frac{B_n}{n} \frac{a}{J_n(va)}. \quad (26)$$

We can assume that  $J_n(va)$  is very large and consequently that the normal vibration of the disk is very small. The potential energy of deformation for a length  $l$  of the cylinder is

$$V = \frac{4\pi\mu\tau^3}{3a} (n^2 - 1)^2 \left[ \frac{\lambda + \mu}{\lambda + 2\mu} \frac{n^2 l^2}{3a^2} + 1 \right] B_n^2. \quad (27)$$

The potential energy of vibration of the disk is given by

$$\frac{E\tau^{13}}{3(1 - \epsilon^2)} \int_0^a \int_0^{2\pi} \left[ (\nabla^2 w)^2 - 2(1 - \epsilon) \left\{ \frac{\partial^2 w}{\partial n^2} \frac{\partial^2 w}{\partial y^2} \right\} - \left( \frac{\partial^2 w}{\partial n \partial y} \right)^2 \right] r d\theta dr.$$

The value of this integral can be easily obtained. But as we regard the vibration of the disk compared to that of the cylindrical surface to be very small, the value of this expression is also very small.

If the volume density be  $\sigma$ , we get the expression for the kinetic energy in the form

$$T = \frac{1}{2} \pi \sigma \tau l a \left[ \frac{1}{3} l^2 (1 + n^2) + n^{-2} a^2 \right] \left( \frac{dB_n}{dt} \right)^2 + \frac{1}{2} \pi \sigma \tau' \frac{a^2}{n^2 [J_n(va)]^2} \int_0^a [J_n(vr)]^2 r dr \left( \frac{dB_n}{dt} \right)^2. \quad (28)$$

If the cylinder contains frictionless incompressible fluid, the motion of the liquid will depend upon a velocity potential  $\Phi$  which satisfies the equation  $\nabla^2 \Phi = 0$ , or in cylindrical coördinates

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0.$$

The solution of this differential equation can be written in either of the

forms

$$\Phi = \alpha_n z r^n \sin n\theta \cos pt, \quad (29)$$

$$\Phi = \beta_n e^{-kz} J_n(kr) \sin n\theta \cos pt. \quad (30)$$

The boundary conditions to be satisfied by  $\Phi$  are

$$\begin{aligned} (i) \quad & \frac{\partial \Phi}{\partial r} = \frac{d\delta r}{dt}, \text{ when } r = a; \\ (ii) \quad & \frac{\partial \Phi}{\partial z} = \frac{dw}{dt}, \text{ when } z = 0; \\ (iii) \quad & \Phi = 0, \text{ at the free surface, } u, \text{ when } z = h. \end{aligned} \quad (31)$$

We assume that

$$\Phi = \alpha_n z r^n \sin n\theta \cos pt + \sum_k J_n(kr) [D_k \cosh kz + E_k \sinh kz] \sin n\theta \cos pt, \quad (32)$$

where the summation extends for all values of  $k$  which are the roots of the equation

$$\frac{d}{da} J_n(ka) = 0. \quad (33)$$

We at once get by condition (i)

$$\alpha_n \cos pt = -\frac{1}{a^{n-1}} \frac{dB_n}{dt}.$$

The condition (ii) gives

$$-\left[ \frac{a}{n} \frac{J_n(vr)}{J_n(va)} - \frac{r^n}{a^{n-1}} \right] \frac{1}{k} \frac{dB_n}{dt} = \sum_k E_k J_n(kr) \cos pt.$$

This equation must be satisfied for all values of  $r$  between the limits ( $0 < r < a$ ) and will give the value of the constant  $E_k$ . Now since

$$\int_0^a r^{n+1} J_n(kr) dr = \frac{na^n}{k^2} J_n(ka),$$

$$\int_0^a J_n(kr) J_n(vr) r dr = \frac{av}{k^2 - v^2} J_n(ka) J_n'(va),$$

and

$$\int_0^a [J_n(kr)]^2 r dr = \frac{1}{2} a^2 \left( 1 - \frac{n^2}{k^2 a^2} \right) [J_n(ka)]^2,$$

we get

$$\begin{aligned} -\frac{1}{k} \frac{dB_n}{dt} \left[ \frac{a}{n J_n(va)} \int_0^a J_n(kr) J_n(vr) r dr - \frac{1}{a^{n-1}} \int_0^a r^{n+1} J_n(kr) dr \right] \\ = E_k \cos pt \int_0^a [J_n(kr)]^2 r dr, \end{aligned}$$

and therefore

$$E_k \cos pt = -\frac{2a}{ka} \frac{dB_n}{dt} \frac{k^2 a^2}{(k^2 a^2 - n^2) J_n(ka)} \left[ \frac{v}{(k^2 - v^2)n} \frac{J_n'(va)}{J_n(va)} - \frac{na}{k^2 a^2} \right]. \quad (35)$$

The condition (iii) gives

$$\alpha_n h r^n + \sum_k (D_k \cosh kh + E_k \sinh kh) J_n(kr) = 0$$

for all values of  $r$  between the limits ( $0 < r < a$ ).

Therefore we get

$$D_k \cosh kh + E_k \sinh kh = -\alpha_n \frac{2na^n h}{(k^2 a^2 - n^2) J_n(ka)}. \quad (36)$$

The equations (35) and (36) give the values of the constants  $D_k$  and  $E_k$ .

To calculate the kinetic energy we have to integrate  $\Phi \times \partial\Phi/\partial n$  over the boundary of the shell. At the free surface  $\Phi = 0$ . We have therefore only to consider the cylindrical surface and the bottom. The expression can be written in the form

$$\begin{aligned} T = \frac{1}{2} \pi \rho \cos^2 pt \left[ n \alpha_n^2 a^{2n} \frac{h^3}{3} + n \alpha_n a^n \sum J_n(ka) \left\{ D_k \left( \frac{h \sinh kh}{k} \right. \right. \right. \\ \left. \left. - \frac{1}{k^2} \cosh kh + \frac{1}{k^2} \right) + E_k \left( \frac{h \cosh kh}{k} - \frac{1}{k^2} \sinh kh \right) \right\} \\ \left. - \sum_k \left\{ \alpha_n D_k \frac{na^n}{k^2} J_n(ka) + \frac{1}{2} ka^2 E_k D_k \left( 1 - \frac{n^2}{k^2 a^2} \right) [J_n(ka)]^2 \right\} \right]. \quad (37) \end{aligned}$$

The constants  $E_k$  and  $D_k$  are very small compared to  $\alpha_n$ . If we neglect  $E_k$  and  $D_k$ , the expression for the kinetic energy reduces to the simple form

$$T = \frac{\pi}{2} \rho n \alpha_n^2 a^{2n} \frac{h^3}{3} \cos^2 pt. \quad (38)$$

In this case the expression for  $\Phi$  reduces to the form

$$\Phi = \alpha_n z r^n \sin n\theta \cos pt. \quad (39)$$

This expression represents the principal mode of vibration of the liquid and all the other coexistent modes are very small compared to this one. Since the expression for the velocity varies as  $(r/a)^{n-1}$ , the velocity is very marked near the margin of the vessel and is almost imperceptible near the center. Using the principle that the sum of the kinetic and potential energies of the solid and liquid together must be independent of the time, we easily obtain an expression for the frequency of vibrations in the most general case from the expressions for the kinetic and potential energies already given. If we neglect  $E_k$  and  $D_k$ , the frequency equation takes a very simple form. The expression in this case is

$$\begin{aligned} [\sigma \tau l a \{ \frac{1}{3} l^2 (1 + n^2) + n^2 a^2 \} + \frac{1}{3} \rho n h^3 a^2] p^2 \\ = \frac{8 \mu \tau^2 l}{3 a} (n^2 - 1)^2 \left[ \frac{\lambda + \mu}{\lambda + 2 \mu} \frac{n^2 l^2}{3 a^2} + 1 \right]. \quad (40) \end{aligned}$$

Thus we see that the law of variation of the frequency with the height of water in the vessel can be expressed in the form

$$p^2 = \frac{1}{A + B(h/l)^3},$$

where  $A$  and  $B$  are two constants for the vessel.

For a glass cylinder whose dimensions are given by  $l/a = 4$ ,  $\tau/a = .02$  and which has the density  $\sigma = 2.6$  and the elastic constants  $\mu = 1.8$  and  $\lambda = 1.53$ , we easily find that the frequency  $p_n$  is given by

$$\left[ .052 \left\{ (1 + n^2) + \frac{.1875}{n^2} \right\} + n \left( \frac{h}{l} \right)^3 \right] p_n^2 = \frac{8\mu\tau^3}{3a^2l^2} (n^2 - 1)^2 [10.4 n^2 + 3].$$

For the three gravest tones given by  $n = 2$ ,  $n = 3$  and  $n = 4$ , the values of the frequencies  $p_2$ ,  $p_3$  and  $p_4$  with different quantities of water in the cylinder are shown in Table II.

TABLE II.

$h/l$	$p_2 \times \text{Const.}$	$p_3 \times \text{Const.}$	$p_4 \times \text{Const.}$
0	12.47	34.44	65.63
.1	12.33	34.40	65.48
.2	12.02	33.98	64.47
.3	11.29	32.05	61.95
.4	10.15	29.45	57.79
.5	8.85	26.26	52.45
.6	7.61	22.99	46.68
.7	6.56	19.97	41.09
.8	5.60	17.34	36.05
.9	4.97	15.13	31.67
1.0	4.21	13.25	27.93

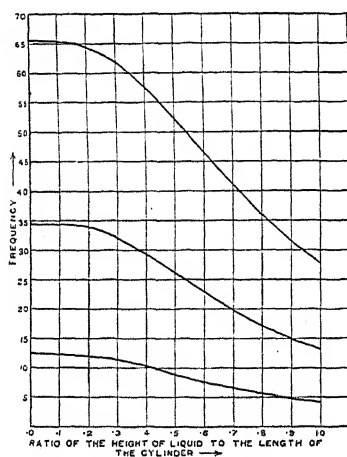


Fig. 2.

The values of the frequencies given in Table II. have been plotted in Fig. 2.

#### 4. CONICAL CUPS.

It is shown in Lord Rayleigh's Theory of Sound in the article already referred to that if a cone for which  $\rho = \tan \gamma \cdot z$ ,  $\gamma$  being the semi-vertical angle, executes flexural vibrations, the displacements  $\delta\rho$ ,  $\delta\phi$ ,  $\delta z$  at any point whose cylindrical co-ordinates are  $(\rho, \phi, z)$  are given by

$$\delta\rho = n \tan \gamma (A_n z + B_n) \sin n\phi,$$

$$\delta\phi = (A_n + B_n z^{-1}) \cos n\phi,$$

$$\delta z = \tan^2 \gamma [n^{-1} B_n - n (A_n z + B_n)] \sin n\phi.$$



If the cone be complete up to the vertex at  $z = 0$ , then  $B_n = 0$ , so that

$$\delta\rho = n \tan \gamma \cdot A_n z \sin n\phi,$$

$$\delta\phi = A_n \cos n\phi,$$

$$\delta z = -n A_n \tan^2 \gamma \cdot z \sin n\phi.$$

If the displacements in polar coördinates  $(r, \theta, \phi)$  be denoted by  $\delta r, \delta\theta, \delta\phi$ , we easily obtain

$$\delta\phi = A_n \cos n\phi$$

$$\delta r = \delta\rho \sin \gamma + \delta z \cos \gamma = 0,$$

$$r\delta\theta = \delta\rho \cos \gamma - \delta z \sin \gamma = n A_n \tan \gamma \cdot r \sin n\phi.$$

It is easy to see that the potential energy of deformation for a length  $l$  of the cone

$$W = \frac{4\pi}{3} \mu \tau^3 \frac{\lambda + \mu}{\lambda + 2\mu} A_n^2 \sin \gamma \left[ \left( -n^3 \frac{\tan \gamma}{\sin^2 \gamma} + n \tan \gamma + n \cot \gamma \right)^2 + \cos^2 \gamma \right] \log \frac{2l}{\tau},$$

where  $\tau$  = thickness.<sup>1</sup>

The expression for the kinetic energy of vibration of the shell can be easily obtained in the form

$$T = \frac{\pi}{8} \sigma \tau l^3 \sin^3 \gamma [n^2 \sec^2 \gamma + 1] \left( \frac{dA_n}{dt} \right)^2. \quad (45)$$

If the cup contains frictionless incompressible fluid, the velocity potential of the fluid must satisfy Laplace's equation. Let us assume that the velocity potential is given by

$$\Phi = C_n r^2 \phi_n (\cos \theta) \sin n\phi \cdot \cos pt \quad (46)$$

where  $\phi_n (\cos \theta)$  is a function of  $\theta$  only. It is easy to see by substitution in the differential equation

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\cot \theta}{r^2} \frac{\partial \Phi}{\partial \theta} + \frac{\operatorname{cosec}^2 \theta}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} = 0,$$

that  $\phi_n (\cos \theta)$  satisfies the equation

$$\frac{\partial^2 \phi_n}{\partial \theta^2} + \cot \theta \frac{\partial \phi_n}{\partial \theta} + (6 - n^2 \operatorname{cosec}^2 \theta) \phi_n = 0.$$

<sup>1</sup> This expression can be readily deduced from a very general expression for the potential energy due to strain in curvilinear coördinates obtained by Prof. Love. (Vide his paper on "The Small Free Vibrations and Deformation of a Thin Elastic Shell," Phil. Trans., Vol. 179, 1888, A.) The expression has been criticized by Prof. Basset (Phil. Trans., Vol. 181, 1890, A) on the ground that Prof. Love has omitted several terms which involve the extension of the middle surface. As the inextensional vibrations only have been considered in this paper, this criticism does not affect us in any way.

A solution of this differential equation can be easily obtained in the form

$$\phi n (\cos \theta) = \tan^n \frac{1}{2} \theta \left[ (1 - n)(2 - n) - 6(2 - n) \cos^2 \frac{\theta}{2} + 12 \cos^4 \frac{\theta}{2} \right].$$

The relation between  $C_n$  and  $A_n$  can be easily obtained by equating the value of  $\partial \Phi / r \partial \theta$ , when  $\theta = \gamma$ , to  $d(r \delta \theta) / dt$ , both of which represent the normal velocity at the boundary. We thus get

$$C_n \cos pt \frac{\partial}{\partial \gamma} \left[ \tan^n \frac{1}{2} \gamma \left\{ (1 - n)(2 - n) - 6(2 - n) \cos^2 \frac{\gamma}{2} + 12 \cos^4 \frac{\gamma}{2} \right\} \right] \\ = n \tan \gamma \frac{dA_n}{dt}. \quad (48)$$

The principal mode of vibration of the liquid will therefore be expressed by (46) except for a small correction to be introduced on account of the existence of a free surface. At the free surface the condition to be satisfied is given by

$$\Phi = 0, \text{ when } z = h,$$

where  $h$  denotes the height of the liquid.

To satisfy this condition, we assume

$$\Phi = C_n r^2 \phi_n (\cos \theta) \sin n\phi \cos pt + \sum_m D_m r^m P_m^n (\cos \theta) \sin n\phi \cos pt, \quad (49)$$

where the summation extends for all values of  $m$  which are the roots of the equation

$$\frac{\partial}{\partial \gamma} P_m^n (\cos \gamma) = 0. \quad (50)$$

The constants  $D_m$ 's have to be determined by means of the equation

$$C_n (h \sec \theta)^2 \phi_n (\cos \theta) + \sum_m D_m (h \sec \theta)^m P_m^n (\cos \theta) = 0, \quad (51)$$

which must be satisfied for all values of  $\theta$  between the limits  $0 < \theta < \gamma$ . Approximate values of the constants  $D_m$ 's can be easily obtained from this equation. To get an idea of the magnitude of the constant  $D_m$ , we shall obtain its value in the particular case when the semi-vertical angle  $\gamma$  of the cone is small and the height  $h$  of the liquid is large compared to the radius of the cross-section of the cone by the free surface. In this case the free surface can be taken to be practically coincident with the surface of the sphere  $r = h$ . The equation for determining  $D_m$  is then

$$C_n h^2 \phi_n (\cos \theta) + \sum_m D_m h^m P_m^n (\cos \theta) = 0.$$

Now since

$$\int_{\cos \gamma}^1 P_m^n (\cos \theta) P_m^n (\cos \theta) \sin \theta d\theta = 0,$$

$m, m^1$  being two different roots of the equation (50),  
and

$$\int_{\cos \gamma}^1 [P_m^n(\cos \theta)]^2 \sin \theta d\theta = \frac{1 - \cos^2 \gamma}{2m + 1} P_m^n(\cos \gamma) \frac{\partial^2}{\partial m \partial \cos \gamma} P_m^n(\cos \gamma),$$

we easily get

$$D_m = - \frac{2m + 1}{1 - \cos^2 \gamma} \frac{C_n h^{2-m} \int_{\cos \gamma}^1 \phi_n(\cos \theta) P_m^n(\cos \theta) \sin \theta d\theta}{P_m^n(\cos \gamma) \frac{\partial^2}{\partial m \partial \cos \gamma} P_m^n(\cos \gamma)}.$$

Neglecting the small correction introduced by the free surface, we see that the kinetic energy of the fluid motion is

$$\frac{\pi}{2} \rho C_n^2 \cos^2 \gamma \dot{\phi}_n(\cos \gamma) \frac{\partial \phi_n(\cos \gamma)}{\partial \gamma} \sin \gamma \sec^5 \gamma \frac{h^5}{5}.$$

Since the sum of the kinetic and potential energies of the solid and liquid together must be independent of the time, we easily obtain, on assuming that  $A_n \propto \cos \gamma \dot{\phi}_n$ , the frequency equation in the form

$$\left[ \frac{1}{4} \sigma \tau h^4 \sin^2 \gamma (n^2 \sec^2 \gamma + 1) + \rho n^2 \tan^2 \gamma \sin \gamma \frac{\phi_n(\cos \gamma)}{\frac{\partial}{\partial \gamma} \phi_n(\cos \gamma)} \frac{H^5}{5} \right] p^2 \\ = \frac{8}{3} \mu \tau^3 \frac{\lambda + \mu}{\lambda + 2\mu} A_n^2 \sin \gamma \left[ \left( -n^3 \frac{\tan \gamma}{\sin^2 \gamma} + n \tan \gamma + n \cot \gamma \right)^2 \right. \\ \left. + \cos^2 \gamma \right] \log \frac{2l}{\tau}.$$

where  $H = h \sec \gamma$ ,  $H$  being the slant height of the liquid. In this case, we see that the law of variation of frequency with the height of liquid can be expressed in the form

$$p^2 = \frac{1}{A + B(h/l)^5},$$

$A$  and  $B$  being two constants for the particular shell.

The frequencies  $p_2$ ,  $p_3$  and  $p_4$  with different quantities of water for the three gravest modes of vibrations given by  $n = 2$ ,  $n = 3$  and  $n = 4$  have been calculated from this expression for a cone of semi-vertical angle  $30^\circ$ , the ratio of the thickness of the sides of the cone to the slant height being equal to .02 and are shown in Table III.

The curves showing the fall of frequency for these three modes of vibrations of the cone when loaded with different quantities are plotted in Fig. 3.

TABLE III.

$H/l$ .	$f_2 \times \text{Const.}$	$f_3 \times \text{Const.}$	$f_4 \times \text{Const.}$
0	5.030	13.58	26.75
.1	5.030	13.58	26.75
.2	5.028	13.57	26.73
.3	5.008	13.54	26.69
.4	4.937	13.41	26.47
.5	4.761	13.08	25.94
.6	4.428	12.43	24.87
.7	3.942	11.64	23.13
.8	3.399	10.07	20.79
.9	2.808	8.63	18.07
1.0	2.310	7.26	15.41

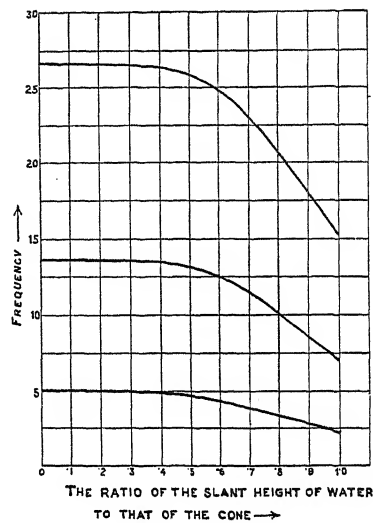


Fig. 3.

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## THE RELATION OF POTENTIAL DISTRIBUTION TO HYSTERESIS EFFECT IN THE WEHNELT TUBE.

BY R. A. PORTER.

### I. INTRODUCTION.

AS a part of his extended study of the discharge in a vacuum tube having a hot oxide as cathode, Wehnelt<sup>1</sup> published a Braun-tube oscillograph curve showing the general character of the voltage and current values in the tube when it is connected to an alternating current generator. Later I made a large number of oscillograph records and from them a study of the relationship of voltage to current in the same form of tube, the cathode being covered with calcium oxide.<sup>2</sup> The curves published at that time show the characteristics for static voltage conditions and for various frequencies of alternating current as gas pressure, maximum current, temperature of cathode, and kind of gas in the tube are varied.

The curves which I obtained showed quite a variety of forms, but one of the marked features was the different current values obtained for a single anode-cathode voltage under both static and alternating current (dynamic) conditions. In general, the static and the dynamic characteristics showed a hysteresis effect, that is, larger current values for decreasing than for increasing voltages.<sup>3</sup> The present paper gives the results of a study of the potential distribution under which three values of current are obtained for the same anode-cathode voltage. The work was undertaken for the purpose of obtaining further knowledge of the conditions producing the hysteresis effect.

### 2. APPARATUS.

The discharge tube used for this study was in form and dimensions almost identical with that of the tube which I used previously in studying the dynamic characteristics, it being essentially the commercial form of Wehnelt tube manufactured by Gundelach, and was obtained from him. It is spherical in shape, 14 cm. in diameter. The only modification from

<sup>1</sup> A. Wehnelt, *Ann. d. Phys.*, 19, p. 138, 1906.

<sup>2</sup> R. A. Porter, *Ann. d. Phys.*, 40, p. 561, 1913.

<sup>3</sup> See figures 4, 15, 10, 11, etc., *l. c.*

the former tube used consists in the addition of a movable sound, shown at *A* in Fig. 1. This sound is made of platinum wire sheathed with glass up to a point within 4 mm. of the end. The other end of the platinum wire is connected electrically to the terminal outside the tube by a coiled strip of bronze. The iron band *L*, attached to the glass tubing,

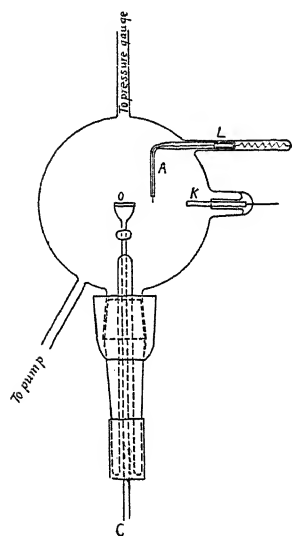


Fig. 1.

allows the sound to be moved magnetically. The anode, *K*, is of iron about 0.22 cm. in diameter and extends about 2.5 cm. within the tube. The end of the anode is 4.28 cm. from the plane of the cathode strip.

The cathode strip, *O*, is platinum,  $1.5 \times 0.25 \times 0.0015$  cm. With a heating current of 11.65 amperes, the temperature as obtained with a Wanner pyrometer was  $1285^{\circ}$  C. with no discharge current, the pressure being 0.010 mm. With a discharge current of 30 milliamperes, the temperature read  $20^{\circ}$  higher. The calcium oxide coat was obtained by placing Kahlbaum c.p. calcium nitrate upon the platinum strip and heating it electrically in the air.

The negative terminal of the storage battery supplying the heating current was grounded at the base of the tube, at *C*. This junction was also the point connected to the electrometer in making measurements of the potential drop from sound to cathode. All potentials were measured finally with the electrometer. A potentiometer method described by Tate and Foote<sup>1</sup> was tried and found to work very well for the larger currents. However, as potentials for the non-luminous discharge had to be measured with an electrometer, it seemed best to use it for all measurements.

A resistance of 1,000 ohms was used in the anode-cathode circuit to stabilize the discharge current, which was furnished by a 300-volt d.c. generator. The applied voltage was measured with a Weston voltmeter, the discharge current, with a Weston millivoltmeter used as an ammeter.

During the observations here recorded, no trap was used to prevent mercury vapor from passing from the pump or the McLeod gauge into the discharge tube.

<sup>1</sup> John T. Tate and Paul D. Foote, Jour. Washington Acad. Sci., 7, p. 482, 1917.

## 3. POTENTIAL DISTRIBUTION AND STATIC CHARACTERISTICS.

After a number of voltage and current measurements had been made with this tube, three characteristic and readily reproducible stages in the discharge were recognizable, the non-luminous discharge and two forms of luminous discharge. Some transient effects were observed and further study may disclose the conditions under which these are obtained.

The wide difference in the characteristics of such a tube for luminous and for non-luminous discharge have already been pointed out.<sup>1</sup> Child<sup>2</sup> has made potential measurements for the luminous and for the non-luminous discharge in a cylindrical tube with CaO cathode. Lester<sup>3</sup> found that with a CaO cathode "there were generally three possible ranges for the current-potential curves" and that "the current broke suddenly from one range to another." A knowledge of the distribution of potential in the space between anode and cathode for the three forms of discharge observed in the Wehnelt type of tube seemed important for the explanation of the phenomenon of hysteresis as well as that of a multiple valued current for a single anode-cathode potential.

The measurements thus far made have been taken at a pressure of

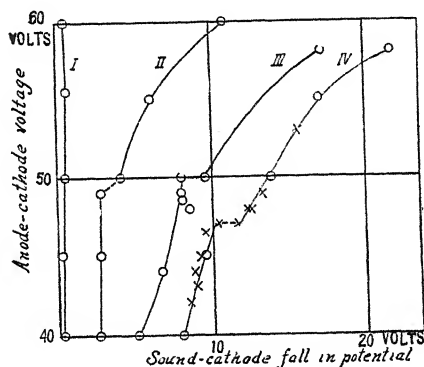


Fig. 2.

Distance of sound from cathode for curve II, 0.60 cm., for curve III, 2.40 cm.; for curve IV, 3.96 cm. These curves are obtained with luminous discharge. Curve I shows approximately the sound-cathode fall for non-luminous discharge for all three positions of the sound. Due to change in position, the sound voltage for non-luminous discharge does not vary more than 0.7 volt.

0.010 mm. With this pressure and a cathode temperature in the neighborhood of 1290° C. (heating current 11.65 amperes), either a luminous or a non-luminous discharge could be obtained for all anode-cathode

<sup>1</sup> R. A. Porter, *Ann. d. Phys.*, 40, p. 583, 1913; Richardson and Bazzoni, *Phil. Mag.*, 32, 426, 1916.

<sup>2</sup> C. D. Child, *PHYS. REV.*, 32, 492, 1911.

<sup>3</sup> H. Lester, *Phil. Mag.*, 31, 549, 1916.

voltages between 40 and 60 volts. Whether the discharge is luminous or non-luminous depends on whether this region of voltage is approached from a voltage above 60 or from one below 40 volts. Curves showing the relationship between anode-cathode voltage and sound-cathode fall in potential for three positions of the sound are given in Fig. 2. The sound lay at a point on a line connecting the anode with the middle of the cathode.

The curves of Fig. 3 show more readily the relation of the sound-

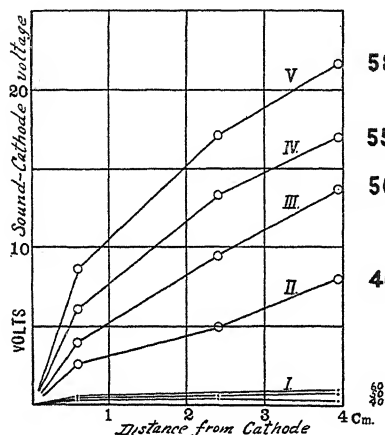


Fig. 3.

Potential distribution for non-luminous and for luminous discharge in Wehnelt tube.

lower than that midway between anode and cathode, a condition which Thomson<sup>1</sup> has observed in a cylindrical form of tube. The apparent irregularity in the readings in the neighborhood of 48 to 50 volts is more pronounced at the higher temperature of the cathode.

Fig. 4 shows the relation between anode-cathode voltage and current for luminous discharge.<sup>2</sup> The relatively large change in current at an applied voltage of about 50 volts suggests a change in the mechanism of the discharge at this point, just as the wide difference in the values of current for the non-luminous and the luminous discharge for the voltage range 40 to 60 volts indicated a difference in the process of conduction for those two forms of discharge. Observation of the appearance of the glow in the tube also indicates a transition as the voltage goes through the region 48-50 volts. When the glow is present with smaller voltages, it is a faint glow extending from anode to cathode, not filling the whole

<sup>1</sup> J. J. Thomson, *Phil. Mag.*, 18, 441, 1909.

<sup>2</sup> In the articles by Richardson and Bazzoni and by myself, *Phil. Mag.*, 32, 428, 1916, and *Ann. d. Phys.*, 40, 583, 1913, respectively, voltage-current curves for tubes with CaO cathodes are given for both the luminous and the non-luminous discharge.



space inside the tube. It is most intense in the space directly between anode and cathode, where one might expect the current density due to the pure electron (non-luminous) discharge to be greatest, and decreases in intensity and disappears in the region where the current density may be less.

At a voltage of 48-50 volts, however, if the applied voltage is being

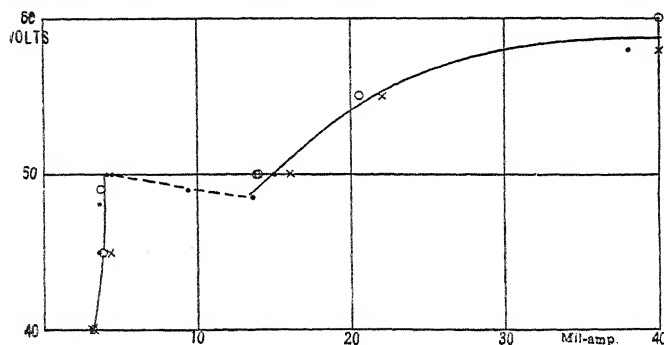


Fig. 4.

Static characteristic for luminous discharge.

increased, a flaming bluish glow appears on the sides of the anode and simultaneously the anode-cathode voltage may decrease slightly and the current increase. As the voltage is raised, the anode light increases in extent and the general glow extends until it fills the whole space inside the walls of the tube; then the anode light contracts and at a higher voltage disappears leaving the whole space inside the tube almost uniformly luminous. After the anode light appears, the current increases very rapidly for small increases in applied voltage. In fact the anode-cathode voltage seemed to remain constant for a considerable range in current. When 50 volts is reached voltage and current rise together.

As the anode-cathode voltage is decreased from 60 volts with a luminous discharge, it may be brought below 50 volts before the anode light disappears. When about 49 volts is reached the discharge changes to the form of a general glow, the anode-cathode voltage rises perhaps a volt and at the same time there is a marked decrease in current.

For one value of pressure and heating current a set of measurements has been made of the anode-cathode voltage, the current, and the potential fall in the line between anode and cathode, for the two stages in the luminous discharge. The curves of Fig. 5 show the relationship of sound-cathode fall to the total anode-cathode fall for three positions of the sound for voltages near 50 volts. Points marked with circles were obtained when the anode-light stage had been reached, those marked by dots show the sound-cathode fall when there is a general glow.

The points on curve I. at 49 volts indicate a considerable variation in the sound-cathode fall for that voltage when the anode light is present. Readings in a single set of observations, however, show regularly that

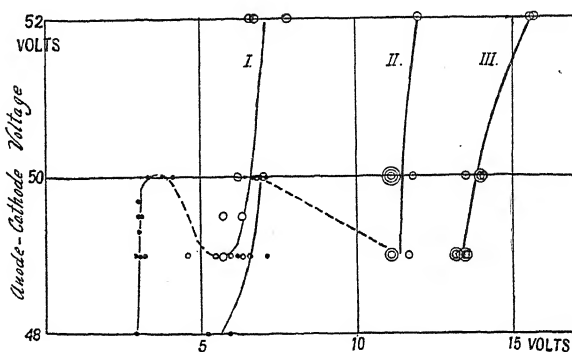


Fig. 5.

Sound—Cathode Voltage. For curves I and II, III the respective distances of sound from cathode were 0.62, 1.96 and 3.93 cm.

there is a difference of two to three volts in this fall for the two forms of discharge with an applied voltage of 49.

The smaller sound-cathode fall occurs when there is a general glow only partly filling the tube. This is the form of luminous discharge occurring first as the voltage is increased and may be called stage II. of the discharge, stage I. being non-luminous. When the region of 50 volts

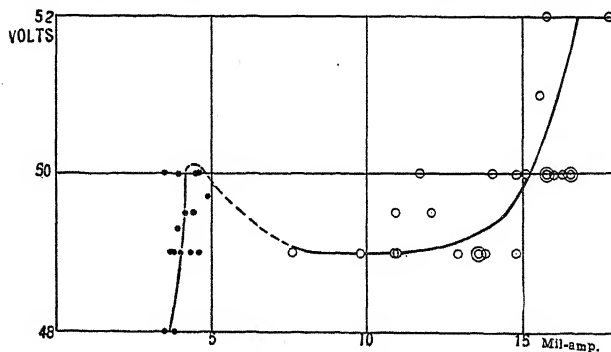


Fig. 6.

Static characteristic of Wehnelt tube in region of 48-52 volts.

is approached from higher voltages, the blue anode light is present and is accompanied by a larger sound-cathode fall and larger currents. We may call this stage III.

The relation of current to voltage for the region 49 to 50 volts is shown in Fig. 6. The potentials existing in the tube for the three stages in the discharge are brought together in Fig. 7.

Readings at higher cathode temperatures indicate an even more sharply marked region of voltage for which the three forms of discharge already noted may occur. At higher temperatures, stages II. and III. apparently have a wider range of voltage in common and there is a larger change in current as the discharge changes from one stage to the other. Modifications of the tube now being used will be necessary in order to obtain a full set of readings at higher cathode temperatures.

When the sound was near the anode, the anode light appeared and disappeared more gradually; no sharp limit of the two forms of discharge

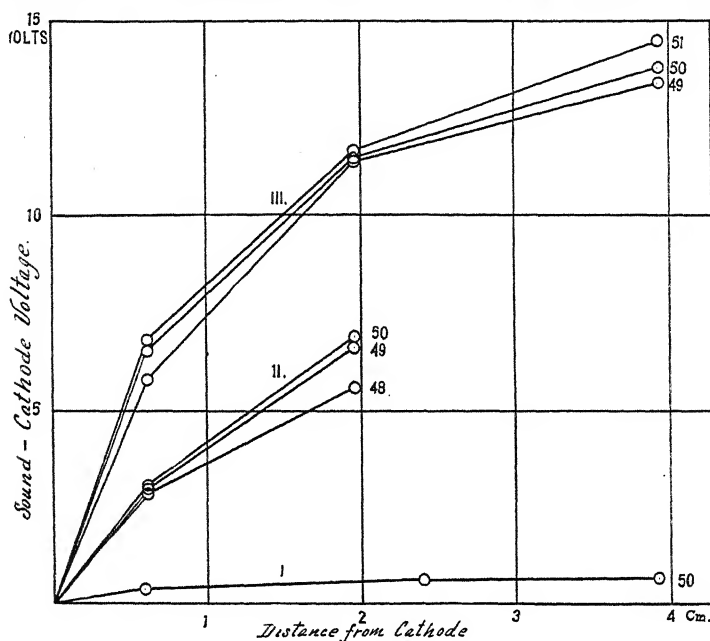


Fig. 7.

Potential distribution for three stages in discharge of Wehnelt tube. The figures at the right of the curves give the anode potentials.

could be distinguished. The curves for this stage in the discharge showing the relation between potential and distance from the cathode in Fig. 7 have therefore been drawn for only about half the anode-cathode distance.

From the results above given it can be seen that the following changes accompany the transition of the discharge from stage I. to stage II. and from stage II. to stage III.: (1) An increase in the cathode fall of 2 to 3 volts; (2) a small increase in the approximately uniform field in the free space between anode and cathode; (3) a decrease in the anode fall; (4) a decrease in the anode-cathode voltage; and (5) an increase in the discharge current. Each of these changes is consistent with the assumption

that there is an increase in the supply of positive ions in the region of the anode, that is, a more vigorous ionization at that terminal, as the current increases.

#### 4. THE HYSTERESIS EFFECT.

The direct observation of the discharge and the study of the dynamic characteristics and of the oscillograph records previously obtained show that during each half-cycle of the alternating current which has a suitable maximum value and takes place under the same conditions of pressure and cathode temperature as here used, all the stages of the discharge here described exist. We may conclude therefore that the widely different values of current at the beginning and at the end of a half-cycle, *i. e.*, just before the beginning and just before the end of the luminous discharge, correspond with entirely different potential distributions of the anode-cathode voltage and hence to different localizations of charges and of ionization.

When the anode light, or stage III., appears in the cycle, there is another abrupt change in the ionization and in the distribution of charge. At each of these transition stages, the old conditions tend to persist. Time and voltage seem to be required to change them.

#### SUMMARY.

Three stages in the discharge of a vacuum tube with CaO cathode have been identified and potential distribution curves for one pressure and heating current have been obtained for each stage. Voltage-current curves for the two stages of luminous discharge are given. The potential distribution curves lie in three groups corresponding to the three stages of the discharge. With the pressure and cathode temperature used, non-luminous discharge may exist for the voltage range 40-50 volts; another stage in the discharge may begin at 48 volts. In the region of 49 volts, therefore, any one of the three stages of the discharge may be obtained. Each of the three stages of the discharge and potential distribution tends to persist as the voltage is varied, or a hysteresis in the current values and in the potential distribution exists when the potential is cyclically varied. This is apparent for very slow cycles as well as for cycles of higher frequency.

The American Association for the Advancement of Science has granted assistance from its research funds in obtaining apparatus for this work. The author gratefully acknowledges this aid.

SYRACUSE UNIVERSITY,  
SYRACUSE, N. Y.,

November 12, 1918.

ELECTRIC DISCHARGE ON THE SURFACE OF A SOLID  
ELECTROLYTE.<sup>1</sup>

BY WAS. ŠULEJKIN.

THE THEORY OF ELECTROLYTIC SAFETIES<sup>2</sup> AGAINST OVERTENSION.

AS is well known, with the electrolysis of some solutions, the aluminum anode is covered with a coating of oxides in a very short space of time after closing the circuit. At the same time the strength of the current falls almost to zero, as the thin layer of oxides and the layer of gas enclosed in the pores offer an enormous amount of ohmic resistance. It is well known also that by increasing the tension, sometimes by hundreds of volts, the strength of the current is increased only inconsiderably, but this phenomenon remains so only until the tension has passed a certain limit, the so-called "critical tension."

On further increasing the tension, even if to only a small extent, the strength of the current suddenly and sharply rises. On the basis of this has been established the practice of applying the so-called "safety appliances against over-tension," the elements of which (practically) do not allow the current to escape to the ground from the line of transmission of electrical energy when the tension in the line is normal, but immediately and rapidly conducts discharges of electromagnetic waves to the ground, before over-tension in the protected part of the line can arise.

It is customary to say that such "safeties" cannot bear the "strain" when the critical tension is passed; the layer is broken through, and so a direct communication is set up between the fluid and the metallic anode.

But one glance at the aluminum electrodes (placed in a glass vessel to facilitate observation) suffices to show that it is not so. At a tension even considerably below the "critical," bright spots appear on the surface of the electrode (anode), here and there little bubbles of gas are given off; this all shows that here we have a case, not of static, but of kinetic equilibrium. In fact, even at low tension the layer is broken through here and there, but the breaks quickly close up; at a tension above the critical, however, such "cicatrizization," for some reason, does not take place.

In order to find out this reason, I tried to use for my experiments the

<sup>1</sup> Reported at the Moscow Institute of Science, May 19, 1918.

<sup>2</sup> Lightning arresters.

same "model" of aluminum electrode as I used in my investigations of the *electrolytic rectifier*<sup>1</sup>. Instead of an aluminum anode, a system of parallelly connected wires of 0.1 mm. was placed into the electrolyte, sealed into glass tubes (Fig. 1a). The part of the breaks in which the fluid comes into contact with the aluminum anode was therefore played here by the extremities of the wires placed in the fluid and projecting out of the glass about 0.5 to 0.75 mm.

It proved that on passing the critical tension each tube immediately

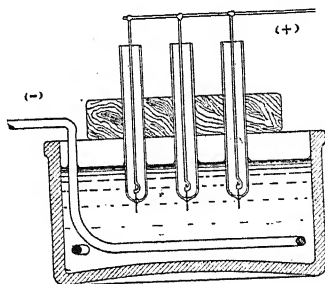


Fig. 1a.

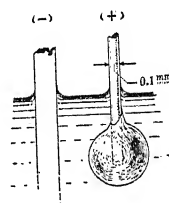


Fig. 1b.

broke. On changing the concentration of the solution, as is known, the critical tension point for the safeties is also changed. It was found that this law is followed when the concentration is changed, and that the attainment of the critical tension coincided with the breaking of the glass tubes.

The moment preceding the breaking of the glass, at the bottom end of the tube at the spot where the platinum is fused in, there is a sudden flash of blindingly bright, white light, quite different from the yellowish, weak light of the discharge between the platinum and the solution. When the glass was removed, and an anode, consisting only of fine platinum wires, was put into the solution, no change at all in the character and appearance of the discharge was observed when passing the former "critical" tension; the strength of the current did not sharply increase.

This refutes the opinion of Günt. Schultze,<sup>2</sup> who supposes that at the critical tension the discharge between the metal and the fluid begins to proceed with the help of quite different ions than up to the critical point (analogous to the Glaser-effect<sup>3</sup>). A critical point for discharge between pure metal and the solution, therefore, does not exist.

But, on passing a certain tension, you have only to touch the end

<sup>1</sup> W. Šulejkin, Archives des Sciences Physiques, I., 1918.

<sup>2</sup> Ann. d. Phys., 34, p. 657, 1911.

<sup>3</sup> H. Nernst, Ber. d. Deutsch. Chem. Ges., p. 1547, 1897.

of the wire-anode *under water* with some piece of glass, marble, jasper, enamel, or the like to produce at the point of contact a blinding white light. Such a flash of light appears, as may easily be observed, also at various points of the aluminum electrode (anode), when passing the critical tension.

If the end of the wire is removed to some distance from the glass (or other substance), in the interval between them an electric arc is formed, the anode of which is the wire, while the cathode is the glass (or other substance), to which, as may then be observed, particles of metal, coming away from the anode in a powdered state, are fused.

From this it is clear that the layer, hitherto an insulator, at critical tension begins to conduct the current. A necessary condition for a discharge of this sort is the near proximity of the metal and the conductor of the second kind, both being immersed in the solution and moistened by it. In aluminum safeties this condition is fulfilled; therefore, whenever at any spot the insulating layer is broken through *at a tension above the critical*, there is immediately a characteristic discharging flash at the adjacent part of the layer. At the same time the insulating layer around it is quite destroyed, in just the same way as the glass of the tube, or, for example, the lamina of the jasper. If on any part exposed in this way a fresh layer of oxides should be formed again by the action of the electrolysis, it will immediately be destroyed with a similar discharge. Meanwhile, by each initial rupture the exposure of the metal extends more and more, by which a sharp increase in the strength of the current in the electrolytic safeties is produced, when passing the critical tension.

But a complete resemblance of the work of the aluminium electrode and its "model" may be determined only by observing them under various conditions.

In order, on the one hand, to demonstrate that the critical point characteristic with electrolytic safeties depends on the peculiarities of the layer of oxides, separate parts of which, as it proves, playing a very essential part in the discharge; on the other hand, to become better acquainted with the processes which go on in the layer itself or on its surface; and, finally, to judge in some way of the movement of the ions in the close proximity of the anode, I would pass on to a description of detailed experiments.

I. CRITICAL TENSION WITH SOLUTIONS OF BI-CARBONATE OF SODA OF DIFFERENT CONCENTRATIONS, INTO WHICH AS AN ANODE IS IMMERSSED AT FIRST AN ALUMINUM LAMINA AND THEN A WIRE, FUSED INTO GLASS (FIG. 1, *a*).

If we take into consideration that aluminum itself is corroded by the solution (this does not occur in the case of the Pt. wire), which introduces a certain error, it may be seen from the series of figures (see Table I.) that the character of the action of the aluminium electrode and the

TABLE I.

Concentration.	0.6.	0.3.	0.15.	0.075.	0.0375.
Critic. t. of the safety .....	180 volt	250	280	350	420
Cr. tens. of Pt. wire .....	164	220	270	340	430

action at the point of contact of our experimental wire with the glass are the same. This gives us a means for determining the critical point even for solutions in which the aluminum anode is not covered with an insoluble layer of oxides (true in the majority of cases); gives us, finally, the possibility of always reproducing the phenomenon in its pure state. The advantage of this will become especially clear if we observe that, for example, with an electrolysis of sulphuric acid between aluminum electrodes the destruction of the insulating layer, *due to the corrosion of the aluminum*, takes place at a tension<sup>1</sup> 100 volts below the real critical tension.

In view of this, further experiments designed to explain the nature of the phenomenon were carried out with "models" of the aluminum electrode.

II. THE DEPENDENCE OF THE VALUE OF CRITICAL TENSION UPON THE CONCENTRATION OF THE ELECTROLYTE FOR DIFFERENT SOLUTIONS.

For experiments, solutions of the following substances have been used: (*a*) Caustic soda (NaOH); (*b*) sodium bicarbonate (NaHCO<sub>3</sub>); (*c*) iron sulphate (FeSO<sub>4</sub>); (*d*) zinc sulphate (ZnSO<sub>4</sub>); (*e*) sulphuric acid (H<sub>2</sub>SO<sub>4</sub>); (*f*) sodium chloride (NaCl); (*g*) ammonium chloride (NH<sub>4</sub>Cl); (*h*) hydrochloric acid (HCl).

Into the electrolyte as an anode were immersed fine wires (platinum or nickel) at the lower ends of which were fused drops of glass of 1 mm. diameter (Fig. 1, *b*).

At first a certain amount of soluble substance was measured out and after each observation the solution was diluted with a double quantity

<sup>1</sup>G. Schultze has called this "seemable critical" tension.



of water, again a double quantity, and so on. In this way a full concentration in gram-equivalents ( $m$ ) to the liter was determined.

With each concentration the tension was observed at which the bright light flashed out round the part of the glass adjacent to the wire.

The values of  $P_{crit}$  were marked off in Descartes coördinates on the axis ordinates; the quantities  $m$  were marked off on the axis of abscissa. As by decreasing the concentration by half, again by half, and so on, the critical tension rises, at first slowly, afterwards very rapidly, for greater precision the points were transferred to logarithmic paper (on the abscissa axis was marked off  $Lgm$ , on the ordinate  $LgP_{cr}$ ).

On doing this, a remarkable law became evident: all the characteristics of all the solutions experimented with, without exception, were represented on the logarithmic paper by straight lines.

Let us examine their peculiarities. (a) First of all it will be noticed from the drawing (Fig. 2) that all the straight lines are similar in appearance, being represented by the equation.

$$A \cdot LgP_{cr} + B \cdot Lgm = C, \quad (1)$$

where  $A$ ,  $B$  and  $C$  are positive coefficients.

Hence:

$$LgP_{cr} = \frac{C}{A} - \frac{B}{A} Lgm = Lg\alpha - \beta \cdot Lgm \quad (2)$$

(For convenience we have introduced the symbols:

$$\frac{C}{A} = Lg\alpha \text{ and } \frac{B}{A} = \beta).$$

Therefore, it is not difficult to see, for all the solutions:

$$P_{cr} = \frac{\alpha}{m^\beta}. \quad (3)$$

(b) As the characteristics of  $ZnSO_4$  and  $FeSO_4$ , of  $NaCl$  and  $NH_4Cl$  respectively coincide, the height of the critical tension, therefore, does not depend on composition of the cation, but depends only on the anion ( $SO_4^{--}$ ,  $Cl^-$ ,  $OH^-$ , etc.).

(c) In the diagram it is seen that the critical tensions of the following characteristics also coincide:

$NaCl$  (or  $NH_4Cl$ ) — to  $HCl$ ; and  $FeSO_4$  (or  $ZnSO_4$ ) — to  $H_2SO_4$ .

But it is known that the dissociated acids ( $HCl$ ,  $H_2SO_4$ ) are incomparably stronger than their salts. With equal full quantities of the dissolved electrolyte ( $m_{acid} = m_{salts}$ ) in the solution of acid there are incomparably more *dissociated* ions than in the salt solutions. Notwithstanding this, as we see, the critical tension in both cases is the same.

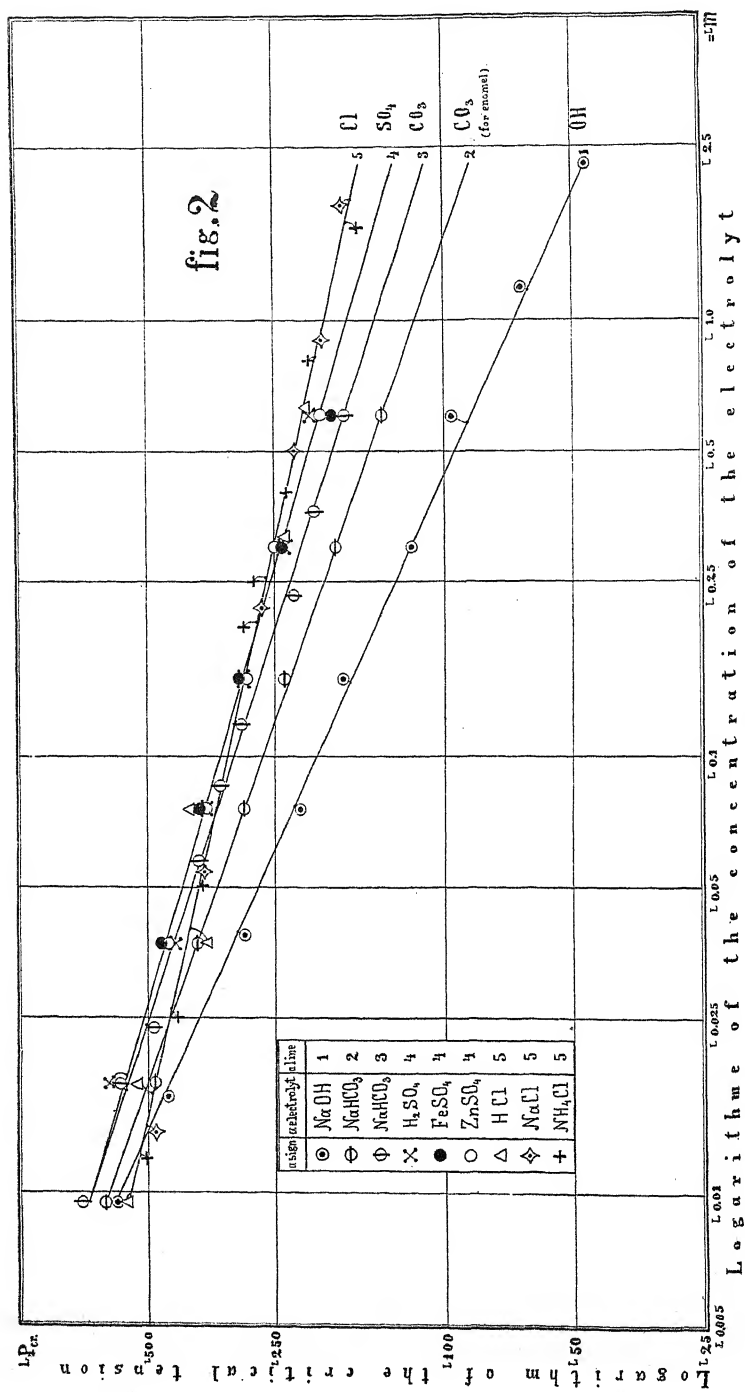


Fig. 2.

Hence it may be concluded that the value of the critical tension does *not depend* on the number of free ions, but depends on the full amount of the dissolved electrolyte. Evidently, the processes at the anode are extremely violent; at its surface new masses of the electrolyte are continually and rapidly dissociated, depending upon the manner in which the discharge takes place.

Upon the number of free ions depends, it is true, the decrease of tension in the solution, but as the critical tension attains hundreds of volts, the decrease of tension in the solution may evidently be neglected, as compared with the decrease of tension in the close proximity of the anode.

(*d*) As regards the mutual relations of the different characteristics (for the different anions), the following peculiarities are clearly noticeable:

The stronger the acid character of the anion, the less is the inclination of the corresponding straight line on the logarithmic paper (the less is  $\beta$ ) and, on the other hand, the higher it intersects the ordinate " $\text{Log } m = 0$ " ( $m = 1$ ) (the greater is  $\alpha$ ).

So, from the observations, the greatest  $\alpha$  (the smallest  $\beta$ ) corresponds to the anion ( $\text{Cl}^-$ ), the smallest  $\alpha$  (the greatest  $\beta$ ) to the anion ( $\text{OH}^-$ ).

TABLE II.

No. of a Line.	Anion.	$\alpha$ .	$\beta$ .
1	$(\text{OH})^-$	68	0.460
3	$(\text{HCO}_3)^-$	141	0.336
4	$(\text{SO}_4)^{--}$	165	0.304
5	$(\text{Cl})^-$	186	0.230

### III. THE DEPENDENCE OF THE HEIGHT OF CRITICAL TENSION ON THE COMPOSITION OF THE SUBSTANCE IN CONTACT WITH THE WIRE.

To the ends of the wires were fused drops of different sorts of glass, of enamel, of jasper, etc. The following results were obtained:

(*a*) The critical tension does not depend on the degree of fusibility or infusibility of the glass in contact with the wire, which was tested by experiments with *six* sorts of glass, from the most fusible to the most insusceptible to fusing.<sup>1</sup>

(*b*) A slight coloring of the glass (transparent glass) does not change the critical tension appreciably. A considerable admixture of metallic oxides (enamel), however, affects the character of the discharge:  $\alpha$  decreases considerably,  $\beta$  slightly increases.

For an example the characteristic has been set down on the diagram

<sup>1</sup> Not to obscure the drawing, the points obtained from these experiments are not marked on the straight line.

(N2), corresponding to the little drops of enamel (dark blue or sky blue) fused to the ends of the fine wires immersed in the solution of bicarbonate of soda ( $\text{NaHCO}_3$ ). For this case we obtained  $\alpha = 114$ ;  $\beta = 0.34$ ; while for glass drops,  $\alpha = 141$ ;  $\beta = 0.336$  (N3). Similar results were obtained from the experiments with drops of jasper.

(c) When the wires come into contact with such a crystal as does not possess electrolytic, but metallic<sup>1</sup> conducting properties, our characteristic discharge is not observed.

#### IV. MICROPHOTOGRAPHS.

In order to be able to determine in some measure the form of the discharge in question, which is not seen by the naked eye, microphotographs were taken. As the strength of the light of the discharge, notwithstanding its comparatively great intensity, is still insufficient to give a highly magnified representation in ordinary microphotographic chambers, it was necessary to take the photograph in a somewhat different way. The object was placed within the range of a small microscope (maximum linear magnifying power—60 times). The weakest object glass was used, giving a full linear magnification of 20 times. After the microscope (with external light) had been focused, the eyeglass was unscrewed, and on the diaphragm beneath it (just at the spot where the real image is formed, was placed (with red light) a small square photographic plate (1.5 cm.  $\times$  1.5 cm.). The current was turned on (for an instant), and the discharged flash was recorded on the plate, magnified, for example, 5 times. From the miniature negative thus obtained, the final enlargement was then made with the assistance of a projecting lantern.

The first photograph (Fig. 3) shows a discharge at the point of contact of the fine wire with the drop of glass fused to it and immersed in the electrolyte. The discharge encircles the wire with a hoop of light (the points of light on the surface of the drop are only gleams from the refracted and reflected light illuminating the drop).

From the drop a fine hair of glass attached to the wire was drawn upwards along the wire. On the photograph it may be seen that a discharge took place on it also.

The second (Fig. 4) photograph represents a part of the aluminum electrode at critical tension. As the layer of oxides is very thin, it is very rapidly destroyed by the discharges, which spread in rings from the points of incidental rupture. The representation, therefore, is blurred and dull.

<sup>1</sup> As, for example, rock salt, as shown by A. F. Joffe.



FIG. 3.



FIG. 4.

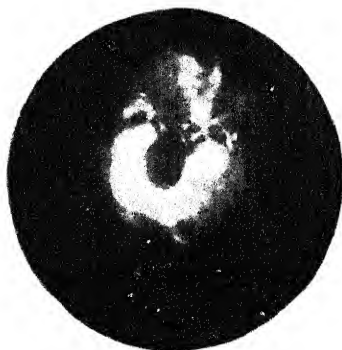


FIG. 5.

WAS. ŠULEJKIN.



Considerably clearer, however, does the discharge appear round the artificial rupture on the *third* photograph (Fig. 5). The black speck in the middle is a bared portion of the metal. In the adjacent layer of oxides (glass!) there was a flashing discharge, encircling the breach like a ring.

From the material just presented some conclusions regarding the nature of the discharge in question may be drawn. First of all, it is apparent that at the critical tension the path of the discharge changes. Namely, at a lower tension its course is "fluid—film of gas—metal" (A), at a higher tension it is "fluid—oxides—metal" . . . (B). The decrease in tension in each path is so great that henceforth we shall consider it equal to the tension between the electrodes, ignoring the fall in tension in the fluid on the way from the metal cathode to the film near the anode. Combined with such a great fall in tension is the generation of a considerable amount of heat at the surface of the anode, and it is quite natural to assume that the discharge's *change of path* is caused by the heating of the layer of oxides, which reaches a "critical" degree at "critical tension." Point III<sub>c</sub> shows that the discharge may take place only at the surface of a solid electrolyte, but not of a crystal possessing metallic conducting properties (rock salt NaCl). Consequently, an essential condition for the formation of a characteristic discharge (change of discharge from path A to path B) is the increase of conducting power in a body by heating. But this condition alone is not sufficient. In fact, at the same temperature, the mobility of ions, and consequently the conducting power, for example, with easily fusible glass is greater than with less easily fusible glass.

Still, however, we see from III<sub>a</sub> that the height of critical tension does not depend on fusibility.

Let us see what indications the external appearance of the discharge may give. The first striking thing of all to observe is that the discharge flashes out suddenly, with a leap, no intermediate stage being noticeable; the portion of glass adjacent to the wire (or the portion of the oxide layer surrounding the point of rupture) all at once becomes heated to white heat. All this reminds us, as nothing could more clearly, of the electric arc phenomenon. This similarity is especially emphasized also by the appearance of the microphotographs 1 and 3 (Fig. 3 and Fig. 5), and the peculiarity III<sub>b</sub>. But even by the microphotographs, taken as they are through a certain thickness of fluid, one cannot definitely be convinced that electric arcs really appeared. As these arcs, if they really exist, can arise only in the interval glass—metal (or oxides—metal), I endeavored to make an experiment of the following character:

At the ends of two platinum (or nickel) wires, placed at a distance of 0.5 to 1 cm. from one another, was fixed a "little bridge" of glass tubing, the ends of which were fused to the wires. If you connect the wires<sup>1</sup> to the source of a constant current with a tension of 100–200 volts and place the "little bridge" over the flame of a Bunsen burner, after some time near the wire-anode will appear bright luminous points.

The points continually move about, going out in one place and appearing in another. Through a magnifying glass it may be distinctly seen that these are miniature electric arcs, seldom reaching the length of 1 mm. If we examine the glass "bridge," after such a discharge, it will be seen that the glass around the wire-anode has acquired a spongy construction. Inside the empty spaces which have been formed move the arcs, which, as it were, "lick" the glass.

If the current is not switched off, but only the heating by the gas flame is discontinued, the arcs for some time do not go out, but the smallest movement of the air is sufficient to extinguish them, after which, without outside heating, they do not appear again.

Now, it seems, we may pass on to an explanation of the critical point of the discharge in question.

The phenomenon, most probably, proceeds thus:

Under the influence of the discharges between the fluid and the metal anode, the layer of glass adjacent to the wire is heated, its conducting power is increased, and a part of the current follows the course of fluid—<sup>+</sup>glass—<sup>+</sup>metal. But, as shown by Nernst,<sup>2</sup> in the glass near the anode a rapid evolution of gas takes place, which "polarizes" the system, separates the glass from the metal. Therefore, at a tension below "critical," the current within the glass may freely be ignored; the discharge proceeds, practically, by way of the fluid—<sup>+</sup>metal.

But if the tension is increased more and more, the edges of the glass at last become heated to such an extent that they begin to emit electrons. At this moment the arc appears between the glass and the metal. The tension, accordingly, at which the arc-path, fluid—<sup>+</sup>glass—<sup>+</sup>metal, is formed is also the critical tension.

For a sufficient heating of the layer of glass, let there be in its proximity, in the gas separating the fluid from the metal, a generation, under the influence of the discharges, of  $W$  joules of heat per second to the unit of surface.

The heat energy of the discharges depends on the fall of tension,  $P$ , and on the number of passing ions,  $q$ .

<sup>1</sup> Essentially in accordance with a rheostat of about 100  $\Omega$ .

<sup>2</sup> Ann. d. Phys.



This latter quantity we cannot determine directly; neither can we judge it by the indications of an ammeter, as its pointer oscillates irregularly; but some observations of the *appearance* of the discharges between the liquid and the metallic wire give reason to *assume* that  $q$  is proportionate to  $P^1$  and  $m$ .<sup>2</sup>

So we suppose:

$$q = k_1 m P, \quad (4)$$

where  $k_1$  is a coefficient of proportionality.

The fundamental condition  $W = \text{const.}$  may be set down thus:

$$W = k_2 \cdot P \cdot q = k_1 \cdot k_2 \cdot m P^2 = \text{const} \quad (5)$$

or

$$m P^2 = \text{const}$$

Hence:

$$P_{cr} = \frac{\alpha}{m^{0.5}} \quad (6)$$

where  $\alpha$  is constant.

If we remember our characteristics of discharges, we see that, for example, the dependence of  $P_{crit}$  on  $m$  for the system glass—metal immersed in a solution NaOH is expressed thus. Instead of an exponent of 0.5, we have only  $\beta = 0.46$ .

Why do all the characteristics, differing from each other in quantity, present nevertheless such remarkable simplicity? Why is there always between  $P_{crit}$  and  $m$  such simple gradual dependence, notwithstanding the fact that  $\alpha$  and  $\beta$  for different substances are different? Our assumption led us to the exponent  $\beta = 0.5$ ; in its turn Fig. 2 and formula (3) perhaps, will give some indication of how *in reality* the ions of the fluid near the surface of a solid electrolyte move.

#### CONCLUSION.

1. The behavior of the aluminum anode *up to* critical tension is explained by a *kinetic*, but not a static equilibrium. In the layer of oxides new ruptures are continually being formed, while the old ones close up.

2. At tensions *above* the critical, the equilibrium is disturbed by the fact that the layer of oxides adjacent to each rupture begins to take part in the discharge, which from the path "fluid—metal" passes over to "fluid—oxides—metal."

As in the first case (A), on the way from fluid to metal, so also in the second (B), on the way from oxides to metal, the discharge takes place

<sup>1</sup> It is easy to observe the increase in number of the cluster of discharges with the increase in  $P$  or  $m$ .

<sup>2</sup> With regard to  $m$ , see II<sub>c</sub>.

in a thin film of gas (oxygen). In the latter case the discharge has the form of an electric arc, heating the edge of the layer of oxides (cathode of the arc) to white heat, and rapidly destroying it.

3. Such a form of discharge may be produced also artificially. To produce it, a metal conductor (anode), touching *below the surface of the fluid* the surface of glass, marble, enamel, or other so-called solid electrolyte, must be immersed in the electrolyte.

4. The critical tension at which this discharge begins, depends on the concentration of the solution, and on the composition of the anion, the dependence between  $P_{crit}$  and  $m$  for all the solutions experimented with being expressed by the formula:

$$P_{cr} = \frac{\alpha}{m^\beta}. \quad (3)$$

For the dependence of  $\alpha$  and  $\beta$  on the character of the anion Table II.

$P_{crit}$  does not depend on the degree of dissociation.

5. In this article it has been possible to explain only roughly the reason for this simple dependence so clearly obtained from the experiments. For the investigation of the actual movement of the ions near the electrode, it may, perhaps, serve as a thread for persons more competent in questions of electro-chemistry.

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Moscow, May, 1918.

## MAGNETO-STRICTION WITH SPECIAL REFERENCE TO PURE COBALT.

BY HOWARD A. PIDGEON.

### PART I. THE WIEDEMANN EFFECT.<sup>1</sup>

THE inter-relations between magnetization and mechanical strain in the ferro-magnetic metals are widely varied in their nature and some of them are extremely complex [in character. Their study has formed such a wide field of investigation that no attempt will be made here to more than briefly outline some of the more salient features of the extensive literature on the subject.<sup>2</sup>

### THE JOULE EFFECT.

In 1847 Joule<sup>3</sup> found that the length of a soft-iron bar was slightly increased when magnetized, and at about the same time Matteucci<sup>4</sup> discovered that the longitudinal magnetism in an iron rod was increased by the application of a longitudinal pull. It was later discovered by Vallari<sup>5</sup> that this effect is reversed in sufficiently strong magnetic fields, and is commonly known as the Vallari reversal. The reciprocal relations between magnetization and mechanical strain discovered by Joule and Matteucci were shown by the work of later investigators to extend throughout the field of magneto-striction and have been dealt with from theoretical considerations by J. J. Thomson,<sup>6</sup> Kirchhoff,<sup>7</sup> Heydweiller,<sup>8</sup> Gans,<sup>9</sup> Houstoun<sup>10</sup> and others, who have derived mathematical relations

<sup>1</sup> This is the first of a series of articles in preparation under the general title given above.

<sup>2</sup> For a fuller discussion of these effects the reader is referred to the following: Poynting and Thomson, *Electricity and Magnetism*; Ewing, *Magnetic Induction in Iron and Other Metals*; Maxwell, *Electricity and Magnetism*, Vol. II., 3d edition, pp. 90-94; Wiedemann, *Electricität*, Vol. 3, p. 519. A fairly comprehensive bibliography of the subject up to that date is given at the end of an article by H. G. Dorsey, *PHYS. REV.*, Vol. 30, p. 718, 1910.

<sup>3</sup> *Phil. Mag.*, Vol. 30, pp. 76, 225, 1847.

<sup>4</sup> *Comptes Rendus*, 1847.

<sup>5</sup> *Pogg. Ann.*, 1868.

<sup>6</sup> Thomson, *Applications of Dynamics to Physics and Chemistry*.

<sup>7</sup> *Wied. Annalen*, 1885, Vol. 24, p. 52.

<sup>8</sup> A. Heydweiller, *Ann. d. Phys.*, Vol. 11, p. 602, 1903.

<sup>9</sup> R. Gans, *Ann. d. Phys.*, Vol. 13, p. 634, 1904.

<sup>10</sup> *Phil. Mag.*, Vol. 21, p. 78, 1911.

for these reciprocal relations, which, however, have been only very roughly verified by data,<sup>1</sup> due no doubt largely at least to magnetic and elastic hysteresis which play a very important role in all magneto-elastic phenomena.

The first really comprehensive view of the Joule effect, as the change in length due to magnetization is called, is due to the extensive work of Shelford Bidwell<sup>2</sup> who worked with specimens of iron, nickel and cobalt, and also studied the change in the effect produced by the application of longitudinal stress. As a result of his work and also that of more recent investigators, especially of Honda and his co-laborers,<sup>3</sup> the general relations between longitudinal magnetization and strain in iron and nickel have been quite definitely established, and with considerably less certainty in cobalt.

The principal features of the work done may be summarized as follows: soft iron elongates when subjected to a magnetic field of weak or moderate strength, retracts in larger fields and in strong fields becomes shorter than its original length. The effect of pull is to increase the intensity of magnetization in weak or moderate fields, and to decrease it in strong fields. The results for steel are similar, but the initial magnetic elongation is less in general.

In the case of nickel, longitudinal magnetization is always accompanied by a decrease in length which approaches an asymptotic value in strong fields, while the effect of longitudinal tension is to diminish the magnetization for all values of the magnetic field.

In the case of cobalt the results have been far less conclusive. Bidwell<sup>4</sup> experimenting with a short rod of cast cobalt found a decrease in length with increasing magnetic field, followed by a retraction reaching the original length at about 750 gauss, and in still stronger fields the specimen became longer than its original length. Thus we see that the effect in cobalt was found to be the exact reverse of that in iron. This result was later confirmed by Nagaoka and Honda<sup>5</sup> for cast cobalt, but a similar annealed specimen was found to decrease in length gradually and did not reach a maximum shortening even in a magnetic field of 2,000 gauss. It was found as expected that the effect of longitudinal tension upon the cast cobalt was to diminish the magnetization in fields of moderate strength and to increase it in very strong fields, while longi-

<sup>1</sup> Honda and Tereda, *Phil. Mag.*, Vol. 14, 1907, pp. 65-107.

<sup>2</sup> *Proc. Roy. Soc.*, 1886, Vol. 40, pp. 109, 257; *Phil. Trans.*, 1888, Vol. 149, p. 205; *Proc. Roy. Soc.*, 1890, Vol. 47, p. 469.

<sup>3</sup> *Phil. Mag.*, 1898, Vol. 46, p. 261; 1900, Vol. 49, p. 329; 1902, S. 6, Vol. 4, pp. 45, 338, 459, 537; 1903, S. 6, Vol. 6, p. 392; 1905, S. 6, Vol. 10, p. 548, 642.

<sup>4</sup> *Phil. Trans.*, vol. 149.

<sup>5</sup> *Phil. Mag.*, Vol. 4, p. 51, 1902.

tudinal pull always diminished the intensity of magnetization in annealed cobalt.

Now all of these specimens were undoubtedly of very impure cobalt, so there has remained some doubt as to what the character of the effect would be in specimens of pure cobalt, since it is known that a marked change may be produced by relatively small quantities of impurities. Bidwell gives no analysis of his specimen but makes the statement that it was soft and easily worked which indicates a considerable amount of impurity since pure cobalt is hard and brittle and not easily worked without special heat treatment. Honda,<sup>1</sup> however, gives an analysis of his specimens which contained from four to five per cent. of nickel, more than one per cent. of iron, and a very high carbon content of from 1.38 to 1.64 per cent.

It was shown by the work of Kelvin,<sup>2</sup> Ewing<sup>3</sup> and others that the effect of longitudinal compression upon magnetization is just the reverse of that produced by longitudinal pull, and that the effect of transverse stress upon longitudinal magnetization is just the reverse of that produced by longitudinal stress.

The work of Nagoaka and Honda<sup>4</sup> has shown that magnetization is also attended by a change in volume which is, in general, of a much smaller order than the change in dimensions.

#### THE WIEDEMANN EFFECT.

Some years previous to Joule's discovery Wiedemann<sup>5</sup> experimenting with iron wires made the interesting discovery that when the specimen was suspended in a very small vertical magnetic field, while at the same time an electric current flowed through the specimen, its free end was observed to twist in such a direction that to an observer looking along the specimen in the direction of the flow of current and also in the direction of the magnetic lines of force, the lines of twist were in the direction of a right-handed screw, that is the twist was positive in direction. The direction of twist was reversed upon the reversal of either the current or the longitudinal field. If the current in the specimen remained constant and the longitudinal field was gradually increased, the twist reached a maximum in fields of from 15 to 36 gauss, gradually decreased in stronger fields until in very strong fields the direction of twist changed and became negative or left handed. This is commonly known as the Wiedemann effect.

<sup>1</sup> Phil. Mag., S. 6, Vol. 4, p. 48.

<sup>2</sup> Kelvin, Reprint of Papers, Vol. II., pp. 332-407.

<sup>3</sup> Loc. cit., p. 202.

<sup>4</sup> Phil. Mag., S. 5, Vol. 46, p. 261; S. 5, Vol. 49, p. 329.

<sup>5</sup> Electricität, Bd. 3.

Wiedemann also found that the reciprocal relations already referred to hold here too, for if a wire in which a current was flowing was twisted it was found that longitudinal magnetization was developed, or if a specimen was subjected to a longitudinal field and then twisted circular magnetization resulted.

Investigation by Knott,<sup>1</sup> and Nagoaka and Honda<sup>2</sup> with nickel wires showed that the twist was always negative in direction and much larger than in iron.

The inability to draw cobalt into the form of wires until recently proved an almost insurmountable obstacle to the study of the Wiedemann effect in that metal. However, Honda and Shimizu<sup>3</sup> did succeed in making some measurements on two cobalt rods 21 cm. in length and approximately one centimeter in diameter. They found that for cast cobalt the twist in small magnetic fields was in the same direction as for nickel. It reached a maximum in a somewhat higher field and then decreased, finally reversing its direction in very strong fields. The magnitude of maximum twist was found to be approximately the same as that in similar specimens of iron. In the case of annealed cobalt the twist was very small and did not reach a maximum until a field of about 200 gauss was reached; it then gradually decreased but did not reverse even in very strong fields.

Beside the uncertainty due to the use of impure cobalt, there is still an additional uncertainty here because it was found that the curves obtained for the twist in similar rods of iron and nickel varied considerably from those obtained when wires approximately one millimeter in diameter were used. It would therefore seem especially desirable in this case to obtain data using specimens of as nearly pure cobalt as possible. Although there are many other interesting magneto-elastic effects we shall not discuss them here since the Joule and Wiedemann effects are the fundamental ones and the only ones studied in this work.

As the result of a detailed study of his own work and that of others, Kelvin was able to explain many of the magneto-elastic effects on the basis of magnetic *æolotropy* produced by stress. In the case of iron in moderate fields he showed that the effect of a simple pulling stress is to produce a greater permeability along than across the lines of strain, while a compressional stress has just the opposite effect. Above the Vallari reversal point the effect is reversed. Kelvin further showed that the idea of magnetic *æolotropy* may be employed to explain the Wiedemann effect and its reciprocal relations. In this case by the super-position of

<sup>1</sup> Trans. Roy. Soc. Edin., Vol. 32, p. 193, 1883.

<sup>2</sup> Phil. Mag., S. 6, Vol. 4, p. 61.

<sup>3</sup> Phil. Mag., S. 6, Vol. 5, p. 650.

the circular field due to the current in the specimen, upon the longitudinal field, the direction of the resultant field in any given element is a diagonal lying in a plane tangent to the element and making an angle with the transverse plane, which varies from zero at the center to a maximum at the circumference. According to Kelvin's theory this must result (for moderate fields) in an elongation in the direction of the resultant field and a shortening in a direction at right angles to it. Since the change in volume is of a smaller order, there must result a shearing strain in a plane making an angle of 45 degrees with the resultant field and, consequently, this strain will have a component in a direction tangent to the element and in a plane perpendicular to the axis of the wire. Such a strain must result in a twist of the specimen. Applying similar reasoning one can predict the nature of the effect produced by stress upon circular or longitudinal magnetization.

Although the conception of magnetic æolotropy is sufficient to explain many of the magneto-elastic effects qualitatively at least, after making due allowance for hysteresis effects, it does not afford a satisfactory explanation of many of the experimental facts.<sup>1</sup> For example, according to this theory the Wiedemann effect may be regarded as only a special case of the Joule effect, and one would accordingly expect to find a comparatively simple relation between the two. However, this is not the case, as has been pointed out by S. R. Williams.<sup>2</sup>

#### OBJECT OF THIS WORK.

The object of this work is two-fold: first, to study the Joule and Wiedemann effects in specimens of pure cobalt wire and if possible to establish a definite relation between the two effects; second, to make a comparative study of these effects in specimens of cobalt, iron and nickel wire previously subjected to exactly the same heat treatment. As has already been indicated, previous investigation of the Wiedemann effect in cobalt has been of such a meager character and all work on cobalt with such impure specimens that it seemed highly desirable to repeat the work with pure specimens now available.

Moreover, as has been pointed out by S. R. Williams,<sup>3</sup> there has been a great lack of coördination in the work done in this field. Much of the experimentation has been upon specimens of whose chemical composition or previous history but little or nothing is known, and since both have a very important influence upon the magneto-elastic effects, much of the data taken has a questionable value so far as making a comparative study

<sup>1</sup> See Ewing's *Magnetic Induction in Iron and Other Metals*, pp. 244 and 246.

<sup>2</sup> *PHYS. REV.*, Vol. 32, p. 295.

<sup>3</sup> *PHYS. REV.*, Vol. 34, p. 258.

is concerned. It is evidently highly desirable that as many as possible of these effects be studied in the same specimens which have previously been subjected to the same heat treatment. It was with this object in view that a considerable portion of this work was undertaken. This paper will deal with the Wiedemann effect in cobalt and for comparison, also in iron and nickel; while a later paper will deal with the Joule effect and a comparative study of these and other magnetic phenomena.

## SPECIMENS.

The specimens of cobalt used were obtained from the laboratory of the School of Mining in Queen's University, through the generosity of Eugene Haanel, director of the Mines Branch, under whose direction H. T. Kalmus and others made an extensive study of cobalt<sup>1</sup> and succeeded not only in producing very pure metal on a commercial basis but also by special heat treatment, in drawing it into wires suitable for many tests otherwise almost impossible. As will be seen from the following analysis given by the School of Mining, the specimens were comparatively pure.

Specimen A.  
Co = 99.73  
Fe = 0.14  
Ni = 0.00  
C = 9.00  
S = 0.019  
Si = 0.02

Specimens B and C.  
Co = 98.71  
Fe = 1.15  
Ni = 0.00  
C = 0.039  
S = 0.012  
P = 0.01  
Si = 0.14

TABLE I.

No. of Specimen.	Material.	Condition.	Length in CM.	Diam. in CM.
A	Cobalt	Annealed in hydrogen.....	25.02	0.0884
B	Cobalt	Unannealed.....	24.92	0.0914
C	Cobalt	Annealed in hydrogen.....	25.06	0.0912
E	Nickel	Annealed in hydrogen.....	25.01	0.1003
F	Nickel	Unannealed.....	25.10	0.1002
H	Iron	Annealed in hydrogen.....	25.00	0.1023

The nickel specimens were from wire labelled "pure nickel" and the iron from the core of an old induction coil, so although no chemical analysis has thus far been made they are undoubtedly of fairly pure material. All of the specimens tested were approximately 25 centimeters in length and varied from 0.85 mm. to 1.00 mm. in diameter. The exact dimensions are given in Table I.

<sup>1</sup> See Bulletin entitled, The Physical Properties of the Metal Cobalt, Canadian Department of Mines, Part II., by H. T. Kalmus and C. Harper.



Specimens A, C, E and H were annealed at 800 degrees centigrade for three hours, after which the temperature was very slowly reduced to that of the room, the process occupying several hours. Oxidation was prevented by keeping a stream of hydrogen flowing through the furnace. After this treatment the iron and nickel were extremely soft and flexible but the cobalt was still hard and showed a decidedly crystalline structure. Cobalt specimen B has the same composition as C but was left in the original condition. Nickel specimen F is the same as E except that it was not annealed. Data was also obtained from specimens of iron and nickel annealed by alternating current, but the results are not given as they did not differ materially from those for specimens E and H.

#### APPARATUS.

The arrangement of apparatus is shown diagrammatically in Fig. 1. The solenoid, *S*, was attached to the plank, *P*<sub>1</sub>, supported firmly between two massive brick pillars about three feet apart. The specimen, *W*, was

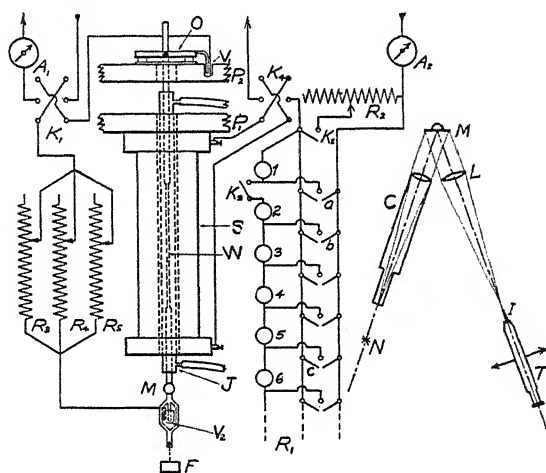


Fig. 1.

soldered to two brass rod extensions and the whole suspended by means of the knife-edge support, *O*, from the plank, *P*<sub>2</sub>, which was supported by the brick piers entirely independent of the plank, *P*<sub>1</sub>. This was to guard against any possible disturbance due to mechanical displacement produced by the field in the solenoid.

The earth's field was determined and the vertical component compensated for by an additional winding on the outside of the solenoid. As it was found in some cases that the rise in temperature of the specimen due to the heating of the solenoid produced quite an appreciable effect

upon the readings, a double-walled water jacket,  $J$ , made of brass tubing was placed inside the solenoid and water from a hydrant kept flowing through it during the experiment. Electrical connection with the specimen was made by means of the mercury cups,  $V_1$  and  $V_2$ , into which dipped copper contacts attached to the specimen. By means of a special construction illustrated in the figure, a small weight,  $F$ , could be suspended from the specimen without interfering with its freedom of motion. The weight was made just large enough to keep the wire accurately vertical in the central line of the solenoid, and to prevent the weight from vibrating it was suspended in oil at the end of a long flexible cord. The entire suspended system was enclosed to prevent the disturbing influence of currents of air in the room.

As it was desired to measure angles varying from zero to considerably more than one degree in magnitude with an accuracy of at least one second for the smaller angles, a special device was necessary for the purpose. After trying several methods, the very simple one illustrated in Fig. 1 was chosen. Lengthwise in the fairly narrow slit of the collimator,  $C$ , was mounted a fine glass fiber. A Nernst glower,  $N$ , brilliantly illuminated the slit, rays from which after being rendered parallel by the collimator lens fell upon the plane mirror,  $M$ , mounted upon the brass rod attached to the lower end of the specimen. The reflected parallel rays were focussed by an achromatic lens,  $L$ , forming an image at  $I$ , which was viewed by the traveling microscope,  $T$ . This image when viewed through the microscope showed a very well-defined diffraction pattern produced by the double slit, and consisted of a large number of alternate light and dark parallel bands. By special adjustment two or three of these lines could be made to appear predominantly clear and well defined, and upon these very good settings could be made by means of double cross-hairs in the eye-piece of the microscope.

The system was calibrated by replacing the specimen and mirror,  $M$ , with another mirror mounted in the axis of a lever approximately 65 cm. in length moved by a micrometer screw. It was found that one division of the micrometer screw of the microscope corresponded to approximately one second of twist of the specimen. By taking an average of two or three settings it was possible to set within one half division, corresponding to  $0.5''$  of arc or  $0.02''$  per centimeter length of specimen, for small angles. With larger deflections the image became less distinct so that the accuracy of setting was not quite as great but in all cases amply sufficient. Considerable difficulty in obtaining measurements was experienced due to vibrations in the building or produced by passing street cars, so that it was found necessary to take data at night when traffic was reduced.

A diagram of the electrical connections is shown in Fig. 1. A special form of rheostat,  $R_1$ , made of carbon lamps was selected, which although it did not have the ease of manipulation nor advantage of continuous change of resistance possessed by a liquid or slide-wire rheostat, had the very decided advantage of certainty of setting permitting readings to be easily repeated, which was very desirable in this work. The rheostat was so constructed that all the lamps could easily be placed in series, in parallel, or in various combinations of lamps in parallel with a group of lamps. For example, with switches  $a$ ,  $b$  and  $c$  closed, lamp  $x$  was connected directly across the line in parallel with 3, 4 and 5 in series. This arrangement made it possible to increase the current from zero to a maximum by as few or as many steps as desired. The adjustable rheostat,  $R_2$ , was connected in parallel with  $R_1$  to obtain the higher values of current.

Current through the specimen was controlled by means of two field rheostats,  $R_3$  and  $R_4$ , in parallel with a slide wire rheostat,  $R_5$ . The latter provided for fine adjustment necessary to keep the current constant during a run.

The current was read by means of Weston milli-voltmeters provided with one and five ampere shunts. They were frequently calibrated by means of a potentiometer, standard cell and standard resistances.

The solenoid was 38 cm. in length and hence the field was not quite uniform throughout the 25 cm. length of the specimen. From approximate dimensions the variation from the maximum, of the average field over the length of the specimen was computed. As no accurate data for the solenoid were obtainable it was necessary to determine its constant experimentally. This was done by means of a ballistic galvanometer and a ballistic coil whose dimensions were accurately known. The galvanometer was calibrated by means of a standard condenser. The constant after applying the correction indicated above was found to be 210.5 gauss per square cm. per ampere. A later determination using a mutual inductance instead of the condenser gave a result differing from that given above by only a small fraction of one per cent.

#### METHOD OF OBSERVATION.

The specimen was thoroughly demagnetized before each run by gradually decreasing an alternating current flowing through the solenoid, which process was repeated several times with decreasing voltages applied. Demagnetization by less frequent reversals made by hand and also by a mechanical commutator constructed for the purpose, was also tried and as no difference was observed it was concluded that the demagnetization

by A.C. was complete in most cases. (Certain exceptions will be noted later.)

Keeping the circular field constant four runs were made as follows: both longitudinal and circular fields direct; both reversed; circular field direct, longitudinal field reversed; circular field reversed, longitudinal field direct. In many cases one or more repetitions of these runs were made. When both fields were direct the relation of circular and longitudinal fields and of twist for iron, was right handed or positive according to the convention previously mentioned.

#### DATA AND RESULTS.

Examination of the plotted results showed that these four runs gave two distinct curves, one when both fields were direct or reversed, the other when either one was reversed. Which one of the curves was obtained evidently depended upon the direction of twist in the specimen since the agreement between the curves of each pair seems to eliminate other possibilities. This lack of agreement has been noticed by other investigators and ascribed to various causes, consequently a study of this effect was made to determine its origin.

Curves 1 and 2, Fig. 2, show typical results for cobalt specimen A, when both fields are direct and when one is reversed respectively. Curve 3 is the average of curves 1 and 2.

Curves 5, 6 and 7 show the same thing for iron specimen H; and curves 9, 10 and 11 for nickel specimen E.

It was found in the case of every specimen that after careful demagnetization the application of either the longitudinal or circular field alone produced a small twist, whose value varied with different specimens and the magnitude of the magnetizing field. The initial twist due to the circular field was, however, extremely small for most of the specimens. The maximum value of the twist produced by the longitudinal field varied from 0.18" to 4.33" per cm. in different specimens and, although its value varied somewhat with different runs on the same specimen, it was always in the same direction and seemed to be practically independent of the previous condition of magnetization or of demagnetization so long as the latter was reasonably thorough. In general the twist was greater in unannealed than in annealed specimens of the same material. Curves 4, 8 and 12 show average results for these correction curves for specimens A, H and E respectively.

In the case of nickel the initial twist due to the circular field varied quite erratically with different runs both in magnitude and direction. However, if the twist, as measured from the original zero when neither

field was operating, be plotted from the correction curve obtained when the longitudinal field alone was operating, as a new zero axis of twist the resulting corrected curves almost coincide. An example of the result of this operation is shown in Fig. 2 by the double row of dots, obtained

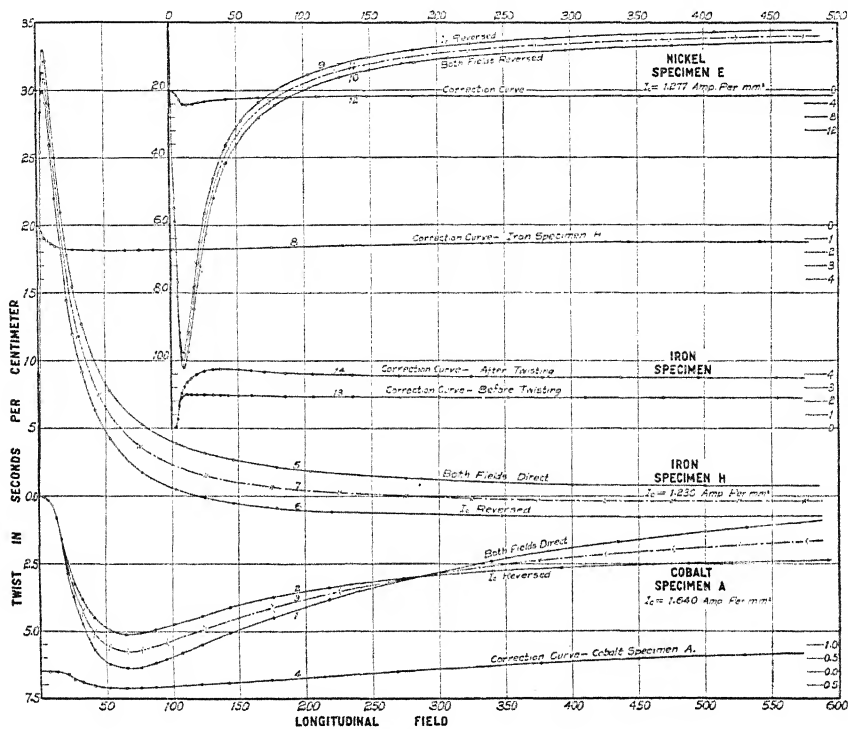


Fig. 2.

by applying curve 12 as a correction to curves 9 and 10. It is seen the dots almost coincide with curve 11 which is the average of curves 9 and 10. In some cases there was some divergence near the peaks of the curves but the agreement was always good for higher values of the longitudinal field.

Treating the data for iron in the same manner gave even better results, an example of which is shown by the dots almost coinciding with curve 7, obtained by applying curve 8 as a correction to curves 5 and 6.

In the case of cobalt, however, the best agreement was obtained by measuring the twist not from the initial zero but from that obtained after the application of the circular field.

An analysis of these rather complex results indicates that the lack of symmetry in twist is apparently due to various combinations of the three

following factors: imperfect demagnetization, change in temperature of the specimen due to the heating effect of the current in it, and æolotropic structure of the specimen.

Imperfect demagnetization was in evidence in the annealed nickel specimens only, which apparently accounts for the initial twist of erratic character due to circular field. This is not at all surprising in this case since for very small values of the longitudinal field, not only is the susceptibility relatively high making perfect demagnetization difficult, but the twist is also very large, thus greatly magnifying the effect of even an extremely small amount of residual magnetism. Indeed, it may be added that this method affords an extremely delicate means of detecting residual magnetism in annealed nickel. No such effect is observed in hard drawn nickel since both the initial susceptibility and the twist are very low for small values of the longitudinal field.

The fact that this initial twist in cobalt and iron due to the circular field alone, was always in the same direction and of about the same magnitude for any given specimen seems to indicate that if residual magnetism were present it was so small that its effect could not be observed. The observed twist was evidently due to heating and æolotropy in these cases.

Twist caused by either of these two effects alone could be distinguished by the fact that æolotropy produces a true magnetic twist for which correction is necessary as in the case of iron; while the effect of heating is merely to change the zero of twist by producing a shift in the equilibrium configuration of the specimen allowing a partial adjustment of residual strains. Such was the case with cobalt. However, if both of the above effects be present, the net twist due to circular field is the algebraic sum of the two components, correction for only one of which should be made. Consequently, the application of the corrections in the manner previously described results, in general, in two parallel curves, the intercept between which is twice the variation of the correction actually made for the circular field (it was zero in the case of cobalt) from the component due to æolotropy alone.

Since the corrected curves for iron and cobalt almost coincide as shown by the curves, we conclude that the twist due to circular field in the case of the former is due almost wholly to æolotropy, and to rise of temperature in the case of the latter.

That this initial effect in cobalt is due primarily to change in temperature is further supported by the following facts. Not only did the maximum twist show a very marked increase with rise of temperature, but in one case after making a run at approximately 60 degrees centi-

grade and demagnetizing, a twist of approximately 4.0" per cm. was observed on reducing the temperature to approximately 15 degrees.

Furthermore, the initial twist in cobalt seemed to increase approximately as a parabolic function of the current which one would expect if due to the heating effect.

On the other hand the effect of temperature change in iron and nickel was so small as to be almost completely masked by the other factors.

The twist produced in all specimens by the application of the longitudinal field alone is apparently due almost wholly to æolotropy caused perhaps by permanent residual strains sustained in drawing, by subsequent coiling of the wire, etc. As we have seen, such an effect may give a circular component to longitudinal magnetism.

This view is supported by the fact that annealing reduced the effect, and by the following experiment. One end of an iron specimen which

TABLE II.

*Annealed Iron Specimen H. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters.*

$I_c = 0.1229.$		$I_c = 0.2951.$		$I_c = 0.615.$		$I_c = 1.230.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
1.50	0.87	1.50	2.50	1.39	8.11	2.93	26.90
2.14	2.32	2.14	6.02	2.14	14.41	4.71	31.39
3.21	3.28	3.21	8.21	3.16	18.12	6.64	30.61
4.28	3.72	4.28	9.16	4.28	19.52	9.33	27.43
5.57	3.74	5.57	9.18	5.56	19.29	12.74	23.49
7.28	3.57	7.28	8.71	7.33	18.01	17.08	19.34
9.21	3.29	9.31	7.96	9.26	16.45	21.41	16.08
11.25	3.01	11.30	7.30	11.30	14.90	25.9	13.61
14.35	2.66	14.35	6.41	14.35	12.94	32.3	10.89
18.42	2.26	18.41	5.43	18.40	10.82	53.3	6.01
23.88	1.83	23.88	4.42	23.92	8.65	77.2	3.49
36.4	1.18	30.4	3.41	30.4	6.76	102.7	2.16
49.5	0.79	49.7	1.89	36.4	5.57	124.5	1.51
69.3	0.50	59.3	1.45	49.7	3.75	146.3	1.06
84.6	0.30	69.4	1.14	59.3	2.92	179.3	0.61
104.8	0.23	84.7	0.81	69.4	2.30	204.1	0.38
133.8	0.15	104.9	0.54	84.7	1.71	275.5	0.00
168.7	0.08	133.9	0.30	104.9	1.19	347	-0.20
196.6	0.07	169	0.16	134.0	0.76	401	-0.30
246.5	0.02	197	0.10	169.1	0.48	450	-0.34
332	0.00	223	0.05	197.0	0.33	501	-0.35
362.5	-0.003	275	-0.02	223	0.23	552	-0.34
450.2	-0.05	334	-0.06	274	0.03	608	-0.33
		392	-0.10	334	-0.03	659	-0.31
		452	-0.13	391	-0.10	708	-0.30
				451	-0.13	800	-0.25
						854	-0.22



$I_c = 2.459.$		$I_c = 3.554.$		$I_c = 5.331$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.89	24.35	2.5	18.37	2.3	12.04
4.6	33.98	4.1	27.71	4.1	20.32
6.6	37.68	5.6	34.55	5.7	28.31
9.4	37.12	7.7	38.85	7.8	35.42
12.8	33.80	10.4	39.62	10.5	39.50
17.1	29.07	14.0	37.27	14.1	40.52
21.5	24.82	17.4	33.92	17.8	39.23
26.1	21.40	21.1	30.25	21.5	36.17
32.6	17.42	26.5	25.80	27.1	31.95
42.1	13.41	34.6	20.62	35.1	26.35
53.4	10.11	44.1	16.01	44.9	20.87
77.7	6.08	54.2	12.61	66.0	13.53
103.0	3.86	65.7	9.95	88.1	8.98
125.3	2.67	73.0	8.59	106.3	6.76
147.4	1.87	83.0	6.39	126.1	4.86
179.7	1.11	106.2	4.68	154.0	3.01
205.7	0.71	126.0	3.36	190.9	1.55
282	-0.05	153.9	2.10	242.5	0.25
406	-0.59	190.7	1.09	300	-0.60
508	-0.68	242	0.24	353.5	-1.05
616	-0.59	329	-0.56	400	-1.33
715	-0.53	397	-0.92	445	-1.45
864	-0.49	490	-1.05	492	-1.45
		591	-0.99	544	-1.42
		685	-0.90	593	-1.37
		778	-0.80	637	-1.35
				700	-1.28
				779	-1.19

had been tested previously, was twisted through 360 degrees, leaving a permanent twist of 180 degrees. Subsequent tests showed that the average maximum twist was reduced and in strong fields the twist was increased in one direction and decreased in the other. The twist due to either circular or longitudinal field alone was greatly increased, the latter being almost doubled as shown by curves 13 and 14, Fig. 2. Data for curve 13 were taken before and for curve 14 after twisting.

One peculiarity still remains unexplained. In some cases the corrected curves showed a considerable divergence in the region of maximum twist. It is perhaps due to a difference in the modulus of rigidity for direct and reversed twist, as magnetic twist must always be accompanied by actual mechanical strain since the tangential component of shear due to the magnetic fields does not vary uniformly from the center to the circumference of the wire.

At any rate it seems that one may safely conclude that the average curve for any given specimen gives very closely at least, the single curve



that would be obtained if experimental conditions could be better controlled and the specimen freed from accidental eccentricities.

In this work the data and curves shown are the result of selecting data from runs giving nearly average results for twist in the two directions and averaging these results. Where runs could not be found very closely approximating the average result an actual average of the different runs is given.

The results for iron are shown in Table II. and in Fig. 3 for several values of current as indicated. In general characteristics the results

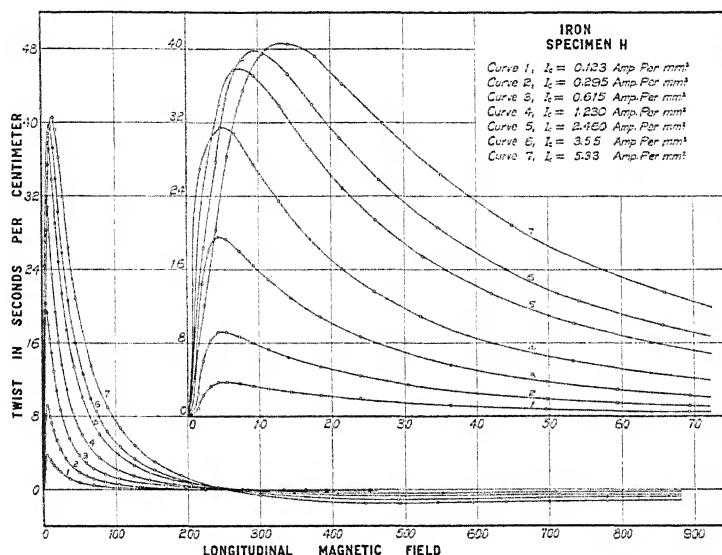


Fig. 3.

agree quite well with those obtained by other observers; in fact they show a striking similarity to results obtained for steel tubes by S. R. Williams.<sup>1</sup> It will be observed that the values of the maximum twist increase roughly proportional to the current in the specimen at first then seemingly approach a maximum. This is what one would expect if the Wiedemann effect is a special case of the Joule effect, since the elongation in iron reaches a maximum in fairly low fields, and consequently with large circular fields the magnitude of the resultant field and its angle with the axis of the wire do not combine to form a condition so favorable for a large twist.

The maxima which gradually move into higher longitudinal fields with increase of circular field, are not only very sharp in this specimen but also occur at somewhat lower fields than observed by others. This is.

<sup>1</sup> *Loc. cit.*

doubtless due to the extremely soft, well-annealed condition of the iron, since in another specimen evidently not so well annealed by alternating current, the maxima were broader and came in stronger fields.

The curves show rather striking characteristics in stronger fields. Not only do they all reverse their direction of twist at approximately the same value of longitudinal field,—a fact the reason for which is not apparent,—but in still higher fields they reach a maximum negative twist and at the highest values of field obtained, seem to be very gradually approaching the axis of zero twist. So far as the author knows this characteristic has never been previously observed. Apparently in only one other instance<sup>1</sup> has the Wiedemann effect been studied with values of longitudinal field above 400 or 500 gauss, and that was with rods about one centimeter in diameter whose characteristics were quite different. A further study of this feature,—for which the Joule effect offers no apparent explanation,—will be necessary to determine its cause and whether it is characteristic of all well annealed iron specimens.

The results for annealed nickel specimen E are given in Table III.

TABLE III.

*Annealed Nickel Specimen E. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters.*

$I_c = 0.306.$		$I_c = 0.639.$		$I_c = 1.277.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.9	16.57	2.9	39.2	2.8	56.8
4.5	27.17	4.5	55.0	4.6	79.0
6.5	32.97	6.5	63.8	6.6	93.7
9.2	33.30	9.2	64.6	9.4	99.8
12.7	28.88	12.7	56.5	12.8	94.5
16.86	23.17	17.1	45.8	17.1	81.5
21.3	18.64	21.4	37.4	21.6	69.0
25.7	15.61	25.8	31.5	26.1	59.4
32.1	12.68	32.2	25.6	32.7	49.0
41.7	9.90	41.8	20.1	42.4	39.0
52.9	7.86	53.2	16.07	66.8	25.85
77.0	5.80	77.4	11.39	78.3	22.43
102.5	4.36	102.8	8.81	103.8	17.28
124.3	3.64	124.8	7.34	126.4	14.32
146.0	3.16	146.6	6.23	148.3	12.18
179.7	2.64	180.2	5.12	182.3	9.83
223.2	2.10	224	4.13	216.4	8.40
279	1.80	280	3.33	289.5	6.30
349	1.38	406	2.32	392	4.69
405	1.18	508	1.86	503	3.66
507	1.00	619	1.54	620	3.03
619	0.84	721	1.36	720	2.59
718	0.80	867	1.15	866	2.19
867	0.66				

<sup>1</sup> Honda and Shimizu, Phil. Mag., Vol. 5, S. 6, p. 650.

$I_c = 2.554.$		$I_c = 3.691.$		$I_c = 5.536.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.7	62.1	2.9	62.9	2.9	56.6
4.5	90.3	4.6	93.4	4.50	88.8
6.4	113.2	6.5	119.5	6.4	116.9
8.9	130.7	9.3	142.4	9.1	146.7
12.4	137.2	12.7	155.4	12.5	170.5
16.5	132.3	17.0	157.7	16.3	186.2
20.8	121.4	21.5	149.7	21.2	189.9
25.1	109.7	25.9	139.2	25.6	186.3
31.3	94.8	32.3	123.5	32.0	174.8
40.5	77.8	41.8	104.0	41.3	154.8
51.5	63.9	53.1	86.4	52.4	132.4
65.4	51.9	67.1	71.2	76.3	98.0
75.0	46.1	77.2	62.8	101.1	75.7
102.0	34.6	102.4	48.2	122.8	63.0
120.8	29.56	124.3	40.1	144.9	53.8
142.0	25.09	146.1	33.9	174.6	44.39
174.1	20.51	179.6	27.7	220.5	35.30
206.47	17.31	222.7	22.40	275.4	28.35
271	13.16	280	17.77	344	22.72
338	10.54	348	14.28	396	19.60
392	9.06	403	12.36	495	15.67
489	7.28	502	9.86	598	13.07
593	6.03	607	8.29	694	11.40
688	5.22	701	7.08	833	9.40
824	4.33	838	5.95		

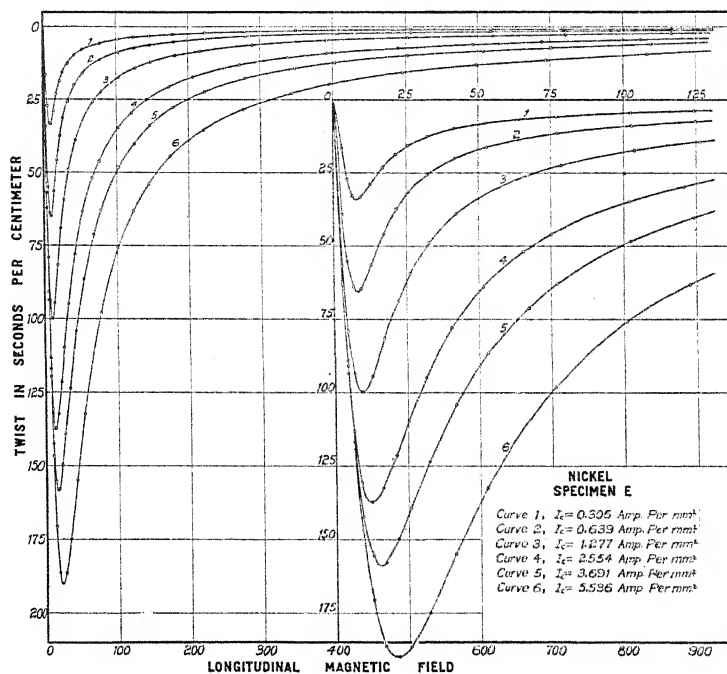


Fig. 4.

TABLE IV.

*Unannealed Nickel Specimen F. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters.*

$I_c = 0.641.$		$I_c = 1.281.$		$I_c = 2.563.$		$I_c = 3.704.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.7	0.02	2.6	0.02	2.5	0.09	2.7	0.14
4.5	0.05	4.3	0.06	4.3	0.17	4.5	0.235
6.4	0.065	6.3	0.09	6.2	0.24	6.4	0.36
12.6	0.14	8.6	0.15	8.6	0.36	9.1	0.51
21.4	0.32	11.8	0.21	11.8	0.49	16.9	1.10
32.2	0.73	15.7	0.34	19.9	1.06	25.8	2.74
41.8	1.08	19.9	0.50	30.1	2.66	41.7	6.54
53.2	1.44	29.9	1.18	38.9	4.19	53.3	8.30
66.2	1.74	38.7	1.98	49.4	5.50	65.9	9.66
77.3	1.92	49.2	2.68	61.4	6.54	77.2	10.62
88.2	2.03	61.2	3.23	71.9	7.21	102.5	11.63
102.6	2.12	82.0	3.83	82.3	7.68	117.6	11.96
115.8	2.16	95.2	4.06	95.9	8.08	124.2	12.04
124.5	2.17	101.5	4.14	101.7	8.24	136.4	12.07
136.9	2.19	109.0	4.22	109.3	8.37	146.0	12.06
146.5	2.19	115.0	4.26	115.9	8.45	163.9	11.96
164.2	2.17	126.4	4.29	127.4	8.54	179.2	11.78
179.9	2.13	135.1	4.30	136.0	8.56	204.0	11.46
204.5	2.06	151.0	4.28	152.0	8.52	222.2	11.16
223	2.01	165.1	4.22	166.4	8.44	278	10.26
256	1.92	187.6	4.14	188.6	8.26	347	9.18
348	1.65	203.5	4.06	205.5	8.08	400	8.49
404	1.52	232	3.90	237	7.72	450	7.91
505	1.32	252	3.80	257.5	7.48	499	7.40
613	1.16	285	3.60	295	7.07	555	6.90
711	1.04	314	3.42	321	6.80	603	6.51
785	0.98	363	3.18	373	6.28	655	6.12
851	0.92	407	2.96	417	5.88	695	5.84
		455	2.78	464	5.50	760	5.44
		498	2.60	508	5.18	825	5.06
		550	2.44	565	4.82		
		595	2.31	609	4.56		
		640	2.19	660	4.28		
		682	2.08	695	4.12		
		682	2.08	777	3.78		
		765	1.90				

and Fig. 4. They agree remarkably well with results obtained by other observers. It will be observed that for nickel the twist is opposite in direction to that in iron, does not reverse its direction and the maximum value is more than four and one half times as great as in iron. The maximum twist in iron for the highest value of current used is approximately 40.5" per cm., while for a slightly higher value of current in nickel it is 190" per cm. As in iron the peaks advance at first slowly, then more

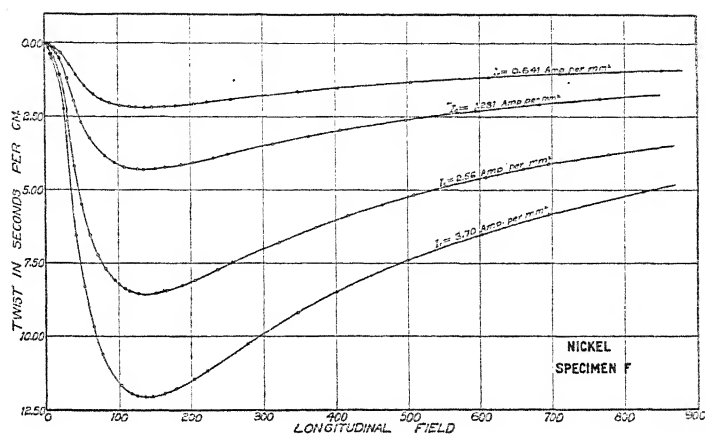


Fig. 5.

TABLE V.

Annealed Cobalt Specimen A. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters.

$I_c = 0.393.$		$I_c = 0.820.$		$I_c = 1.640.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
4.3	0.01	4.5	0.06	2.7	0.04
6.4	0.02	6.4	0.10	4.5	0.08
9.0	0.04	9.1	0.22	6.5	0.17
12.6	0.18	12.6	0.41	9.2	0.34
16.9	0.37	16.9	0.83	12.6	0.82
21.2	0.59	21.2	1.29	16.9	1.62
25.6	0.76	25.5	1.69	21.2	2.63
32.0	0.97	31.9	2.13	25.6	3.48
41.6	1.19	41.3	2.55	32.0	4.27
53.0	1.31	52.5	2.84	41.4	5.09
65.9	1.33	65.0	2.94	52.9	5.57
74.5	1.31	76.1	2.92	65.3	5.72
87.2	1.29	86.7	2.86	76.4	5.69
101.5	1.22	100.3	2.77	87.0	5.55
123.1	1.14	121.8	2.58	101.0	5.35
144.7	1.05	143.0	2.39	122.4	4.96
177.5	0.95	175.6	2.18	143.8	4.57
201.9	0.87	216.5	1.91	176.0	4.11
275	0.65	272.5	1.66	217.5	3.62
341	0.55	339	1.41	273.0	3.10
394	0.49	391	1.30	340	2.59
490	0.42	486	1.10	392	2.30
594	0.33	533	1.01	488	1.95
686	0.26	679	0.85	592	1.64
819	0.20	811	0.75	685	1.42
				825	1.13

$I_c = 3.279.$		$I_c = 4.734.$		$I_c = 7.102.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.7	0.10	2.5	0.24	2.5	0.72
4.6	0.20	4.3	0.52	4.3	1.48
6.5	0.43	6.4	0.98	6.4	2.66
9.1	0.95	8.9	1.97	12.5	7.77
12.7	2.03	12.5	3.79	20.1	15.45
16.9	3.92	16.9	6.41	25.5	18.99
21.2	5.80	21.2	9.07	31.8	22.58
25.6	7.25	25.6	11.27	41.2	25.83
32.1	9.06	32.0	13.77	52.3	27.48
41.6	10.63	41.6	15.96	64.8	27.69
52.9	11.55	53.0	17.10	75.8	27.17
65.8	11.83	65.7	17.38	86.5	26.40
77.6	11.70	76.6	17.14	100.5	25.19
87.1	11.46	87.4	16.70	122.1	23.25
95.5	11.19	101.7	16.00	143.8	21.45
101.5	11.00	123.4	14.84	176.4	18.98
123.3	10.20	145.0	13.67	200.5	17.47
144.5	9.45	178.1	12.18	275	13.94
177.5	8.48	202.8	11.26	341	11.76
219.8	7.40	276	9.04	393	10.45
276	6.36	344	7.66	489	8.61
342	5.37	399	6.82	590	7.30
396	4.77	498	5.66	680	6.37
494	4.00	603	4.83	807	5.42
596	3.40	695	4.24		
687	3.00	827	3.57	8.9	4.78
817	2.55			16.8	11.93

rapidly into higher longitudinal fields with increasing circular field. However, the peaks of the curves seem to approach a maximum value of twist much less rapidly than in the case of iron. Here again the sharply defined maxima occurring in comparatively low fields are an indication of good annealing. A specimen of the same wire not so well annealed by alternating current gave curves (not shown) having flatter maxima of less magnitude and occurring in higher longitudinal fields as in the case of iron.

This is very strikingly illustrated by the results for specimen F, shown in Table IV. and Fig. 5, which was a hard drawn nickel wire exactly the same as specimen E except that it was not annealed. Here the broad flat maxima, not only occur in very much higher fields, but have only about one thirteenth the magnitude of the former. It clearly shows the importance in work on magneto-striction of knowing the previous history of the specimen, especially the heat treatment.

In Table V. and Fig. 6 are shown the results for cobalt specimen, A, and in Table VI. and Fig. 7 for the less pure specimen, C. A comparison

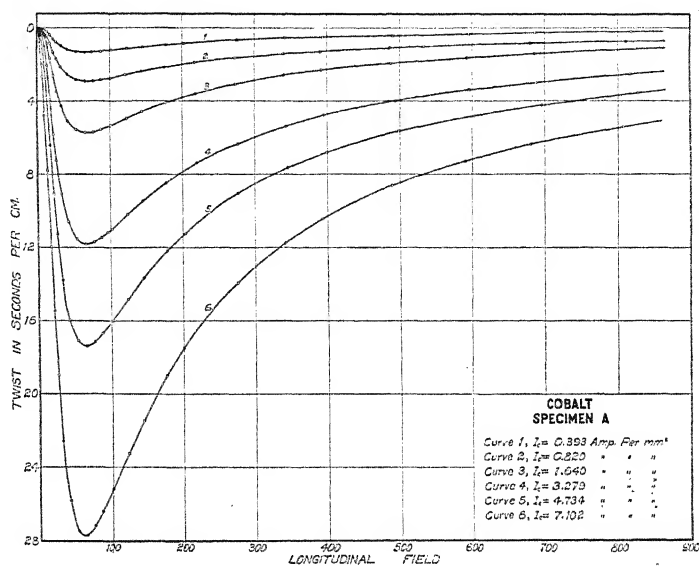


Fig. 6.

TABLE VI.

Annealed Cobalt Specimen C. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters.

$I_c = 0.308.$		$I_c = 0.771.$		$I_c = 1.542.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.7	0.02	2.7	0.04	2.7	0.04
4.3	0.03	4.3	0.06	4.3	0.10
6.4	0.05	6.4	0.09	6.4	0.17
9.0	0.06	9.0	0.16	9.0	0.31
12.5	0.12	12.54	0.32	12.5	0.65
16.8	0.35	16.90	0.92	16.8	1.92
21.1	0.75	21.2	1.90	21.2	4.01
25.5	1.13	25.5	2.83	25.5	5.78
31.8	1.51	31.8	3.77	31.8	7.61
41.2	1.77	41.2	4.40	41.3	8.92
52.3	1.84	52.4	4.62	52.6	9.22
65.0	1.80	65.1	4.52	65.3	9.02
76.1	1.74	76.3	4.34	76.4	8.63
86.9	1.62	87.0	4.12	87.2	8.21
100.9	1.54	101.2	3.83	101.2	7.66
122.6	1.37	123.1	3.43	123.1	6.84
144.2	1.24	144.7	3.06	144.7	6.14
177.2	1.07	177.5	2.62	177.7	5.28
201.5	0.94	202.1	2.37	202.2	4.74
275.7	0.73	276.2	1.83	276.2	3.60
343.6	0.57	345	1.47	346	2.92
399	0.49	400	1.27	399	2.54
499	0.41	500	1.01	498	2.03
608	0.32	608	0.84	608	1.65
705	0.28	704	0.71	703	1.41
852	0.23	850	0.59	847	1.17

$I_c = 3.083.$		$I_c = 4.451.$		$I_c = 6.677.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.7	0.12	2.7	0.27	2.7	1.20
4.5	0.23	4.5	0.53	4.5	2.52
6.4	0.42	6.4	0.94	6.4	4.66
9.1	0.80	9.1	1.83	9.08	9.00
12.6	1.78	12.6	4.23	13.5	16.32
16.9	5.00	16.96	10.20	16.9	26.55
21.2	9.33	21.2	16.58	21.2	35.60
25.6	12.93	25.6	21.40	25.5	42.25
31.9	16.68	32.0	25.85	31.85	48.42
41.5	18.53	41.6	28.69	41.3	51.88
52.7	19.07	52.9	29.13	52.4	51.68
65.5	18.57	65.6	28.08	65.1	49.10
76.5	17.72	76.5	26.67	76.3	46.10
97.3	16.81	87.2	25.25	87.0	43.18
101.6	15.62	101.7	23.32	101.1	39.40
123.2	13.93	123.3	20.73	122.9	34.49
144.7	12.46	144.8	18.41	144.4	30.15
177.8	10.64	177.9	15.65	177.1	25.23
210.8	9.27	202.2	14.01	201.6	22.34
276	7.25	276.1	10.51	275.8	16.28
341	5.89	344	8.49	343	12.83
397	5.06	398	7.29	398	10.92
497	4.01	496	5.78	498	8.46
602	3.31	601	4.73	604	6.78
695	2.82	694	4.01	696	5.71
832	2.32	828	3.28	833	4.60

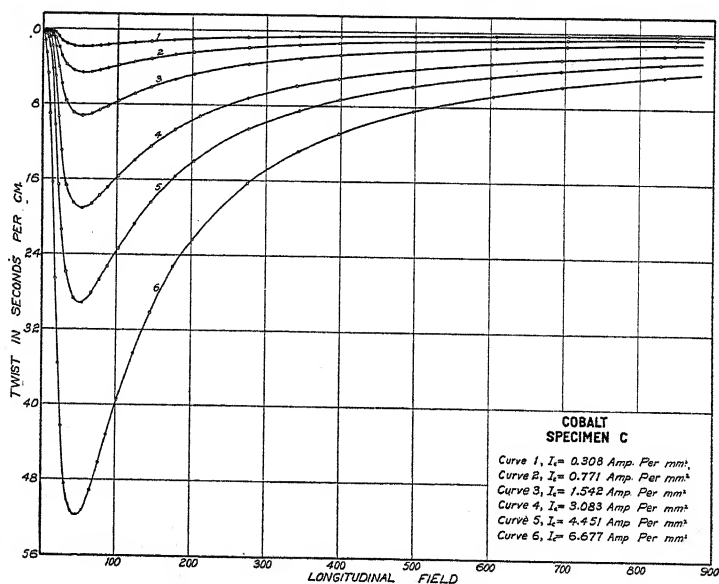


Fig 7.



of the curves shows that the maximum twist for the purer specimen is much less than for the other and occurs at higher values of the longitudinal field. This is not surprising when we consider that the purer cobalt is more highly crystalline and consequently exhibits a greater magnetic hardness and a correspondingly lower susceptibility; properties which as we have seen in the case of nickel, produce exactly this sort of an effect.

These results for pure cobalt differ very materially from those obtained

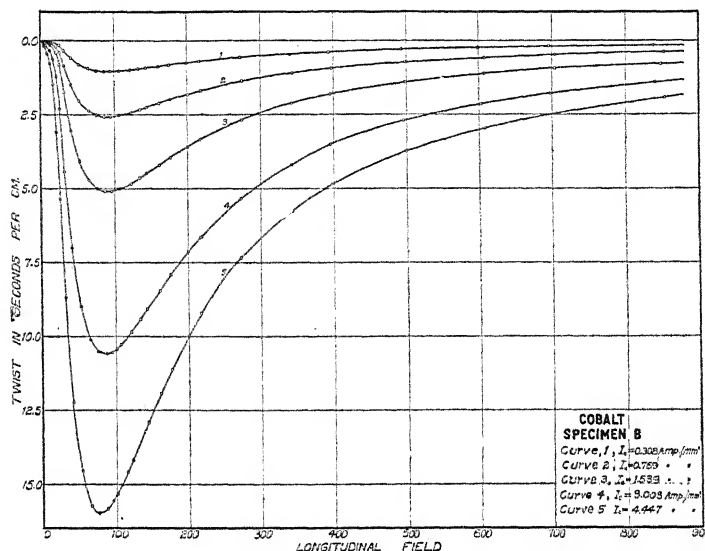


Fig. 8.

by Honda,<sup>1</sup> for not only is there no reversal of twist in strong fields such as he found for a cast cobalt rod, but the curves show very little resemblance to those he obtained for an annealed cobalt rod. The difference is doubtless due to the difference in purity of cobalt.

Examination of the curves also shows that there is a very marked similarity between the results for cobalt and nickel, making due allowance for the difference in the magnitude of twist in the two cases. In fact, there is actually less difference between the curves for annealed specimens of cobalt and nickel than between those for annealed and unannealed nickel. (It will be shown in a later publication that this similarity holds also for the Joule effect.) This suggests the possibility that, if a more satisfactory method of annealing cobalt could be found reducing it to a less marked crystalline condition or at least to a finer crystalline texture,

<sup>1</sup> Honda and Shimizu, Phil. Mag., Vol. 5, S. 6, p. 650.

the results might compare even more favorably with those for nickel. Such a suggestion receives some support from the results for cobalt specimen B, given in Table VII. and Fig. 8. This specimen differs from specimen C, only in not being subjected to further annealing.

Here again we see the marked effect of annealing and that the result corresponds exactly with that found in iron and nickel. Indeed it may be stated as a general characteristic of all three metals that, the more thorough the annealing the greater is the magnitude of the maximum twist, the sharper the peaks of the curves, and the lower the values of the longitudinal field in which they occur. Now this is exactly the sort of effect that annealing has upon the susceptibility. So, qualitatively

TABLE VII.

*Unannealed Cobalt Specimen B. Current,  $I_c$ , in Specimen in Amperes per Square Millimeters*

$I_c = 0.308.$		$I_c = 0.769.$		$I_c = 1.539.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
4.3	0.008	2.6	0.01	2.6	0.02
9.0	0.02	4.4	0.03	4.5	0.04
16.6	0.04	6.4	0.04	6.4	0.06
20.9	0.07	9.0	0.05	9.0	0.08
25.4	0.15	12.4	0.06	12.5	0.13
31.7	0.34	16.7	0.11	16.7	0.21
40.8	0.58	21.0	0.20	21.1	0.40
52.0	0.80	25.4	0.40	25.5	0.82
64.5	0.94	31.8	0.84	31.8	1.74
75.7	1.02	40.97	1.49	40.9	3.05
86.2	1.03	52.1	2.04	52.2	4.07
94.5	1.04	64.7	2.37	64.7	4.71
100.2	1.02	75.9	2.52	76.1	5.01
106.8	1.02	86.5	2.57	86.6	5.09
121.7	0.98	94.8	2.57	95.0	5.08
133.7	0.95	100.5	2.55	100.6	5.05
143.0	0.92	107.1	2.52	107.1	5.00
160.4	0.86	122.1	2.44	122.2	4.84
175.5	0.81	134.1	2.34	134.1	4.64
199.7	0.74	143.7	2.26	145.6	4.49
216.4	0.69	161.2	2.11	161.0	4.20
272.5	0.56	176.2	1.98	176.5	3.95
340	0.45	200.6	1.81	200.6	3.58
393	0.39	218.7	1.69	217.9	3.34
493	0.28	274.5	1.36	275	2.69
599	0.24	343	1.10	342	2.12
695	0.20	397	0.93	395	1.82
839	0.16	498	0.73	497	1.41
		607	0.58	606	1.12
		704	0.47	704	0.93
		853	0.38	849	0.78

$I_c = 3.008.$		$I_c = 4.447.$		$I_c = 3.008.$		$I_c = 4.447.$	
$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$	$H.$	$\theta.$
2.6	0.05	2.6	0.10	106.9	10.27	107.1	15.00
4.4	0.08	4.3	0.18	121.8	9.83	115.3	14.57
6.4	0.13	6.4	0.26	133.8	9.42	122.2	14.17
9.00	0.22	9.0	0.44	143.2	9.07	143.7	12.90
12.4	0.35	12.5	0.75	160.7	8.45	161.2	11.93
16.8	0.61	16.7	1.46	176.0	7.90	176.5	11.12
21.0	1.20	21.0	3.08	200.2	7.11	200.9	9.96
25.5	2.30	25.4	5.35	217.9	6.64	218.7	9.22
31.7	4.42	31.7	8.68	273.6	5.34	275	7.34
40.9	7.02	40.97	12.21	342	4.20	343	5.75
52.1	8.99	52.1	14.55	396	3.50	397	4.87
64.7	10.11	64.7	15.73	495	2.72	498	3.74
75.7	10.50	76.0	15.98	602	2.15	608	2.97
86.5	10.59	86.6	15.82	697	1.79	705	2.46
94.6	10.54	95.0	15.55	841	1.40	854	1.93
100.4	10.42	100.7	15.31				

at least, there seems to be a close connection between the susceptibility and the Wiedemann effect. However, the latter is so complex that the relation is far from being a simple one. It is hoped that further study may bring out more definite relations.

The results for iron specimen H, nickel specimen E, and cobalt speci-

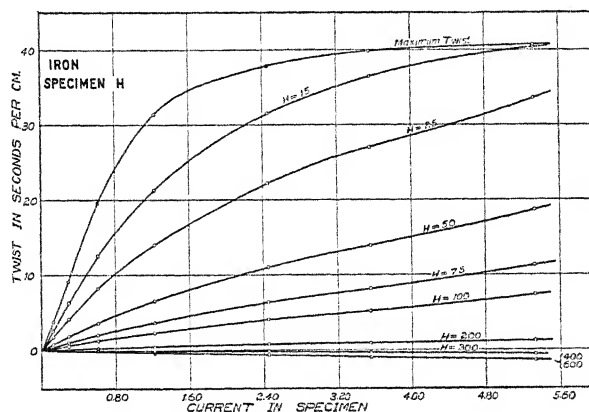


Fig. 9.

men C are presented in a somewhat different way in Figs. 9, 10 and 11 respectively, where for several values of the longitudinal field, values of current in the specimen are plotted as abscissæ and the corresponding twist as ordinates. In the case of nickel and cobalt it is seen that for values of the longitudinal field very large compared with the circular

field, the twist is approximately proportional to the latter. This is exactly what would be expected if the Wiedemann effect be a modification of the Joule effect. For, let  $P$ , Fig. 12, be any particle in a cylin-

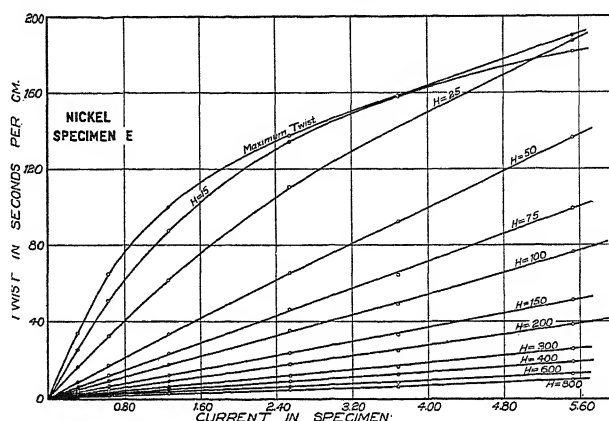


Fig. 10.

drical element of the wire. The approximate net effect of combined longitudinal and circular magnetization is to displace the particle through the distance  $\lambda$ , in the direction  $PP'$  of the resultant magnetic field, making

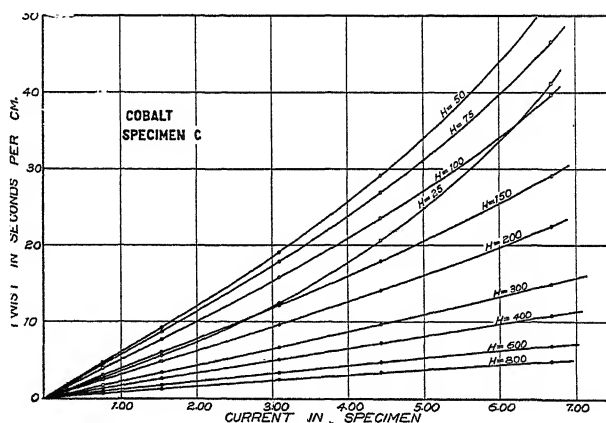


Fig. 11.

the angle  $\theta$  with the length of the element in the direction of the axis of the wire. The twist,  $\theta$ , is given by,

$$\theta = \frac{\lambda}{r} \sin \theta$$

$$= \frac{\lambda}{r} \frac{H_c}{\sqrt{H^2 + H_c^2}}$$

where  $H_c$  is the circular field,  $H$  the longitudinal field and  $r$  the distance of the particle from the axis. If  $H$  is very large compared to  $H_c$  then

$$\theta = \frac{\lambda}{rH} H_c.$$

Previous study of the Joule effect has shown that  $\lambda$  in nickel, reaches practically a constant value in moderately strong magnetic fields. From the equation it is seen that for constant values of  $H$ ,  $\theta$  is very nearly a linear function of  $H_c$ . This result, derived only for high values of magnetic field in a cylindrical element, at least gives an indication of what may be expected in a wire. For any given value of  $H_c$  the twist varies inversely as  $H$ , if  $H$  is very large, and consequently approaches zero as a

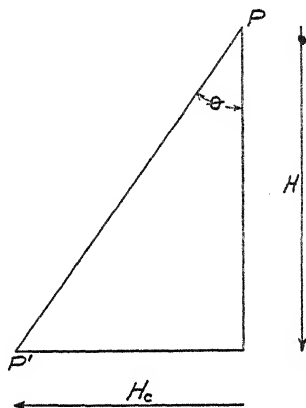


Fig. 12.

limit as  $H$  is increased indefinitely. The curves indicate that this is perhaps the case for nickel and cobalt.

These results correspond quite well with what one might predict from the molecular theory of magnetism; for in strong longitudinal fields which magnetize the material almost to saturation, the molecular magnets are almost all aligned in the direction of the field so that the only effect of the circular field is to produce a deflection of these molecular magnets proportional to the deflecting field with a corresponding deformation of the magnetized material.

Cobalt becomes saturated only in much stronger fields than nickel and consequently the agreement is not so good.

In this case as in that of all the magneto-elastic effects in iron, the results are very complex and any attempt to explain them without further study is fruitless.

Any analysis of magneto-elastic effects is made much more difficult by

the marked hysteresis occurring in all of them. In Figs. 13, 14 and 15 are shown hysteresis curves for iron, nickel and cobalt respectively, obtained by keeping the circular field constant and cyclically varying the longitudinal field. The comparatively large area inclosed in the hys-

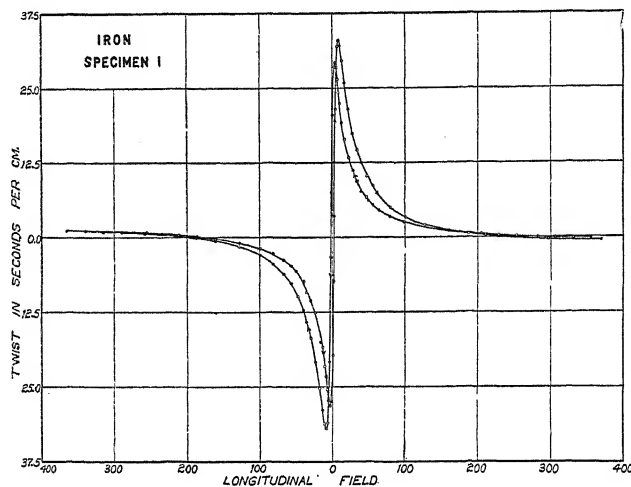


Fig. 13.

teresis loop for cobalt indicates its rather extreme magnetic hardness and low susceptibility, which has been previously mentioned.

Some of the features of the Wiedemann effect will be still further dis-

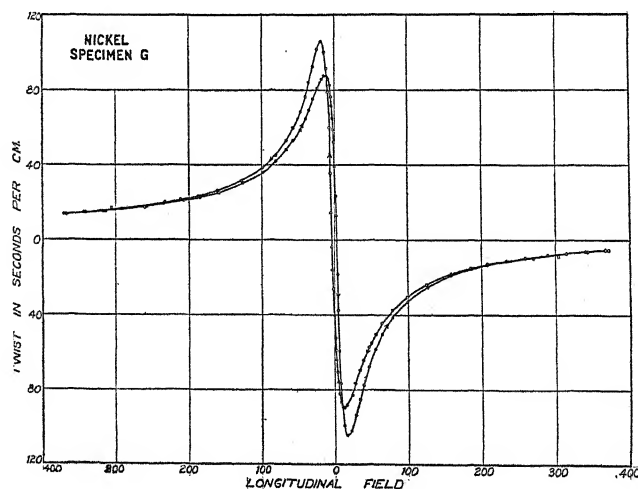


Fig. 14.

cussed in a later publication in connection with the Joule effect and other magnetic phenomena.

## SUMMARY.

Experimental curves are shown for the Wiedemann effect in specimens of pure cobalt wire, and for the purpose of comparison, results are also shown for specimens of iron and nickel subjected to the same heat treatment. A comparison of the results is made and certain features of the Wiedemann effect discussed.

The results for pure cobalt differ materially from those obtained by

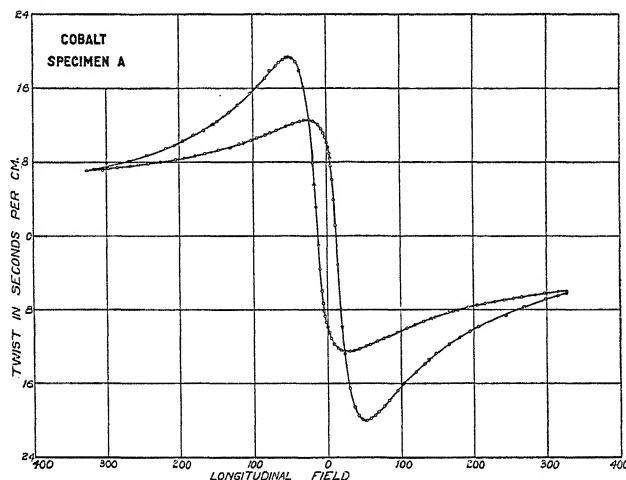


Fig. 15.

other observers working with impure specimens, and show a close similarity to the results for nickel, the twist, however being much less.

Certain eccentricities of twist observed in all specimens are studied and an explanation given which seems to account for nearly all of the observed facts.

Hysteresis curves for iron, nickel and cobalt are shown.

Experimental results for the Joule effect and other magnetic phenomena will be presented in a later publication.

The writer wishes to express his thanks to various members of the department staff for helpful suggestions given, and especially to Professor E. L. Nichols through whom the specimens were obtained and under whose direction the work was done.

PHYSICAL LABORATORY,  
CORNELL UNIVERSITY,  
September, 1918.

## ON THE MECHANICAL AND ELECTRODYNAMICAL PROPERTIES OF THE ELECTRON: ADDENDUM.

BY MEGH NAD SAHA.

It appears that the relation

$$-\mu = -\frac{m_0 c^2}{e} = \frac{f_{12}}{\Omega_{12}} = \frac{f_{23}}{\Omega_{23}} = \frac{f_{31}}{\Omega_{31}} = \frac{f_{14}}{\Omega_{14}} = \frac{f_{24}}{\Omega_{24}} = \frac{f_{34}}{\Omega_{34}},$$

which was given in my paper<sup>1</sup> "On the Mechanical and Electrodynamical Properties of the Electron" and which was obtained from the determinant

$$\begin{vmatrix} f_{12} + \mu\Omega_{12} & f_{13} + \mu\Omega_{13} & f_{14} + \mu\Omega_{14} \\ f_{21} + \mu\Omega_{21} & f_{23} + \mu\Omega_{23} & f_{24} + \mu\Omega_{24} \\ f_{31} + \mu\Omega_{31} & f_{32} + \mu\Omega_{32} & f_{34} + \mu\Omega_{34} \\ f_{41} + \mu\Omega_{41} & f_{42} + \mu\Omega_{42} & f_{43} + \mu\Omega_{43} \end{vmatrix} = 0$$

or

$$(f_{12} + \mu\Omega_{12})(f_{34} + \mu\Omega_{34}) + (f_{23} + \mu\Omega_{23})(f_{14} + \mu\Omega_{14}) + (f_{31} + \mu\Omega_{31})(f_{24} + \mu\Omega_{24}) = 0,$$

cannot hold. For, from the principle of least action written in Minkowski's form,

$$\delta \int m_0 c^2 ds - \int \delta W \cdot ds = 0,$$

we obtain the relation

$$\int [(f_{12} + \mu\Omega_{12})(dx\delta y - \delta x dy)] + 5 \text{ other similar terms} = 0.$$

But it is not possible to equate to zero the coefficients of the six-components  $(dx\delta y - \delta x dy)$  of the area-six-vector  $(dSx\delta s)$  as was done in that paper, for though  $(\delta x, \delta y, \delta z, \delta l)$  represent an arbitrary displacement  $(dx, dy, dz, dl)$ , is not so, but represent the actual displacements. We have, therefore, to collect the coefficients of  $(\delta x, \delta y, \delta z, \delta l)$  and put them separately equal to zero. In this way we obtain

\* PHYSICAL REVIEW, January, 1919.



$$\begin{aligned}
 -\frac{m_0 c^2}{e} &= \frac{f_{12}\omega_2 + f_{13}\omega_3 + f_{14}\omega_4}{\Omega_{12}\omega_2 + \Omega_{13}\omega_3 + \Omega_{14}\omega_4} = \frac{f_{21}\omega_1 + f_{23}\omega_3 + f_{24}\omega_4}{\Omega_{21}\omega_1 + \Omega_{23}\omega_3 + \Omega_{24}\omega_4} \\
 &= \frac{f_{31}\omega_1 + f_{32}\omega_2 + f_{34}\omega_4}{\Omega_{31}\omega_1 + \Omega_{32}\omega_2 + \Omega_{34}\omega_4} = \frac{f_{41}\omega_1 + f_{42}\omega_2 + f_{43}\omega_3}{\Omega_{41}\omega_1 + \Omega_{42}\omega_2 + \Omega_{43}\omega_3}, \quad (A)
 \end{aligned}$$

which are simply another form of the Minkowskian equations

$$m_0 c^2 \frac{d^2 x}{ds^2} = X, \quad m_0 c^2 \frac{d^2 y}{ds^2} = Y, \quad m_0 c^2 \frac{d^2 z}{ds^2} = Z, \quad m_0 c^2 \frac{d^2 l}{ds^2} = L \quad (A')$$

for

$$\frac{d^2 x}{ds^2} = -(\omega_2 \Omega_{12} + \omega_3 \Omega_{13} + \omega_4 \Omega_{14}), \text{ etc.}$$

The form (A) as it involves the acceleration six-vector may for certain purposes prove more convenient than the form (A').

## NEW BOOKS.

*The Theory of the Relativity of Motion.* By RICHARD C. TOLMAN. Berkeley: University of California Press, 1917. Pp. ix + 225.

This is an introduction to Einstein's physical theory of relativity and its developments, of a scope about like that of Cunningham's book.

The first chapter gives a historical review, a mere sketch but enough to hint at the development of the physical setting from Galileo and Newton to Michelson and Morley. Chapters two to six give an elementary account of the new kinematics and dynamics of a particle together with the optical matters of aberration, Doppler effect, and Fresnel convection. The material here is about that of Einstein's pioneer papers with one important and undoubtedly wise change, in that the introduction of the concept of mass in connection with phenomena of impact gives direct access to dynamics without requiring reference to particles electrically charged, though the field of a moving charge and the relations of mass and energy are reached as corollaries. This earlier portion of the book, nearly half, uses only elementary mathematical notions.

The rest of the volume is naturally less elementary. Chapters seven to eleven deal with dynamics of systems, some aspects of thermodynamics and statistical mechanics, and elastic bodies. In each case the principle of least action and Lagrange's equations have a fundamental place, in the last case with a frank avowal of the questions left open because of the lack of unique determination of the kinetic potential. Chapter twelve outlines the theory of the electromagnetic field, including brief reference to moving dielectrics. Here the whole development is based on an assumed form for the electromagnetic kinetic potential, giving an elegance of treatment that one would not like to miss. It may be asked however whether it would not have been more appealing to most readers to exhibit first, as Einstein did, the invariance of the Maxwell-Lorentz equations, and then pass over to the variational deduction, giving an order of thought more like that used for the dynamics. The final chapter is an outline of vector analysis in four dimensions, with a few examples of its use in formulating the theory previously given.

The book deserves hearty commendation as an interesting introduction to a subject often considered difficult of access, and will win the reader's thanks after he has pruned away a few obvious blemishes. Some passing remarks are distinctly irritating; for example, the reference to physical "model-making," one of the glories of physics, especially in the nineteenth century, as a "debauch." The reviewer was once reminded however by a biological friend that irritation and stimulation are not sharply distinct.

A. C. L.

## THE PHYSICAL REVIEW.

### ON THE IRREGULARITIES OF MOTION OF THE FOUCAULT PENDULUM.

By A. C. LONGDEN.

**SYNOPSIS.** In a brief historical statement, emphasis is put upon the current opinion that a Foucault pendulum must be very long and very heavy in order to be successful. It is then shown that the elliptical motion so common in Foucault pendulum experiments is not due to insufficient length or weight, or even to atmospheric disturbances, but simply and solely to unequal freedom of motion in different directions. The disturbing influences are within the pendulum itself. Different forms of support and suspension are discussed, and methods of eliminating the disturbing influences are suggested. What the author describes as a double roller support is shown to be superior to any other form of support thus far proposed. With this support and a perfectly annealed double bifilar suspension, a  $2\frac{1}{2}$ -meter pendulum weighing a single kilogram succeeds perfectly.

THE pendulum experiment of Léon Foucault, by means of which he demonstrated the axial rotation of the earth, is held in reverence by physicists and astronomers, not simply as one of the successful experiments of the nineteenth century, but as one of the brilliant experiments of all time.

Foucault began his work on this problem with a pendulum not more than 2 meters long, but finding the results unsatisfactory, he increased the length to 11 meters, and finally, under the great dome of the Pantheon, to 67 meters. Taking advantage of the height of the great cathedral towers of Europe, the experiment was more or less successfully repeated at Cologne, Reims, Amiens, St. Jacques, and elsewhere, where towers of great height could be made available.

Many attempts have been made to reduce this classic experiment to lecture room dimensions, but by common consent the short wire and small mass have been condemned as unsatisfactory. More than a score of well-known physicists and astronomers are on record as affirming that the Foucault pendulum must be very long and very heavy in order to give satisfactory results—some say, to prevent cross vibrations or to keep the pendulum from falling into an elliptical path. Many insist

that it must be set in motion with great care and protected from external disturbances, which, it is said, develop irregularities in its motion. In a recent article in a scientific journal on "A Laboratory Method of Demonstrating the Earth's Rotation," it is stated that the Foucault pendulum method is inapplicable in many laboratories because there is no convenient place to hang a sufficiently long and heavy pendulum.

We shall probably never be quite certain of the exact conditions under which Foucault's celebrated experiment was performed. Our sources of information, in addition to the very brief records of the French Academy of Sciences,<sup>1</sup> are the papers published by Mme. Foucault,<sup>2</sup> ten years after Foucault's death, and the contributions of Lissajous<sup>3</sup> and Flammarion,<sup>4</sup> published still later.

Most of the important information concerning the pendulum of the Pantheon in its final form, is found in one of the papers in Mme. Foucault's "Recuile," entitled "Explications sur l'experience relative au mouvement de la terre." This paper was without date and had not been published by Foucault himself, though he is said to have published a similar paper in le Journal des Debats in March, 1851.

That the work was rushed through in great haste is indicated by Flammarion's statement that it was undertaken in January, 1851, and Mme. Foucault's statement that it was presented to the French Academy of Sciences, February 3 of the same year.

The records are not in perfect agreement concerning the dimensions of the pendulum in its final form, but the evidence is abundant that it consisted of a mass of 28 kilograms, suspended by a steel wire about 67 meters long, strained somewhat beyond its elastic limit. The fact that the wire was greatly strained is important later on.

A few years ago I set up a 60-foot Foucault pendulum consisting of an iron ball weighing about 8 kilograms, suspended by a No. 18 brass wire, from a tolerably good though by no means perfect double knife-edge support. The ball was provided with a spike at the bottom to enable the observer to determine its position, as it moved back and forth over a chart provided with a circular scale calibrated in degrees.

The performances of this pendulum were not very satisfactory and I attributed its partial failure to the presence of air currents in the shaft. However, after trying the pendulum a number of times, I found that its eccentric behavior was repeated over and over again in much the same

<sup>1</sup> *Compte Rendu*, t. XXXII., p. 135.

<sup>2</sup> *Recuile des Travaux Scientifiques de Léon Foucault*. Publié par Madame Veuve Foucault, Sa Mère. Paris, 1878.

<sup>3</sup> *Notice Historique sur la vie et les travaux de Léon Foucault*.

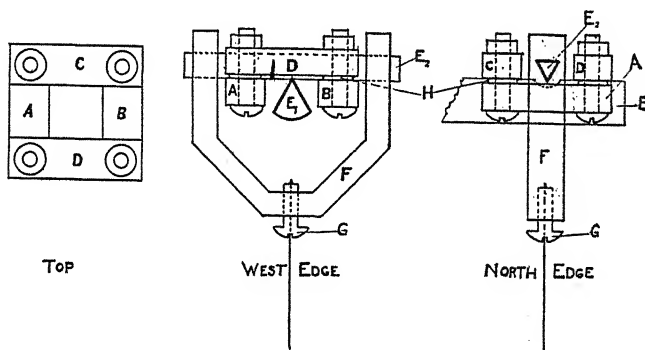
<sup>4</sup> *Notice scientifique sur la pendule du Panthéon*.

way. I could not convince myself that such *regular* "irregularities" were accidental, and I was quite sure they were not due to the uncertain shifting of air currents.

For the purpose of making a closer study of the Foucault pendulum and its behavior, I moved this pendulum to the lecture room, suspending the 8 kilogram ball from a more carefully constructed double knife-edge support attached to one of the ceiling beams over the lecture table. The pendulum was now a little more than  $2\frac{1}{2}$  meters long. It was started swinging by tying the ball back and burning the thread, in the usual way. It was allowed to swing through an arc of about  $10^\circ$ .

A few preliminary experiments gave me reason to believe that none of the seemingly irregular movements of the pendulum were in any way chance or accidental, but that they could all be produced and reproduced under perfectly definite conditions.

The double knife-edge support used is shown in detail in Figs. 1 and 2.



Figs. 1 and 2.

$E_1$  and  $E_2$  are the knife-edges,  $ABCD$ , the rocker, consisting of hardened and polished steel plates bolted together at the corners, and  $F$ , a brass hanger, carrying the wire in a machine screw,  $G$ , which facilitates substituting one wire for another.

The upper knife-edge could be raised or lowered by varying the vertical distance between the plates of which the rocker was made. This might be done by means of micrometer screws, but in these experiments it was done by inserting thin metal plates between the upper and lower halves of the rocker, as at  $H$ , in Figs. 1 and 2.

With this support the behavior was that of a Blackburn pendulum with slightly different periods in two directions at right angles to each other. Since the elliptical motion of a Blackburn pendulum depends upon the ratio of these two periods, it ought to be possible to adjust

the distance between the planes of the double knife-edge support in such a way as to get any desired amount of eccentricity, or to eliminate it altogether.

In one of the preliminary experiments with the double knife-edge support adjusted at random, the pendulum started on its elliptical path, rotating clockwise as viewed from above; and I noticed that as the minor axis of the ellipse increased in length, the shift of the plane of vibration of the pendulum, as marked by the position of the major axis of the ellipse, increased at an abnormal rate. After verifying this fact by repeated observations, I undertook to investigate the relation of the length of the minor axis of the ellipse, to the position of the major axis.

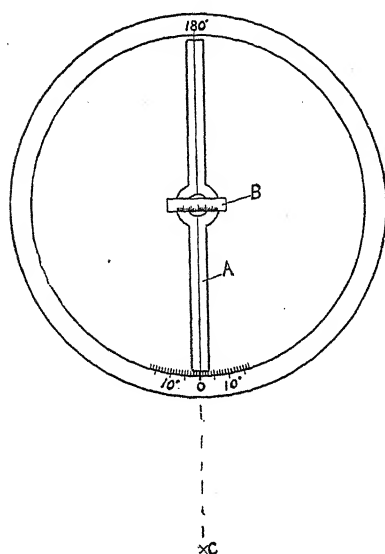


Fig. 3.

I placed a movable index, *A*, Fig. 3, upon the circular scale over which the pendulum was swinging and mounted a scale of millimeters, *B*, at right angles to it at the center of the circle. The index consisted of a

light wooden bar with a screw running through it at the center, and with a fine black line running lengthwise along the bar from zero to 180° on the scale.

With a pendulum provided with a sharp spike travelling close to the black line on the movable index, and with a suitable lens placed as at

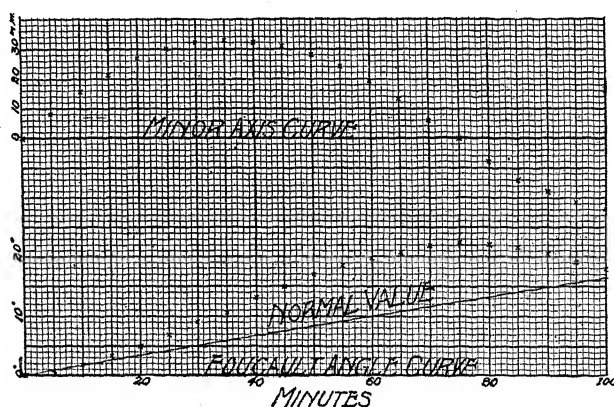


Fig. 4.

*C*, in Fig. 3, it is always possible to read the length of the minor axis of the ellipse to single millimeters and in many cases to approximate tenths of millimeters; while the position of the major axis can be read to within  $\frac{1}{10}^\circ$  with ease and certainty, or to  $\frac{1}{100}^\circ$  approximately.

Starting the pendulum swinging from north to south, with an amplitude of 20 centimeters (an arc of about  $9^\circ$ ), I read the length of the minor axis of the ellipse, and the angle between the major axis and the meridian every 5 minutes for an hour and three quarters. The results are tabulated below and plotted in the curves shown in Fig. 4. In the column marked "Direction of Rotation," clockwise or direct is marked "*D*," and counterclockwise or reverse is marked "*R*." In the plot, the minor axis curve is above and the curve representing the position of the major axis in angular displacement from the meridian, and which we shall hereafter call the Foucault angle curve, is below. The length of the minor axis is in millimeters and the Foucault angle is in degrees, on the *Y* axis. Both are in minutes on the *X* axis. The full curves are not actually drawn, because the points plotted are so close together that it was not thought necessary to connect them.

Time in Minutes.	Minor Axis in Mm.	Direction of Rotation.	Foucault Angle in Degrees.
0	0		0
5	8.0	D	.65
10	15.5	"	1.8
15	21.0	"	3.1
20	26.2	"	5.0
25	30.0	"	6.8
30	32.0	"	9.0
35	32.5	"	10.5
40	32.0	"	13.0
45	30.5	"	14.9
50	28.0	"	16.9
55	24.0	"	18.5
60	19.2	"	19.85
65	13.0	"	20.8
70	6.0	"	21.8
75	0		22.2
80	8.0	R	22.1
85	14.0	"	21.6
90	18.0	"	20.3
95	21.5	"	18.9
100	24.0	"	17.7

The minor axis curve is certainly a close approach to a simple sine curve. It probably would be a perfect sine curve if it were not for the fact that the amplitude of the pendulum is diminishing all the time and this affects the length of the minor axis of the ellipse as well as the major axis.



Note that the length of the minor axis increased to 32.5 millimeters in 35 minutes and then diminished to zero at the end of 75 minutes, when it crossed the  $X$  axis of the plot and the pendulum began to rotate counterclockwise. Note also that while the pendulum was rotating clockwise, the Foucault angle increased abnormally; but that when the pendulum began to rotate counterclockwise, the Foucault angle began to diminish.

The calculated value of the Foucault angle for the latitude of Galesburg is  $9.83^\circ$  per hour. The straight line in the Foucault angle plot, running from the origin to the  $9.83^\circ$  point at the 60-minute line, represents this value. It will be noted that the plotted curve is much above this line most of the time, but that it reaches its greatest height and starts down again when the minor axis curve falls below the  $X$  axis of the plot, that is, when the pendulum begins to rotate counterclockwise.

At the end of this series of observations the pendulum was accidentally thrown out of adjustment. After readjusting, it was started swinging in the usual way, but this time it started rotating counterclockwise, and continued rotating in that direction during the entire series of

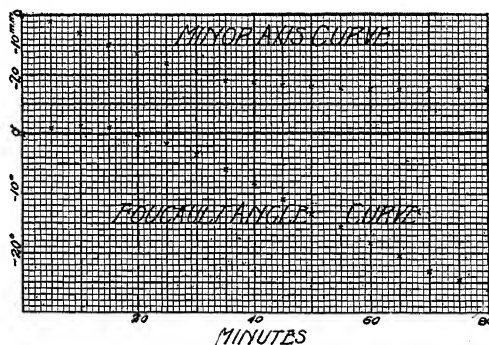


Fig. 5.

observations which followed. That is, it did not complete a half cycle and reverse its direction of rotation as it did in the preceding series.

The curves for this series are shown in Fig. 5.

Our first glance at these curves failed to reveal the fact that they had anything in common with those in Fig. 4, but further inspection shows that they are alike in the fact that they both show that counterclockwise rotation of the pendulum tends to reverse the direction in which its plane of vibration is shifting.

However, I had in Fig. 4 a part of a cycle in which the rotation of the pendulum was mostly clockwise, and in Fig. 5 a part of a cycle in which the rotation was entirely counterclockwise.



It then occurred to me to shorten the cycle, so that I might have an opportunity to compare the minor axis curve with the Foucault angle curve through several complete cycles, before the amplitude of the pendulum became very greatly reduced. This was easily accomplished by readjusting the distance between the planes of the double knife-edge support, so as to change the length of the pendulum in one direction without changing it in the other.

After adjusting the support for about 3 or 4 cycles per hour, I made the series of observations represented in the curves shown in Fig. 6.

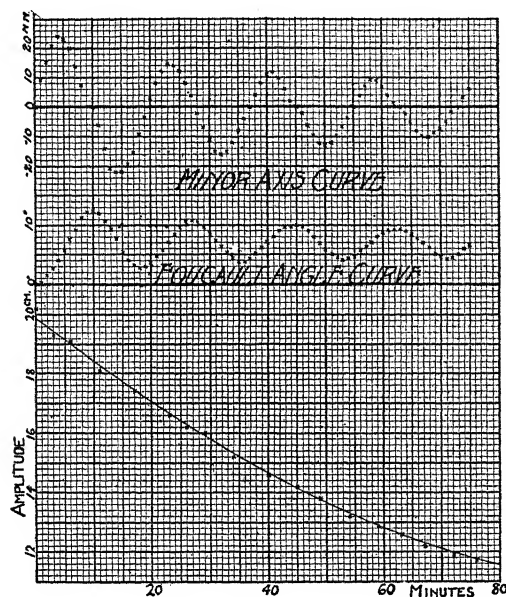


Fig. 6.

In this series, both readings were made *every minute* for an hour and a quarter. Also occasional readings of the amplitude of vibration of the pendulum, along the major axis of the ellipse, for the purpose of determining the decrement.

These curves show in a most striking manner, the relation of the elliptical motion of the pendulum to the shift in its plane of vibration. In every case, when the direction of rotation changes (when the minor axis curve changes sign) the direction of the shift in the plane of vibration also changes. Expressing it in another way, whenever the rotation of the pendulum is clockwise, the Foucault angle is increasing, and whenever the rotation is counterclockwise, the Foucault angle is decreasing. There is a slight time lag as the Foucault angle curve rises, and the

reverse as it falls, because the net effect plotted is the minor axis effect plus or minus the real Foucault effect.

At this point I thought my troubles were over, for it now seemed that in order to get a normal Foucault angle, it would only be necessary to adjust the double knife-edge support for equal lengths of the pendulum in both directions. Then there would be no elliptical motion, and consequently no shifting of the plane of vibration except that due to the rotation of the earth on its axis,—the normal Foucault angle.

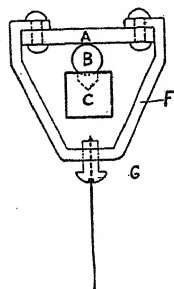


Fig. 7.

I expected to arrive with very little difficulty at an adjustment which would eliminate the elliptical motion, but all my efforts in this direction were futile. In sheer desperation I abandoned the double knife-edge and designed the following ball and plane support:

In Fig. 7, *A*, is the edge of a hardened steel plate 30 millimeters square. The lower surface of this plate is plane and highly polished. It rests at its center upon a polished steel ball, *B*, which in turn rests in a depression drilled in the upper side of a square steel rod, the end of which is shown at *C*. The hanger, *F*, is symmetrical, having four arms (only two are shown in the drawing) running obliquely down from the four corners of the plate to the small brass plate carrying the machine screw, *G*, through which the suspension wire passes.

As this hanger was perfectly symmetrical, and the pendulum free to swing in any direction from the upper surface of the ball as its point of suspension, it seemed that there could be no possibility of elliptical motion due to difference of length in two directions. Still the elliptical motion persisted.

In the first experiment with the ball and plane support, the minor axis of the elliptical path of the pendulum would increase to a maximum, the rotation being in one direction, then the axis would shorten to zero and rotation would begin in the opposite direction. These changes in the length of the minor axis of the ellipse, accompanied by corresponding changes in the direction of rotation of the pendulum, followed each other with great regularity.

This elliptical path seemed to be a splendid example of a resultant of two simple harmonic motions at right angles to each other and differing in period by a small amount which could be easily calculated, or which could be determined experimentally. Since I had eliminated the possibility of the pendulum having two lengths in different directions, by the use of the ball and plane support, I wrongly attributed the elliptical

motion to unevenness in the wire, although numerous measurements failed to reveal any great lack of roundness and there was nothing to indicate imperfections of any kind. I could not see that it would have made any difference if the wire had not been round or even if it had been more flexible in one direction than in the other, for with the ball and plane support, the pendulum was perfectly free to swing in any direction and the wire was not expected to bend at all. It was expected to swing with perfect freedom from the point of contact between the ball and plane. The correctness of this assumption was demonstrated later by substituting a flat steel ribbon for a round wire.

Nevertheless the pendulum behaved as if it had two periods. Furthermore it *always started* rotating clockwise, *never* counterclockwise. This fact remained a puzzle until another wire was substituted for the wire which I had been using. The two wires were cut from the same piece, and were certainly as nearly alike as two wires could well be, and yet the pendulum now *invariably* started rotating *counterclockwise—never clockwise*.

I immediately hit upon the curl in the wire as the cause of the elliptical motion. The wire was brass spring wire, and had been lying in a coil about 25 centimeters in diameter, until the "set" in it was very pronounced.

After starting the pendulum a number of times with the new wire and getting always the same results, *counterclockwise* motion at the start, I rotated the *wire* 180 degrees on its own axis, without disturbing either the pendulum ball or the support. The wire may be rotated by turning the screw G, Fig. 7, without disturbing the hanger, and a similar arrangement at the bottom enables us to rotate the wire without rotating the ball. Now, the pendulum started rotating *clockwise*.

In a series of experiments which followed, for the purpose of studying the effects of the curl in the wire, I decided not to use the ball and plane support on account of its tendency to rotate at the upper surface of the ball.

The ball and plane support has no advantage over a perfectly constructed double knife-edge support. It even has some disadvantages; but it at least proved valuable in detecting some of the defects in the wire, which perhaps would not have been detected otherwise.

In order to eliminate the effects due to the rotation of the hanger in the ball and plane support, and also the possibility of two periods, resulting from lack of perfect adjustment of the double knife-edge support, and yet to retain a rotary adjustment of the wire, I designed the following torsion head plate support.

A, in Fig. 8, is a cast iron cylinder, 30 millimeters long and having an internal diameter of 30 millimeters. It is bored true on the inside and faced at the ends. B, B, are steel rods, 13 millimeters in diameter, screwed into shoulders on opposite sides of the cylinder. These rods are held in strong clamps attached to the ceiling beam. C is a brass head plate 10 millimeters thick, and turned in the lathe to fit into the cylinder as shown. A circular scale, not shown in the drawing, is cut

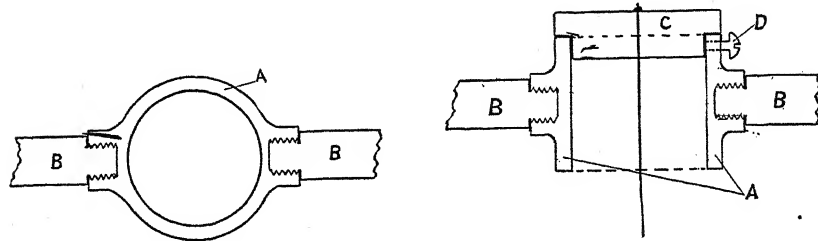


Fig. 8.

upon the edge of the head plate, and a reference mark is placed on the side of the cylinder, so that when the head plate, carrying the wire, is rotated, its position can be read on the scale on its edge. D is a set screw, to hold the head plate firmly in any desired position. There are three of these set screws, placed 120 degrees apart, in order to prevent any slight tendency to wobble.

As the wire must bend at the lower edge of the head plate when the

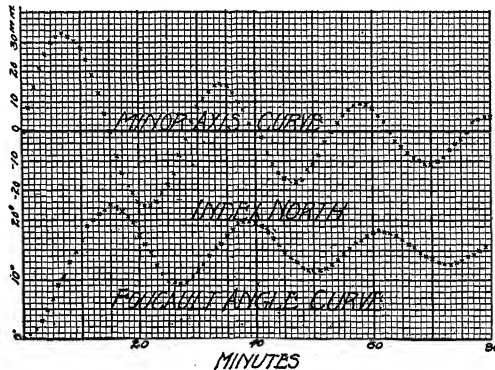


Fig. 9.

pendulum swings, it is important that there should be no looseness at this point, and particularly that there should be no greater freedom of motion in one direction than in another.

To realize these conditions, the hole in the head plate was drilled

from the upper side and the plate was faced on the lower side afterwards, leaving the edges of the hole perfectly sharp at the circumference of the wire. The hole was made with a drill a size smaller than the wire. The plate was then heated and put in place on the wire by the "shrinking on" process. Finally, the wire was bent over and soldered at the upper surface of the plate, to prevent its being pulled out when the weight was placed upon it.

It is also important that the ceiling beam or other structural element to which this support is clamped, should be as rigid as possible, for it is obvious that any lack of rigidity in the support would be likely to offer greater freedom of motion in one direction than in another.

The experiments with this form of support were conducted with a steel piano wire a trifle more than a millimeter in diameter, as a suspension. The wire had been lying in a closely bound coil, 10 centimeters in diameter, and when opened up, the coil expanded to a diameter of 19 centimeters.

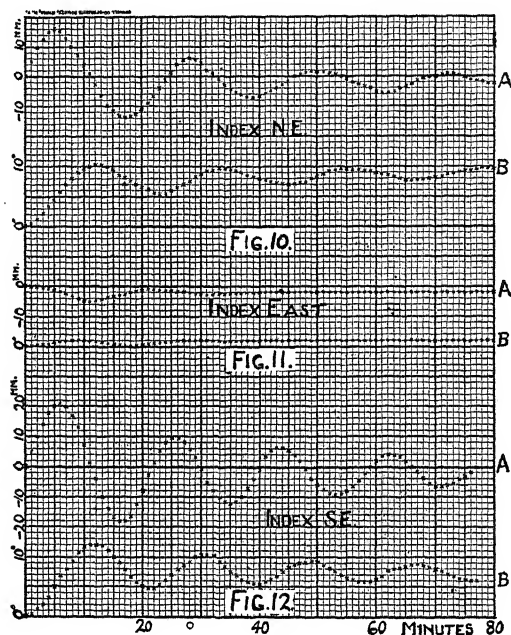
In the first experiment it was my intention to adjust the torsion head so that the convex side of the wire would be in some particular direction,—say north; but, as the wire,  $2\frac{1}{2}$  meters long, curled several times between its extremities, the position of the convex side was different at different points. It would be difficult to determine the direction in which the resultant of all the forces exerted by the wire acted upon the pendulum. However, assuming that such a resultant acted in some direction and produced a definite effect upon the motion of the pendulum, an index was placed upon the torsion head, so that any position arbitrarily chosen could be noted.

A series of observations was then made, on the position of the plane of vibration of the pendulum, and the length of the minor axis of its elliptical path, with the index on the torsion head pointing north. The results are shown in Fig. 9. The wire was then rotated  $45^\circ$  clockwise (index northeast), and another series of observations made. The results are shown in Fig. 10. In Fig. 10 and the succeeding figures, the minor axis curve is marked *A*, and the Foucault angle curve, *B*. Other observations were made with the index in other positions as shown in the curves down to and including Fig. 16.

A simple inspection of these curves in pairs leaves no room for doubt about the correctness of our inference from the curves in Fig. 6. In every case, the dependence of the Foucault angle curve upon the minor axis curve is obvious. Whatever the *cause* of the elliptical motion, the *effect* is a perfectly definite shift in the plane of vibration. In the present case, the elliptical motion was due to elastic reactions in a highly tempered steel wire.

To produce a perfect Foucault pendulum the disturbing effect dependent upon the double knife-edge is easily controlled by adjustment. The effect due to the elastic reactions within the wire, ought to be eliminated by *annealing the wire*. A number of more or less successful attempts in this direction were made.

In one experiment with the torsion head plate support I used a rather poorly annealed iron wire. The observations on this wire are shown in Fig. 17. I could not believe my observations had been so careless as the kinks in these curves indicated, but I repeated the series about



Figs. 10, 11, 12.

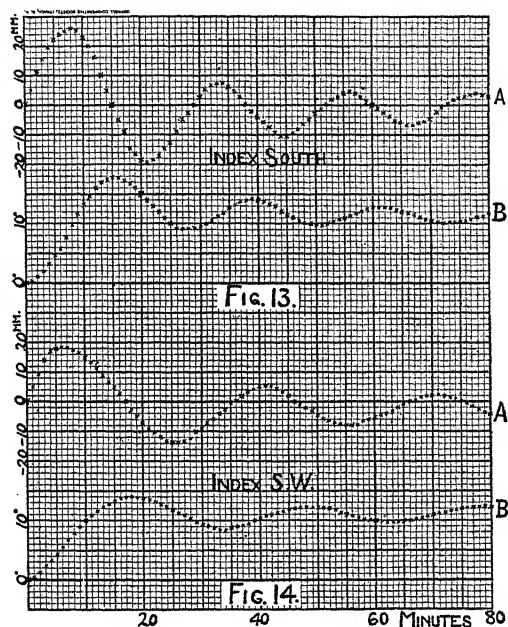
sixteen hours later, the pendulum having remained undisturbed during the interval. This time I got the results indicated in Fig. 18.

The curves in Fig. 18 show the same irregularities as those in Fig. 17, but in a less pronounced degree. The wire was gradually straightening out, under the influence of the weight which was upon it. The irregularities are less conspicuous in the Foucault angle curves than in the minor axis curves. In fact they are not perceptible in the Foucault angle curve in Fig. 18. I have drawn a smooth curve through both the Foucault angle curves from the 39 minute point to the 82 minute point. This hits practically every point on the Foucault angle curve in Fig. 18, but shows perceptible variations from it in Fig. 17, at about 58, 72, and 78 minutes.



I then ran a Bunsen burner flame up and down the entire length of the wire several times as it hung with the 8 kilogram weight upon it; after which, a third series of observations resulted as shown in Fig. 19. The kinks are entirely smoothed out of both curves. Evidently I had finished annealing the wire. If this wire had not been required to bend at the point of suspension, that is, if it had been used with a double knife-edge support instead of a torsion head plate support, it might have been a good Foucault pendulum after it was perfectly annealed.

To illustrate the effect of continued bending upon very soft wire I cite the following case: The 8 kilogram ball was suspended by a No. 18



Figs. 13, 14.

(B. & S. gauge) soft copper wire capable of sustaining a steady pull of about 30 kilograms. The upper end of this wire was soldered into a No. 60 hole (drill gauge size) in a brass torsion head plate. This pendulum was started swinging with an amplitude of 20 centimeters and ran with no perceptible tendency toward elliptical motion and with no perceptible variation from the true Foucault angle, but the wire became brittle and broke off close up to the plate after running about five minutes.

Any wire which is required to bend at the point of suspension must not only be perfectly annealed and perfectly round and perfectly homogeneous in all respects, but it must also be free from wear at the point of

suspension and it must not be allowed to become brittle at the point where it bends. There are so many chances of violating one or more of these requirements that we may hardly expect success with a torsion head plate support.

The double knife-edge support is ideal in some respects but it has one defect which must be looked upon as serious. The knife-edges must be *perfectly sharp* and the planes upon which they rest must be so *hard* that the knife-edges will not cut them *in any degree whatever*. These are conditions which can not be fully realized, and if realized, could not be maintained. Any wear on the knife-edges converts them into cylindrical bearings, and any wear on the planes upon which they

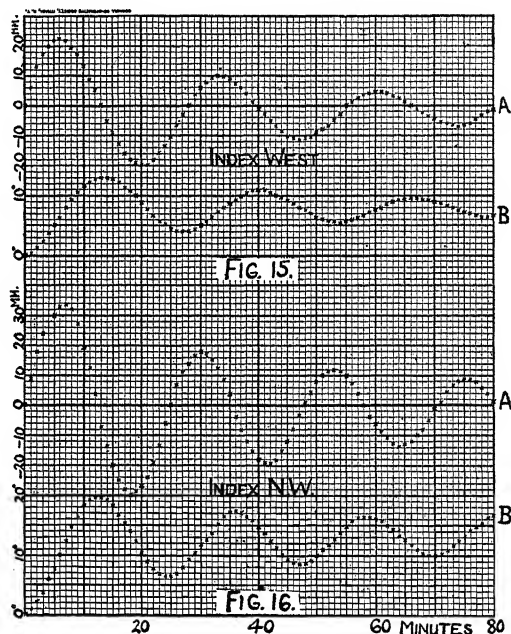


Fig. 15, 16.

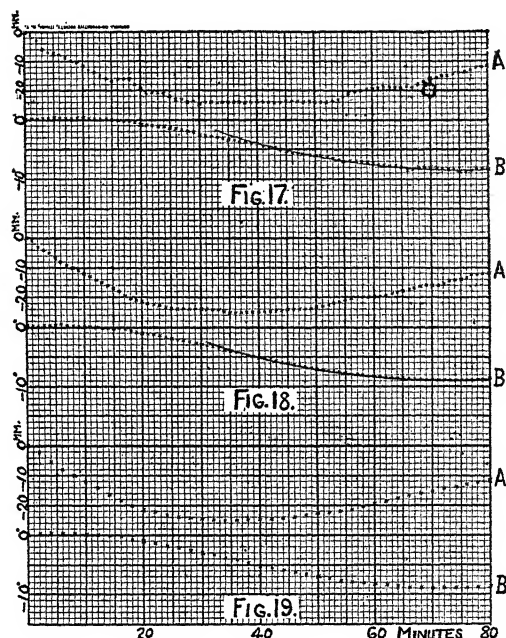
rest converts them into cylindrical troughs, so that after a time we have a cylinder rolling inside of a larger cylinder instead of a knife-edge resting upon a plane. The axis of suspension of a pendulum with a knife-edge support is at the surface of the plane upon which the knife-edge rests; but if a cylindrical roller be substituted for the knife-edge, the axis of suspension of the pendulum is no longer at the surface of the plane but at the center of the roller. We have therefore changed the length of the pendulum by an amount equal to the radius of the cylindrical roller. As the knife-edges wear, the effective length of the pendulum is



increased in one direction and diminished in the other and our Foucault pendulum is gradually converted into a Blackburn pendulum.

Of course the wear can be compensated for by readjusting the distance between the planes, or in some cases by means of a counterpoise above the support; but these adjustments are really somewhat difficult to make, and as the wear continues the pendulum is never in a perfectly satisfactory condition.

The most satisfactory support which I have been able to construct is a double roller support, in which hard polished steel rollers of considerable size have been deliberately substituted for knife-edges. The wear



Figs. 17, 18, 19.

is thus reduced to an inappreciable minimum, the radius of curvature is perfectly definite and constant and the adjustment when once made is permanent.

This support is simple and easily constructed and possesses a number of features which ought to recommend it. The details of construction are shown in Fig. 20.

*A* is a hard steel plate, 25 millimeters square and 3 millimeters thick, with shallow grooves in the upper and lower surfaces at right angles to each other as shown at *B*. The bearing surfaces, *C*, and the hardened steel rollers, *D* and *E*, which are in contact with them, are highly polished.

A double bifilar suspension,  $F$ , is used, consisting of soft copper wire small enough to be easily stretched when the weight of the ball,  $L$ , rests upon it. No. 30 (B. & S. gauge) is a suitable size for a 1 kilogram ball. The wire is simply looped over the ends of the upper roller, and *under* the ends of a short crossbar,  $K$ , at the top of the ball,  $L$ . Shallow grooves may be cut near the ends of the roller and crossbar, to keep the wire from slipping. If the wires are not quite equal in length, they may be equalized by carefully administering a gentle pressure directly downward upon the ball, while the upper roller is held rigidly in a horizontal

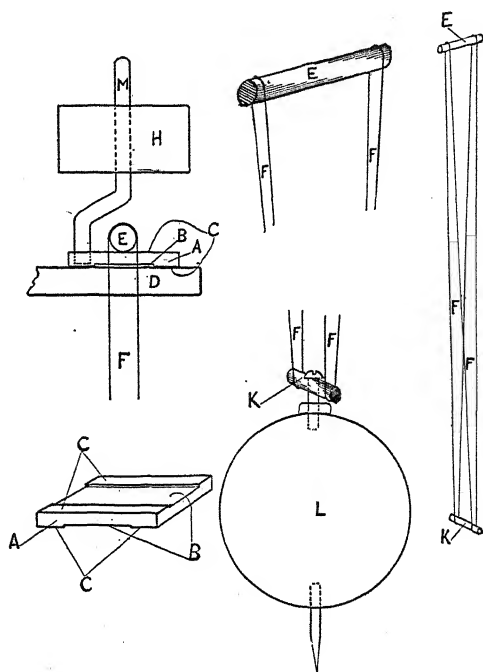


Fig. 20.

position. The lower roller,  $D$ , acts as fixed support and should be rigidly held in its position by means of a strong clamp of some kind. If  $D$  is mounted in an east and west position, when the pendulum is swinging east and west, the upper roller,  $E$ , rolls on the plate,  $A$ . When the pendulum is swinging north and south, the plate,  $A$ , rocks upon the lower roller,  $D$ .  $H$  is a counterpoise to compensate for the difference of length of the pendulum in the two directions. This counterpoise may be run up or down on the threaded rod,  $M$ . (Threads not shown in the drawing.) When  $H$  is properly adjusted, the period of the pendulum is the same in both directions. There is then no elliptical motion and no

shift in the plane of vibration of the pendulum, except that due to the rotation of the earth upon its axis.

In the apparatus which I have just described, the rollers are 6.35 millimeters in diameter and the counterpoise weighs 140 grams. The counterpoise is provided with a circular scale to facilitate fine adjustment. Of course the adjustment is affected not only by the mass and position of the counterpoise, but also by the thickness of the rollers and the thickness of the plate. It is also true that changing the angle between the rollers will disturb the adjustment. For this reason, great care should be exercised in making other adjustments, not to shift the position of the upper roller or the plate. Within small limits, canting the upper roller might be used as a method of adjusting.

It has been shown that the tendency of the pendulum to run in an elliptical path may be the result of any one or more of a variety of causes, but whatever the cause, we may always develop a counter tendency by raising or lowering the counterpoise in the device now under consideration, and perfect compensation is always possible. Of course, the final adjustment must always be made by experiment, and although the process may be a little tedious, it is not difficult, and when once made, the results are all that could be desired.

If the amplitude of the pendulum is large, there is a tendency of the rollers to slip on the polished surfaces of the plate. I find an amplitude of 20 centimeters (40 centimeter path) is very satisfactory for a pendulum  $2\frac{1}{2}$  meters long.

The double roller support used in connection with a double bifilar suspension of soft copper wire is so satisfactory that I do not find it necessary to use a pendulum having a length of more than  $2\frac{1}{2}$  meters or a mass of more than 1 kilogram. In fact I have been quite successful with a pendulum only one meter long and weighing only 240 grams, but of course there is no need of going to such an extreme as this, except to demonstrate that it can be done. A pendulum weighing a kilogram, and swinging over the lecture table, from a rigid beam in the ceiling of the lecture room, is ideal.

Foucault unquestionably succeeded in minimizing his difficulties by using a very long and very heavy pendulum, but his results may not have been as exact as some of us have supposed. I find nothing in any of the original papers or early comments upon them, to indicate a high degree of accuracy. Indeed the only statements bearing upon this point indicate that the results of the experiment were only approximately correct. For example: "Quoique ces oscillations diminuent d'amplitude assez rapidement, au bout de cinq à six heures elles sont encore assez

grandes pour permettre d'observer la déviation que est alors de 60 à 70 degrés." Or, "La montre à la main, ou voit que, à Paris, la déviation est du un degré en cinq minutes."

Such statements can not be thought of as representing precise measurements. They are good enough to leave no doubt in regard to the general proposition that the earth rotates on its axis, but they are not by any means exact. If success depends upon great length and great mass, Foucault's results ought to have been very exact. The fact that they were not, indicates that there was something else to be reckoned with. I have already referred to the fact that his wire was strained beyond its elastic limit, and I have no doubt this condition of the wire contributed to his success quite as much as its great length. Of course the great mass of the ball was responsible for the strain upon the wire, but if the wire had been *annealed*, the great mass would not have been necessary.

To use a pendulum of very great mass in order to prevent atmospheric disturbances is also quite unnecessary. Air currents, if forcible enough, do disturb the pendulum slightly, but such disturbances never "develop large irregularities" in the motion of the pendulum, in spite of certain ingenious theories intended to explain how they do it. In fact they never *develop* any irregularities at all. They always die out gradually, and at the same rate as the decrement in the principal motion of the pendulum along the major axis of the ellipse, when the amplitude has been reduced to the same value. Would you dare present any other view of the case to your students if you were discussing the second law of motion?

How unimportant slight atmospheric disturbances are, may be indicated by the fact that I have succeeded perfectly with a 1 kilogram pendulum in an ordinary lecture room with all the doors and windows open (2 doors and 7 windows) and a good breeze flowing through the room.

The majestic oscillations of the Pendulum of the Pantheon, as Flammarion fittingly describes them, must have produced a profound impression upon those who had the good fortune to observe them; but it is not possible to repeat the experiment in every lecture room on such a magnificent scale. To reduce the experiment to convenient lecture room dimensions, ought to greatly extend its field of usefulness.

KNOX COLLEGE,  
GALESBURG, ILL.,  
October, 1918.

## ON THE DIFFRACTION-FIGURES DUE TO AN ELLIPTIC APERTURE.

BY C. V. RAMAN.

**SYNOPSIS.** Photographs are shown of the diffraction pattern obtained near the focus of a converging pencil of rays from an elliptic aperture. The transition from the Fresnel to the Fraunhofer class of diffraction figure is traced and attention drawn to the geometric law to which the pattern conforms, namely, that the brightest part of the diffraction pattern outside the elliptic cross-section of the beam lies within the geometric evolute of this cross-section and is bounded by it.

IT is well known that the diffraction pattern at the focus of a convergent pencil of rays limited by an elliptic aperture is made up of a central spot of light surrounded by alternate dark and bright rings which are all elliptical in shape, the direction of the major axis of the rings being the same as that of the minor axis of the aperture and *vice versa*.<sup>1</sup> The distribution of luminosity in the focal plane may be readily deduced from the known results for the case of the circular aperture by a simple transformation of coördinates. The theory of the phenomena observed in *ultra-focal* planes is, however, considerably more complicated, and so far as the present writer is aware, has never been fully worked out for the case of elliptic apertures. Recently, while working in collaboration with Mr. R. S. Deoras, the writer made some observations on the diffraction of light by elliptic apertures for convergent and also for divergent pencils, and has noticed that the configuration of the fringes presents some rather striking geometrical features, particularly in the cases in which the eccentricity of the elliptic aperture is considerable. It is thought that a brief account of the observations, and the reproductions of some of the photographs of the diffraction-figures secured in the course of the work (Figs. 2 to 11) may be of interest to the readers of the **PHYSICAL REVIEW**.

Figs. 2 to 9 represent the gradual transition from the Fresnel to the Fraunhofer class as the focus of the convergent pencil is approached. In all these cases, the aperture limiting the pencil was of considerable eccentricity and had its major axis vertical. The interesting feature to which the writer wishes to draw attention is the following simple geometrical law to which the diffraction-pattern is found to conform. In

<sup>1</sup> Airy's Tract on the Undulatory Theory of Optics, Section 86.

Fig. 1 the heavy lines represent the elliptic cross-section of the geometric pencil of rays by the plane of observation, and also the geometric evolute of this cross-section. [The ratio of the major to the minor axis of the ellipse is taken to be greater than  $\sqrt{2}$ , so that two of the cusps of the evolute lie outside the cross-section.] Observation shows that the brightest part of the diffraction-pattern outside the elliptic cross-section lies within the evolute and is, in fact, bounded by it. The complete figure of the evolute may be observed visually or even photographed with sufficiently long exposures. Beyond the two cusps of the evolute, a horizontal brush or extension may also be seen on each side, as shown in Fig. 1. The evolute boundary is clearly seen in Figs. 3, 4 and 6, though in all these cases, the size of the photographic plate and the exposure were insufficient to record the whole of it.

Obviously, as the plane of observation approaches the focus of the convergent pencil, the cross-section of the pencil and the evolute both contract and ultimately reduce to a point. The horizontal brushes extending from the two cusps of the evolute, however, persist (vide Figs. 7 and 8) and break up into beads on both sides which, when the focal plane is further approached, ultimately join up and form the elliptic rings observed in the Fraunhofer pattern. Altogether, the case presents unusually interesting features in the transition from the Fresnel to the Fraunhofer class of diffraction-figure.

Fig. 10 represents the elliptic rings at the focus, for the case in which the eccentricity is small. Fig. 11 represents the rings observed in the focal plane when a *circular* reflecting surface held very obliquely forms the diffracting aperture. Some degree of asymmetry is observable in this figure.

The writer hopes to be able to present a fuller study and mathematical treatment in due course.

210 BOWBAZAR STREET,  
CALCUTTA, INDIA,  
October 20, 1918.

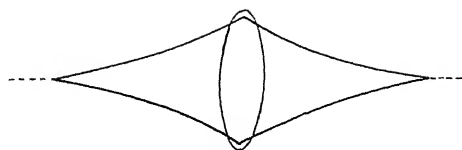


FIG. 1.

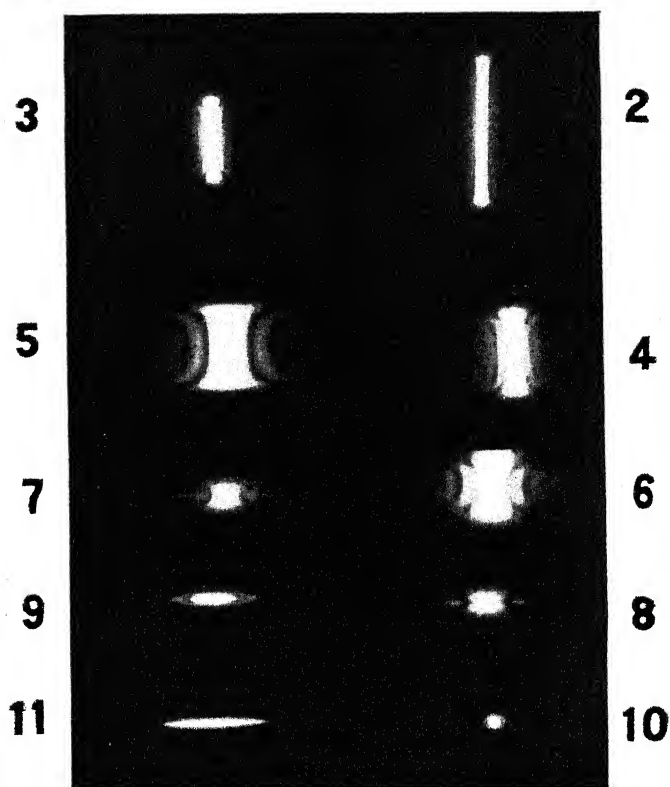


FIG. 2.

C. V. RAMAN.







## REFINEMENTS IN SPHEROMETRY.<sup>1</sup>

By G. W. MOFFITT.

**SYNOPSIS.** *Mechanical Methods.*—The shortcomings of the ring spherometer are discussed and a new type of contact ring with selected ball bearings for contact members, giving greater precision and reliability, is described. A modified instrument having but one contact is also described. It has certain advantages in some cases.

*Optical Methods.*—The auto-collimating method, as often used, is limited in its precision because of the depth of focus of the optical system. The improvement described consists in placing the lens on a nodal slide and adjusting until lateral motion of the image in the field of view is no longer noticeable when the slide is rotated through an angle. In the caustic curve methods use is made of the aberrations introduced on oblique reflection from a spherical surface. By measuring the distance from the primary to the secondary image and multiplying by a factor depending upon the arrangement of the apparatus the radius is determined with high precision for concave surfaces or for convex surfaces of short radius. This method is especially valuable when the element of surface is small.

*Summary.*—Surface measured with precision by the various methods described are as follows: (1) Mechanical Methods: Concave and convex surfaces of short and medium radius, the precision decreasing rapidly as the radius becomes larger. (2) Auto-collimating Method: Concave and convex surfaces of short or medium radius up to the limit of the slide on the turntable, the precision being high and practically independent of the radius. (3) Caustic Curve Methods: Concave surfaces of any radius to a high degree of precision, even when the element of surface available is too small to be measured by other methods. Convex surfaces of short and medium radius to a degree of precision decreasing with increase of radius.

**I**MPROVEMENTS in methods for the measurement of spherical lens surfaces do not seem to have kept pace with the development of processes for shaping the surfaces. To verify the execution of a formula, or to ascertain the constructional data of a given lens system without destroying it, requires a precise determination of the thicknesses, radii, and focal lengths of the individual elements of which the system is composed. One can measure the thicknesses with an ordinary micrometer gauge. The focal lengths and color curves may be found by the use of a good optical bench equipped with a nodal slide. But with methods now in common use it is usually difficult, often impossible, to determine the radii with a satisfactory degree of precision. I have been led, therefore, to a study of existing methods in spherometry, and in consequence, make a few suggestions for their improvement. Some

<sup>1</sup> Communication No. 78 from the Research Laboratory of the Eastman Kodak Company.

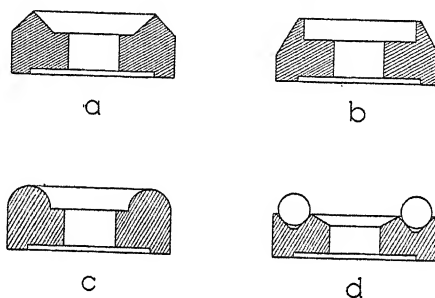
methods are also given in this paper which seem to possess elements of novelty and a limit of precision surpassing that of those now in use.

# I. MECHANICAL METHODS.

## *The Ring Spherometer.*

Those who may have attempted to verify the curvatures of a lens system are familiar with the shortcomings of spherometers even of the best ring type, such as that illustrated in Fig. 1.

A vertical section through the axis of a theoretically ideal spherometer ring is shown in Fig. 2a. But for mechanical reasons such a ring cannot be made. The cross-section usually employed is that shown in Fig. 2b.



Figs. 2a, 2b, 2c, 2d.

There are really two diameters, one for convex surfaces, the other for concave. But the corners are always more or less rounded and irregular, making it practically impossible to determine the true value of the diameter of the contact circle because it varies in an unknown manner with the radius being measured. Being made of soft metal to prevent injury to the lens surfaces the ring is easily damaged unless the edge be especially protected against rough usage. Even if carefully handled the ring will soon show indentations and bends under the microscope and these defects introduce further uncertainty in the constants of the ring as there is no way of telling to what extent they affect a given determination.

## *The Ball Ring Spherometer.*

The question naturally arises whether it is possible to give the spherometer ring such form that the size and position of the contact circle may be accurately stated in every case.

One answer to the question would be to make the vertical section of the ring circular in form as in Fig. 2c. The formula for reducing the readings would be of the same general type as before, but the ring would be of sturdy form not easily damaged, and not detrimental to the surfaces

measured upon it. On the other hand the mechanical difficulties standing in the way of making such a ring of truly circular form and cross-section make its realization hardly possible. One can, however, accomplish the same end by the use of three or four selected ball bearings whose sphericity and uniformity of size fall within the required limits of exactness, conditions which may be easily met.

A V-shaped circular groove may be turned in a disk of metal whose base fits the spherometer as does the usual type of ring (Fig. 2*d*). After the ring has been ground until true the balls may be laid in the groove with the disk in place and the instrument is ready for use. It may be desirable to fasten the balls in position in the groove by some means which will not distort the upper part of the balls.

With a ring of this type the determination of the constants involves nothing but readings that can be made with a high degree of precision. In the measurement of the radius of the ball ring there is no guessing as to the point on which to set the reading microscope—no setting on ill-defined edges whose position seems to change with every change in lighting. The personal error in bringing the sharply defined image of a ball tangent to the cross wire may be easily determined by comparing the results for the diameter of a ball with the results of direct micrometer gauge measurements. But by taking the distance from one side of a ball to the same side of one on the other end of the diameter of the ring the personal equation vanishes in the determination of the diameter of the circle through the center of the balls.

For the ordinary ring spherometer the equation is

$$R = \frac{d^2}{2h} + \frac{h}{2},$$

where  $R$  is the radius to be determined,  $d$  the radius of the contact circle—different for concave and for convex surfaces—and  $h$  is the sagitta of the arc of radius  $R$  subtended by the diameter of the contact circle.

For the ball-bearing contact ring the equation becomes

$$R = \frac{d^2}{2h} + \frac{h}{2} \pm r,$$

where  $r$  is the radius of the ball bearings and the double sign is to be interpreted as meaning two separate cases in which the negative sign is used for convex and the positive sign for concave surfaces. Here  $d$  represents the radius of the circle passing through the centers of the balls.

For either type of ring the error in a determination may be written

$$\Delta R = \frac{d}{h} \Delta d + \left( \frac{1}{2} - \frac{d^2}{2h^2} \right) \Delta h \pm \Delta r,$$

for concave and convex surfaces respectively. For the ordinary type of ring the last term becomes zero. Other sources of instrumental error have been considered, but their effect on the results with any well made instrument are small compared to the error from the sources already considered. Among those considered were the errors due to lack of centering of the ring with respect to the center contact, obliquity of the plane of the ring with respect to the direction of motion of the center contact, and form of the center contact. All these sources of error would be negligible in any instrument constructed with a reasonable amount of care.

The relative merits of the two rings may be compared by assuming an average case and determining the maximum error to be expected in a determination, obtaining the maximum error by summing the numerical values of all the terms on the right-hand side of the above equation. Take, for example, the case where  $d = 10$  mm.,  $h = 1$  mm. For the ordinary ring  $d$  may be determined to 0.01 mm., or 0.02 mm. Assume that the settings for  $h$  can be read to 0.0005 mm. Then the error in the determination of  $h$  would probably be about 0.001 mm. In the ball ring instrument the error in the determinations of  $d$  and  $r$  need not exceed 0.001 mm. and the error in the determination of  $h$  would be as in the first case about 0.001 mm. On the basis of these assumptions the maximum error to be expected in the determination of the radius of average magnitude would be about 0.15 mm. for the ordinary ring and about 0.06 mm. for the ball-bearing ring.

The gain in precision is not all that could be desired, but it is only one of the advantages realized by this modification of the instrument. When it is remembered that the error has been reduced to less than half its former value, and that the ball-bearing ring will maintain its constants for a long time and even stand considerable rough usage without injury, the gain is seen to be considerable.

The error equation shows that for a given value of  $R$  the error will be smaller the smaller  $d/h$  becomes, or for a given lens the larger the value of  $d$ . But this is limited by the aperture of the lens while  $h$  is limited by the radius of the lens face. For a given contact circle the size of the ball bearings used does not affect the precision of the measurements. Therefore it is possible to use the balls most easily obtained or for other reasons most suitable for the purpose. If the lenses to be measured are small the balls must be small in order that a ball ring of small diameter may be constructed.

While it may be best to fasten the balls rigidly in place a very serviceable ring may be made by simply placing the loose balls in the groove

if care be taken to see that balls and groove are well cleaned. One set of balls and a disk with several concentric grooves will serve for practically all lenses whose curvatures are adapted to measurement on the spherometer.

*The Single Contact Sliding Stage Spherometer.*

A modification of the ball-bearing ring spherometer which is excellent for nearly all measurements made with a spherometer is what I have called the single contact slide spherometer. The details of this instrument are shown in Fig. 3. The troublesome contact ring has been entirely dispensed with and the center contact rod is inverted so that the contact member is at the lower end. In the base of the instrument is a sliding stage actuated by a micrometer screw capable of a range of motion about as great as the aperture of the largest lens to be measured on the instrument. On the stage is mounted the lens carriage which should have a short screw-actuated cross slide which need not be calibrated in any way, as it is used only to bring the center of curvature of the lens face into line with the axis of the contact shaft. The center contact of this instrument must be of truly spherical form. A carefully selected ball bearing of small diameter serves very well. As a means of reading the instrument the microscope method is entirely satisfactory.

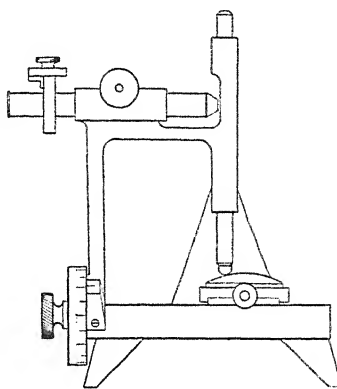


Fig. 3.

To use the instrument place the lens on the carriage and clamp in place, paying little attention to centering or leveling. Adjust the cross-slide until the contact rod reading in the microscope is a maximum—or minimum. After this has been done any movement of the main slide will cause the contact to move along the circumference of a great circle of the lens face. The main slide is then set so that the contact is at some point near the edge of the lens and slide and microscope read. The slide is then moved until the microscope reading becomes a maximum or a minimum and slide and microscope readings again taken. Then the slide is an additional distance equal to that already moved and the readings taken again. The radius may be computed by means of the ball-bearing ring spherometer formula. The last reading made was not essential but served as a check on those already taken. On the other hand a number of distributed readings may be taken and several determinations of the radius made from one set of readings.

The advantages of this type of instrument are:

1. In construction it is much easier to select one ball of precise sphericity than it is to select three or four balls of the same size and of sufficiently precise sphericity.
2. There are no instrumental constants to be determined, or rather, the constants are determined in the making of the readings.
3. The instrument affords a means of studying the deviation of a surface from the spherical form.
4. Will measure the radius of curvature when the area of the lens is too small to be determined on the ring spherometer.

Against these advantages may be placed the following disadvantages:

1. Not as rapid as the ring-type instrument.
2. Side motion of the contact shaft must be reduced to a minimum.

## II. OPTICAL METHODS.

### *Auto-Collimating Methods.*

A number of variations of the auto-collimating methods have been proposed from time to time. Of these the one shown in Fig. 4 is perhaps the most satisfactory. A short distance in front of the objective of a simple reading telescope is placed a lens similar to the objective of the telescope itself but in a reversed position. Between the two lenses is placed a partly silvered mirror which may be adjusted to reflect the light from a distant small source through the front lens onto the face of the lens to be measured which is carried in a holder at *L*. This

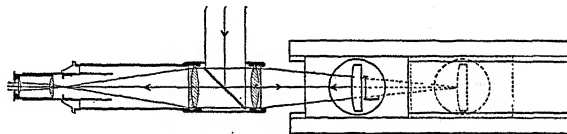


Fig. 4.

arrangement gives a better field of view than can be had by the use of an auto-collimating eyepiece. It may be shown that the illumination at the eyepiece will be a maximum when the product of the transmission and the reflection of the mirror is a maximum. If the point of convergence of the light pencil and the center of curvature of the lens face coincide a part of the light will be reflected back through the telescope and an image of the source formed at the cross-wires. The lens is carried on a slide fitted with scale and vernier by means of which the displacement of the lens between settings may be determined.

In use the slide is first adjusted so that fine dust particles on the face of the lens are in sharp focus. The slide is then moved until the source

is sharply imaged on the cross-wires. The displacement of the slide from one reading to the next is equal to the radius of curvature of the lens face.

Several difficulties are encountered in the application of this method. The presence of spherical aberration in the lenses of the instrument may cause considerable uncertainty in the settings. The spherical aberration may be reduced to a minimum by correct design of the lenses, but even then the depth of focus naturally inherent in an optical system of this kind renders precise focusing practically impossible. With this in mind I have modified the method of determining the correct setting, and have investigated the precision obtainable with what may be called the

*Auto-Collimating Nodal Slide Method.*

The lens to be measured is mounted on a nodal slide arrangement similar to that used for determining the position of the gauss points of a lens (see Fig. 5). In other respects the apparatus is the same as before except that the scale and vernier for measuring the displacement between settings are now on the turntable of the nodal slide whose support is stationary. The method may be applied to a T testing bench by placing the telescope on a carriage mounted on one of the cross-tracks of the bench and using the regular turntable.

In use the apparatus is first adjusted as well as may be by focusing. Then the turntable is rotated back and forth through an angle limited by the aperture of the lens and further adjustment made until there is no side shift of the image with respect to the cross-wires during the rotation. When this adjustment has been completed the center of curvature of the lens face coincides with the axis of rotation of the turntable. To find the reading when the face of the lens is tangent to the axis of rotation of the turntable the carriage is moved until fine dust on the surface of the lens is in focus. The final adjustment is made by rotating the turntable back and forth and further adjusting the slide until there is no relative motion of image and cross-wires at the center of the field. If this setting cannot be made with sufficient accuracy by means of the telescope, because of its low magnification, it may be made with the regular high-power microscope on the main track of the bench. The distance through which the slide has been moved is, as in the preceding case, equal to the radius of curvature of the lens face.

The error in a determination of a radius by this method will be due almost entirely to the error in the setting for the center of curvature provided the setting on the axis of rotation has been made with a high power microscope. To evaluate this error assume that the center of



curvature,  $C$ , fails to coincide with the axis of rotation of the turntable by a small distance  $e$  (Fig. 6). Let  $R$  be the true radius of curvature of the lens face and  $A$  the maximum angle through which the turntable may be rotated before the image disappears. Since  $e$  will always be small in comparison to  $R$  the position of the image of  $O$  formed by reflection from the lens face may be assumed to be at a distance  $2e$  from  $O$ . Then if  $x$  represent the lateral shift for a rotation through an angle  $A$

$$x = 4e \sin \frac{A}{2}, \quad \text{or} \quad e = \frac{x}{4} \csc \frac{A}{2}.$$

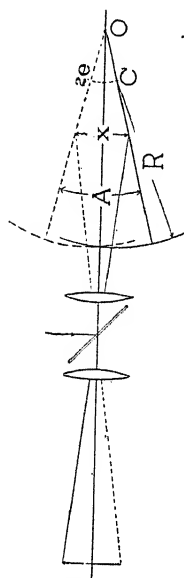


Fig. 6.

If the two lenses of the apparatus be of the same focal length  $x$  is also the lateral shift of the image with respect to the cross-wires. Assuming that a shift of 0.02 mm. may be detected and that a rotation of  $30^\circ$  is possible the smallest error in a setting which could be detected would be about 0.02 mm. Since the lens surface may be placed very accurately tangent to the axis of rotation of the turntable by means of a high-power microscope the above value may be taken as a fair estimate of the precision attainable with this method for any surface measurable on the apparatus. It is interesting to note that the magnitude of the radius has little effect on the precision for all surfaces measurable on the instrument.

#### *Caustic Curve Methods.*

Unlike the auto-collimating methods just described where spherical aberration in the instrument itself entered as a disturbing factor, these methods are based upon the aberrations introduced by oblique reflection from a spherical surface. I have devised several variations of the general method to suit different sets of apparatus from the elaborate lens testing bench to a simple travelling microscope, pivot stand, and distant source, but the one whose description follows seems best suited to the purpose as it involves only measurements of distances. Caustic curve methods are especially valuable in cases where the element of lens surface is too small to be measured by other methods. In favorable cases the precision seems to be limited more by the errors of adjustment of the apparatus than by those of making the readings.

Fig. 7 shows the general arrangement. The light is assumed to be coming from the left and to be reflected from the lens face whose center



of curvature is at  $C$ . The continuation of the spherical surface is represented by the dotted line.  $FN$  is the caustic curve of reflection in this case. It is a portion of a two-arched epicycloid. The axis of the reading microscope is perpendicular to the incident light. Lens, light, and microscope are so arranged that the chief incident ray is reflected along the axis of the microscope. Now if the microscope be moved along its axis the two astigmatic images formed by the reflection from the spherical surface will be seen one after the other. The primary image is perpendicular to the plane of the paper at  $PP'$ , while the secondary image lies in the plane of the paper and parallel to the incident rays at  $SS'$ , that is, the secondary image is located along the principal axis of the concave mirror of which the lens surface forms an element. The measurement consists in finding the distance between the primary and the secondary images. It will be seen that this distance multiplied by the proper constant, which comes out to be  $2\sqrt{2}$ , gives directly the radius sought. It may also be easily shown from the properties of the curves that the distance from the primary image to the lens surface, measured along the axis of the microscope produced, is equal to the distance between the primary and the secondary images. This valuable check relation is one of the advantages realized by choosing the perpendicular relation between chief incident and reflected rays. If a moderate magnification be chosen, and there seems to be little gain in precision by pushing the magnification beyond a reasonable limit, one is able to focus on the lens face and so measure both these distances.

The procedure in making a determination by means of this method would be about as follows: Having placed the lens in rough position the secondary image is thrown into the field of the microscope and examined. If the image rises or falls in the field of view of the instrument as the sharp focus position is passed it is an indication that the center of curvature of the lens face does not lie in the plane defined by the axis of the microscope and the chief incident ray. The lens must be adjusted by tipping it in its holder until the secondary image does not rise or fall in

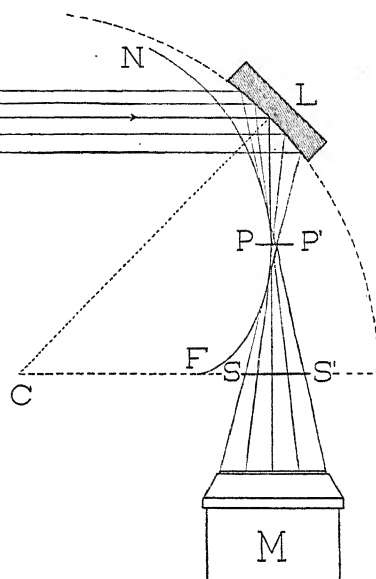


Fig. 7.

the field as the working plane of the microscope is moved through the position of sharp focus, but simply diminishes in width and fuzziness, passes through sharp focus, and widens again. As will be seen later the microscope should be raised or lowered until the sharp image falls across the middle of the field. When so adjusted the center of curvature of the lens face lies in the plane defined by the incident and reflected chief ray. The microscope is next placed so as to receive the primary image. This may require a bodily movement of the lens carriage toward or away from the source if a simple rotation does not suffice. The focusing of the primary is not at all the finding of the best image of a sharp line. The line is never sharp on one side. The case is illustrated in Fig. 7, in which the image sought is in the plane  $PP'$ . The image will be quite sharp and bright on the edge touching the caustic and will gradually fade off toward the other edge which will be limited by the aperture of the microscope. As the microscope is moved either way from the position of correct focus it is evident that the image will move across the field of view, the sharp bright edge leading. Therefore as the true position of focus is approached and passed the image moves into the field of view with its faint side leading, then, as the true position is passed, it reverses its motion and the bright edge leads the procession back. The true position of the focus is not easy to locate precisely by a simple estimate of the position at which the reversal of motion occurs for at this position the lateral movement of the image for a given movement of the microscope is small. But at a short distance to either side of the point of reversal the motion is easily detected. By making a reading when the bright edge, or the first fringe if diffraction fringes show, is on a certain mark in the micrometer, and then moving the microscope until the true position has been passed and the edge, or bright line, is again on the chosen mark the mean of the two readings will give a good determination of the setting. The adjustment of the apparatus should be so made that the true focus on the primary image occurs when the bright edge of the image is in the center of the field of view. The microscope is then moved back and the reading for the secondary taken. The focusing in this case is simply the determination of the sharpest focus of a well-defined straight line which should be in focus all the way across the field since the secondary, being parallel to the incident light, lies parallel to the working plane of the microscope. The microscope is then moved up and focused on fine dust on the surface of the lens. As the lens surface is at an angle to the working plane of the microscope only a part of the field will be in focus at a time. The proper vertical part of the field to focus sharply is that in which the bright edge of the primary

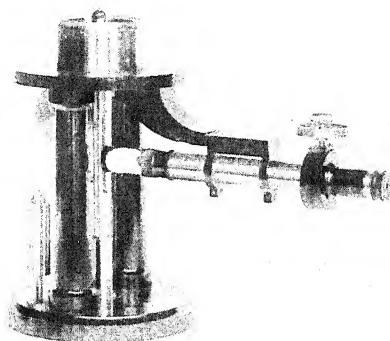


Fig. 1.

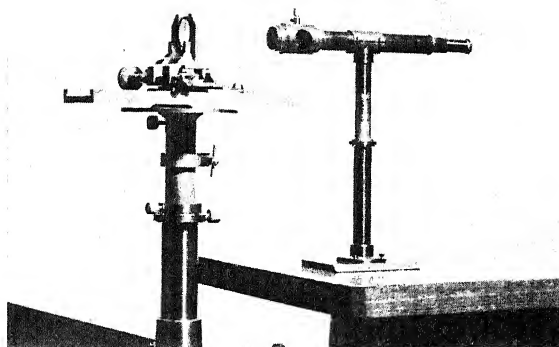


Fig. 5.

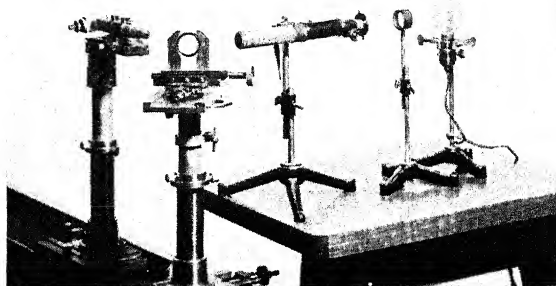


Fig. 8.

G. W. MOFFITT.



image was focused. Since the lens is curved in a vertical plane as well as in the horizontal care is also required in choosing the horizontal portion of the field on which to focus. Evidently the correct portion is that in which the secondary was focused when the setting was made on it. If, as above noted, the center of the field was used then the focusing on the lens face should be made for the center of the field also.

The radius may be computed from the measured distances between the primary and the secondary images by simply multiplying by  $2\sqrt{2}$ , using the distance from the lens to the primary as a check on the work. Or the reading on the primary may be disregarded, using only the readings on the lens face and on the secondary and multiplying the difference by  $\sqrt{2}$ . Care must be taken, however, that the setting on the lens face is focused in the proper part of the field and this can only be determined by a careful observation of the primary focus. The work of making the settings is, therefore, not reduced. Various factors affect the precision obtainable with the two methods and the one giving the best results in any case can be determined only by trial.

On concave faces where high magnification may be used results of high precision may be obtained. A radius of 100 mm. may be determined to within a few hundredths of a millimeter. On convex faces where the images are virtual and lie at a considerable distance back of the surface the working distance of the microscope must be almost as great as the radius to be determined. The precision of focusing would be considerably reduced. For such surfaces the auto-collimating method previously described would probably be preferable.

Several other adaptations of the caustic curve method have been tried. One in particular in which the angle between incident and reflected chief rays was so chosen that the distance between the primary and the secondary images was equal to the radius sought. But errors introduced by a small error in the adjustment of the angle were much greater than in the right-angled case. This, and other limitations seemed to indicate that the right-angled position would give better results and at the same time afford the check readings on the face of the lens. Fig. 8 shows the arrangement of the apparatus used in the tests on caustic curve methods.

By the use of the methods described above one may determine the radius of curvature of a polished spherical surface to a degree of precision usually quite satisfactory for the needs of optical design. In the case of convex surfaces of long radius the precision is sometimes not all that could be desired.

ON THE AUTOMATIC REGISTRATION OF  $\alpha$ -PARTICLES,  
 $\beta$ -PARTICLES AND  $\gamma$ -RAY AND X-RAY PULSES.

BY ALOIS F. KOVARIK.

SYNOPSIS. A device is described which makes it possible for  $\alpha$ -particles,  $\beta$ -particles and  $\gamma$ -ray and X-rays pulses to make records automatically on chronograph paper, which records can be conveniently and accurately studied at leisure. Historically, the method developed from Rutherford's method of counting  $\alpha$ -particles and the last previously described development was by the writer in which he used a telephone receiver whose diaphragm was actuated by the current to a needle electrode, after amplification by means of a 3-electrode vacuum tube. In the present method the amplified current actuates a sensitive relay which in turn operates the chronograph pen by means of a local circuit. The accuracy of the records was checked up for the  $\alpha$ -particles by the scintillation method of counting and for the  $\beta$ -particles by determining the charge carried by the  $\beta$ -particles. Records of  $\gamma$ -rays and X-rays are similar to those of the  $\alpha$ - and  $\beta$ -particles and some arguments are given supporting the idea that the records represent individual pulses or short trains of pulses. A record of  $\beta$ -particles taken over a 7-hour period was studied and conclusion deduced that the variations in the number of  $\beta$ -particles emitted in a given time interval follow the probability law.

## INTRODUCTION.

THE method of counting individual  $\alpha$ -particles by the scintillations they produce on a fluorescent screen is well established. The  $\alpha$ -particles have also been counted by the method of ionization by collision at low pressures, devised by Rutherford<sup>1</sup> in connection with his experiments on the charge of an  $\alpha$ -particle. Rutherford and Geiger<sup>2</sup> modified this method, later, by using a spherical electrode in helium at a pressure of about one third atmosphere instead of the wire electrode used at very low pressure in air. In this case they substituted a string electrometer for the quadrant electrometer which was a marked improvement. Later, Geiger<sup>3</sup> used a sharp needle inside of the chamber and was able to detect at atmospheric pressure not only  $\alpha$ -particles entering the chamber but  $\beta$ -particles also. This method has been used successfully in a number of researches by Kovarik and McKeehan,<sup>4</sup> Chadwick,<sup>5</sup> and

<sup>1</sup> Rutherford, E., and Geiger, H., Proc. Roy. Soc., A 81, 162, 1908.

<sup>2</sup> Geiger, H., and Rutherford, E., Phil. Mag., 56, 24, 618, 1912.

<sup>3</sup> Geiger, H., Verh. D. Phys. Ges., 15, 534, 1913.

<sup>4</sup> Kovarik, Alois F., and McKeehan, L. W., Phys. Zs., 15, 434, 1914; Phys. Rev., N. S., 6, 426, 1915; Phys. Rev., N. S., 8, 574, 1916.

<sup>5</sup> Chadwick, J., Verh. D. Phys. Ges., 16, 383, 1914.

Vennes.<sup>1</sup> The writer<sup>2</sup> has used various modifications of the method mainly in the observing instruments, Zeleny's electroscope, C. T. R. Wilson's electroscope, a telephone receiver, and a ballistic galvanometer had been used with success for this purpose. These visual or audible methods of counting are quite trying on the nerves because close attention is necessary on account of the probability law of emission of the particles; and long periods of counting are essential because the method is a statistical one. A self-recording device would therefore be an obvious improvement. Rutherford and Geiger,<sup>3</sup> by moving a photographic film past the illuminated fiber of a string-electrometer, obtained a photograph of the fiber showing deflection of the fiber caused by the discharges between an electrode and the surrounding chamber when ionization by collision took place in the chamber upon the entrance of individual  $\alpha$ -particles into the chamber. This has been the only method described thus far for obtaining a record of  $\alpha$ -particles.

The method described in this paper makes it possible to get a record on a chronograph paper, of individual  $\alpha$ -particles or  $\beta$ -particles or of  $\gamma$ -ray pulses (?), which may then be studied at one's convenience and the count made with a greater accuracy than is obtainable with the visual or audible methods. The accuracy of the method was tested both for  $\alpha$ -particles and  $\beta$ -particles. The applicability of the law of probability to the emission of  $\beta$ -particles was proved and some experiments were made with  $\gamma$ -rays.

#### THE SELF-REGISTRATION METHOD.

In the visual method the counting chamber is connected to a fairly high positive voltage (about 2,000 volts) and the needle electrode is connected to earth through a very high resistance (ink on paper) and also to the fiber of the string electrometer or to the gold leaves of the other electrometers above mentioned. In the audible method a telephone receiver is placed between the needle and earth, if the discharge current in the counting chamber is sufficiently large to actuate the diaphragm of the telephone receiver; or else, it is placed in the plate-filament circuit of an audion amplifier, the needle electrode being connected directly to the grid or to earth through the primary of an induction coil, the secondary coil being connected to the grid and filament. In the auto-registration method, the current between the needle electrode and the walls of the counting chamber is amplified by a three electrode

<sup>1</sup> Vennes, H. J., Am. J. Sc., L. 4, 44, 69, 1917.

<sup>2</sup> Kovarik, Alois F., PHYS. REV., S. 2, 9, 567, 1917.

<sup>3</sup> Rutherford and Geiger, *loc. cit.*

vacuum tube amplifier and this current is used to operate a very sensitive relay which in turn operates a local battery circuit actuating the pen on a chronograph. Fig. 1 shows diagrammatically the apparatus used and the necessary connections.  $B$  is a high potential battery (about 2,000 volts) whose positive pole is connected to the counting chamber,  $W$ , the other pole being earthed. If  $W$  is made negative a much higher voltage is necessary for any given point electrode.  $E$  is the needle point electrode which is connected through a high resistance, H.R., to earth. The high resistance consisted of India ink on paper and was

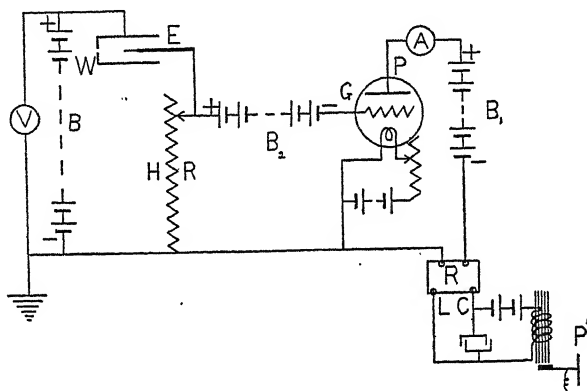


Fig. 1.

adjustable to different values. The needle electrode is also connected to the positive pole of a battery,  $B_2$  (about 80 volts) the negative pole of which is connected to the grid of the audion amplifier.  $B_1$  is another battery (about 160 volts) whose positive pole was connected to the plate of the amplifier and the negative pole to earth through the sensitive relay,  $R$ . The relay operated a local circuit, L.C., which actuated the chronograph pen,  $P'$ . As indicated in the figure another battery was used to heat the filament. A static voltmeter,  $V$ , indicated the voltage on the counting chamber, and a milliammeter,  $A$ , registered the plate-filament current. The battery  $B_2$  on the grid was used to oppose the battery  $B_1$  on the plate and when properly adjusted only a negligible current would flow in the plate-filament circuit. When a discharge takes place in the counting chamber, the needle electrode becomes positively charged and because of the high resistance between it and earth, the grid comes momentarily to a higher potential and an increased electron current flows to the plate  $P$  which, passing through the relay  $R$  causes the local battery circuit to be closed and thus makes the pen register the discharge. The high resistance regulates the duration and



partly the amount of the momentary potential on the grid. The sensitive relay operated on a current of a fraction of a milampere. Fig. 2 shows a photograph of a portion of a chronograph record automatically registered by  $\beta$ -particles entering the counting chamber at the rate of 160 per minute; and also in the lower portion of the natural discharges which occurred at the rate of about 8 per minute. The natural discharges are caused by the  $\alpha$ -particles from the emanation in the air,

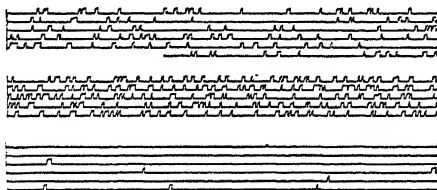


Fig. 2.

by radiations from the material of the counting chamber,<sup>1</sup> by penetrating  $\gamma$  radiations from the earth and other sources, and are also perhaps partly due to imperfect needle points.

The needle point electrodes require special attention in preparation. Ordinary points may cause continuous discharge in the intense electric field. The needle electrode for our purposes must respond only when ionization is produced by radiations. The writer has experimented on many kinds of electrodes: pointed platinum wires, platinum wires with minute globules at the end produced by heating the end to melting temperature, steel points of various shapes and sizes, tungsten filaments, etc. Steel needles, ground to quite fine cylindrical ends and these treated in a flame (to slight redness) so as to form a smooth and uniform surface, have been found to serve best. The tungsten filament can easily be made into a point of any shape by heating (oxidizing) in a flame and these points serve quite well in a dry atmosphere and can easily be renewed; but humidity seems to affect the surface readily and destroy the requisite properties. A point electrode of lasting qualities has not yet been discovered. Some point electrodes may be used for more than 100 hours continuously while others break down after an hour's use.

#### TESTS OF THE METHOD.

The accuracy of the record is necessarily a function of the mechanical contrivances used. The sensitive relay used was tested and found to respond 100 times per second when the intervals were uniform but the

<sup>1</sup> It may be noted that a counting chamber of lead gives so large a number of natural discharges due to rays from Radium *DEF* in the lead as to make it useless for this purpose.

electromagnet used to operate the pen was less rapid in its action. Consequently, particles following each other in a very short interval of time may not be recorded. The apparatus as used was found to give accurate records up to 400 per minute. Particles entering simultaneously would give one record, but the number of these as well as those following each other in a very short space of time becomes negligibly small for a rate of 100 or 200 per minute. These facts were obtained by varying the distance, in vacuo, between the source and the window of the counting chamber and finding that the number of  $\alpha$ - or  $\beta$ -particles per minute which reached the window varied inversely as the square of the distance; also, by varying the size of the window in which case the number varied directly with the area. In the latter case the largest area was six times the smallest area.

The counts for  $\alpha$ -particles had been made on various polonium plates and the check was made by counting the scintillations on a good zinc sulphide screen and also by the ionization method at low pressures used by Geiger,<sup>1</sup> which method is, however, based on the scintillation method. Checks were always good and some have already been referred to in previous papers. Working with the  $\beta$ -particles in previous researches, Kovarik and McKeehan assumed that all  $\beta$ -particles were counted when the point gave an accurate check for the  $\alpha$ -particles and when the number of  $\beta$ -particles counted remained constant with further increase of the potential on the counting chamber. This procedure has been found correct by the following experiments.

A check on the count of the  $\beta$ -particles from a given source was made by determining the total number of  $\beta$ -particles by measuring the charge carried by them and assuming each to carry one elementary charge. The source used was a radium DEF deposit on aluminum of very small area and only the more penetrating rays were used by placing the source between two aluminum plates. Radium DEF was used because the  $\gamma$ -ray effect is small. The plate so employed was flat and consequently the radiations through the aluminum envelope were not absorbed equally in all directions. To get the count of the total number of particles emitted per second by the plate it was necessary to count the  $\beta$ -particles in various directions and integrating. The source was placed axially in a brass cylinder lined with thick paper to reduce reflection. At one end of this vessel was placed a steel or brass plate having a window covered with aluminum of smallest thickness (0.0003 cm.) capable of withstanding atmospheric pressure. The counting chamber was placed against this window, being insulated from it and the brass cylinder.

<sup>1</sup> Geiger, H., Proc. Roy. Soc., A 82, 486, 1909.

The brass vessel was exhausted to a pressure of less than 2 mm. of Hg in every case to eliminate absorption and scattering by air which is very important, the effect being about 50 per cent. when the source is 10 cm. from the counting chamber, for 1 atmosphere of air. The counting chamber was at atmospheric pressure. The source was set 10 cm. from the window and its plane could be rotated about an axis perpendicular to the axis of the cylinder and could thus be set for any angular position for which the count was to be made. The source was set at various angular positions from  $0^\circ$  to  $360^\circ$  but since the envelope was of uniform thickness on both sides the results for corresponding angular positions were averaged, and a curve drawn for positions within a quadrant. This curve is shown in Fig. 3, where the ordinate for  $0^\circ$ , *i. e.*, rays coming

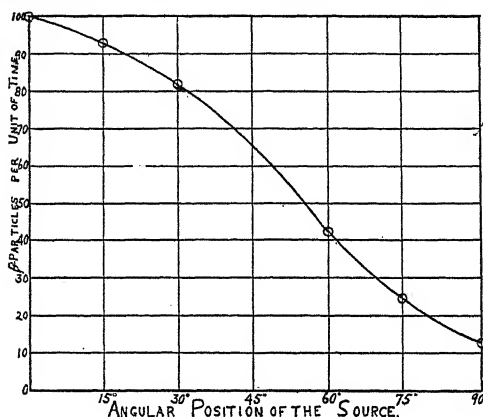


Fig. 3.

normally through the envelope, is taken as 100 and represents relatively the count of  $\beta$ -particles in a unit time through the window of the counting chamber. With the aid of this curve, the actual number counted at  $0^\circ$ , the area of the window, and the distance of source from the window one can determine the number of  $\beta$ -particles emitted into the various zones about the source as center. This integration can be made with sufficient accuracy by dividing the space into 3 or 4 zones. If 3 zones are considered with the source as center and the window of the counting chamber at 10 cm. from the source considered as the pole,  $a$  being the area ( $3.95 \times 10^{-3}$  cm.<sup>2</sup>) of the window and  $N$  (1.23 per second) the number of  $\beta$ -particles counted at  $0^\circ$ , the total number of  $\beta$ -particles per sec. is found to be

$$n = 2 \frac{N}{\text{sec}} \cdot \frac{1}{a} [X \cdot P_1 + Y \cdot P_2 + Z \cdot P_3]$$

$$= 2 \left\{ \frac{N}{\text{sec}} \cdot \frac{1}{a} \cdot \pi r^2 [0.268 P_1 + 0.732 P_2 + 1. P_3] \right\}$$

where  $X$ ,  $Y$ ,  $Z$  are the areas of the zones (for a hemisphere) and  $P_1$ ,  $P_2$ ,  $P_3$  are the average values of the ratio of the number of  $\beta$ -particles per sec. for the various zones to the number counted at  $0^\circ$ . These ratios were obtained graphically from the accurate curve (Fig. 3). A small correction had to be made for the  $\gamma$  rays of radium D and the natural discharges. This was done by obtaining records when a steel or brass plate without a window, but otherwise similar to the one used in front of the counting chamber, was substituted. The total number of  $\beta$ -particles per sec. for the specimen used came out  $1.89 \times 10^5$ .

The source was then placed inside of, but insulated from a metal vessel lined with thick aluminum completely enclosing the source except for a small hole for the electrode holding the source and the whole was then placed inside another vessel which could be evacuated to  $10^{-4}$  mm. Hg. The vessel surrounding the source absorbed the  $\beta$ -particles and the rate at which it charged up was measured by an electrometer, the source being alternately connected to positive and negative potential varying from 0 to 500 volts. The delta ray effects from the metal of the source and the enclosing vessel were found different, as recorded by Moseley<sup>1</sup> in his experiments, but were not so marked in these experiments as in his, owing to the small  $\gamma$ -ray effect of radium D. A magnetic field used similarly as by Moseley showed less marked influence on the results. Therefore, the mean of the rate of charging up of the electrometer with alternately positive and negative electrostatic fields between the source and the charge absorbing vessel, was used. Determinations were also made of the negative charge lost by the source and this checked with the charge absorbed by the surrounding vessel. The greatest difference between the various determinations made was 3 per cent. Assuming the electronic charge to be  $4.77 \times 10^{-11}$  e.s.u., the total number of  $\beta$ -particles by the charge method came to  $1.92 \times 10^5$   $\beta$ -particles per second. The number by the self recording method was  $1.89 \times 10^5$   $\beta$ -particles per second. The check is much closer than expected and warrants the writer to say that the  $\beta$ -particles are accurately recorded. It is to be noted the  $\beta$ -particles counted were the more penetrating rays of radium E ( $\mu = 43.3 \text{ cm}^{-1} \text{ Al}$ ). Experiments were also performed with the very soft rays of radium D and E and the check was far from satisfactory and the results indicated that there are some soft rays which do not ionize but which nevertheless carry a negative charge. This particular point is being investigated.

<sup>1</sup> Moseley, H. G. J., Proc. Roy. Soc., A 87, 230, 1912.

EMISSION OF  $\beta$ -PARTICLES FOLLOWS THE PROBABILITY LAW.

Since the record obtained on the chronograph is made by the  $\beta$ -particles themselves, such a record if made over a sufficiently long time would give the law of emission. A similar study was made by Rutherford and Geiger<sup>1</sup> of the law of emission of  $\alpha$ -particles in which case the observer had to record the scintillations observed. In the present case the  $\beta$ -particles themselves made the record. A record was obtained of 37,266  $\beta$ -particles in 430.6 minutes or a little over seven hours. The record was divided into intervals of 0.2 minutes and the number of  $\beta$ -particles in each of the 2,153 intervals was counted. The smallest number per interval was 7 and the largest number 31. The numbers of intervals containing 7, 8,  $\dots$ , 31, were tabulated and plotted as ordinates against the number per interval as abscissas. To show that the experimental results follow a probability curve, the number of intervals which should contain any given number of particles was calculated from the total number of intervals and the average number of  $\beta$ -particles per interval assuming a probability distribution and using the formula of Bateman.<sup>2</sup> If  $x$  = average number per interval, and  $P$  = the probability that  $n$  enter per interval, then,

$$P = \frac{x^n}{n!} e^{-x}$$

from which the number of intervals containing  $n$   $\beta$ -particles readily

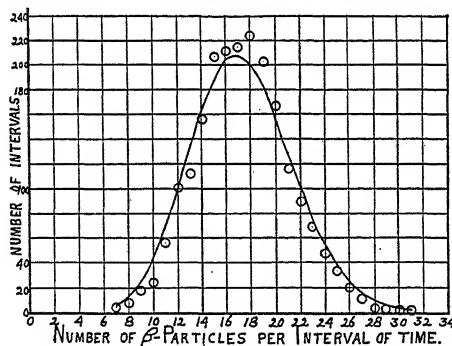


Fig. 4.

follows. Plotting these values against  $n$  we get the curve shown in Fig. 4. The observed values are plotted as circles. It will be noticed that these points follow the curve fairly closely, falling a little under at the low and high values of  $n$  and coming somewhat above at the mean

<sup>1</sup> Rutherford, E., and Geiger, H., *Phil. Mag.*, S. 6, 20, 698, 1910.

<sup>2</sup> Rutherford, E., and Geiger, H., *Phil. Mag.*, S. 6, 20, 698, 1910.

values. It may be concluded that the variations in the number of  $\beta$ -particles emitted in a given interval of time follow the probability law.

#### $\gamma$ -RAYS AND X-RAYS.

If the counting chamber is made without a window and of material to permit no  $\alpha$ - or  $\beta$ -particles to enter from the outside, and some radium is brought into the neighborhood, discharges between the point and the chamber take place and records are obtained which are similar to those obtained in the case of the  $\alpha$ - or  $\beta$ -particles. The same is true when X-rays are passed through the chamber. The number of these discharges decreases with increase of distance of the source and for the  $\gamma$ -rays it was shown to follow the law of inverse squares, the distance being changed from 10 feet to a couple of inches. If lead screens are interposed between the source of  $\gamma$ -rays or X-rays and the counting chamber, the numbers are decreased. The effect is therefore directly connected with the energy of the  $\gamma$ -ray and X-ray pulses. It seems likely that the  $\beta$ -particles produced on the inside of the counting chamber from the metal (mainly) and also from the air, are directly responsible for the point discharges. Whether a pulse produces one or a large number of such  $\beta$ -particles seems immaterial for causing the point discharge provided that these  $\beta$ -particles ionize. To all intents and purposes the  $\beta$ -particles produced by the pulses will act at the same time and therefore one discharge will result per pulse or train of pulses. Since the records appear similar to those of the  $\alpha$ - or  $\beta$ -particles, in so far as distribution with time is concerned and do very likely (although no calculations have been made) follow the probability law, it seems likely that the  $\gamma$ -ray or X-ray pulses or short trains of pulses, are actually registered.

Experiments bearing on this point as well as some unsettled questions regarding the  $\gamma$ -ray and X-ray pulses are at present in progress and will be reported later.

I desire to express my sincere thanks to the Western Electric Company, or the special sensitive relay and the audion bulb, both of which they kindly loaned me for my experiments.

I also wish to acknowledge the assistance of Mr. Walter B. Lang in some of the observations.

SHEFFIELD SCIENTIFIC SCHOOL,  
YALE UNIVERSITY,  
NEW HAVEN, CONN.,  
January 25, 1919.

PROCEEDINGS  
OF THE  
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE NINETY-SIXTH MEETING.

THE ninety-sixth meeting of the American Physical Society was held in Fayerweather Hall of Columbia University, New York, on February 28 and March 1, 1919.

A regular meeting of the Council was held on February 28, five members being present, and President Ames presiding. Professor Theodore Lyman, of Harvard University, was elected vice-president of the society for the unexpired term caused by the death of Professor Wallace C. Sabine. The following were recommended to the National Research Council as representatives of the American Physical Society in the Division of Physical Sciences: H. A. Bumstead, Wm. Duane, Irving Langmuir, Earnest Merritt, R. A. Millikan, and E. B. Wilson. The following were appointed representatives of the American Physical Society in the International Astronomical Union: J. S. Ames, Henry Crew, and Theodore Lyman.

Elections to membership were made as follows: to *regular membership*, J. A. Fleming, Carl Hering, Paul W. Merrill, Henry Norris Russell; to *associate membership*, Carl Darnell, J. P. Delaney, Carl F. Eyring, Joseph Foster, Clifford G. Formd, John G. Homan, Horton W. Hall, G. A. Johnstone, Edwin W. Kelly, C. N. Moore, A. M. Mahon, B. A. Orndorff, K. G. Szlupas, Paul B. Taylor, Phillips Thomas; *transferred from associate to regular membership*, S. J. Mauchly, Peter I. Wold.

Sessions for the reading of papers were held in the morning and in the afternoon of Saturday, March 1, 1919, at which twenty-seven papers were presented, six being read by title. The program was as follows:

Water Vapor Pressure in Units of Force. ALEXANDER MCADIE. (Read by title.)

The Thermal Expansion of Living Tree Trunks. C. C. TROWBRIDGE (deceased) and MABEL WEIL.

Pyrometer Absorption Glasses. PAUL D. FOOTE, C. O. FAIRCHILD and F. L. MOHLER. (Read by title.)

The Function of Phase Difference in the Binaural Location of Pure Tones. R. V. L. HARTLEY.

Temperature Coefficient of Resistance of Molybdenum. E. C. BLOM.

Measurement of Small Alternating Currents by Means of a Rayleigh Resonator. F. R. WATSON. (Read by title.)

An Addition to the Theory of the Quadrant Electrometer. A. H. COMPTON and K. T. COMPTON.

Permanent-Contact Crystal Detectors. LOUISE S. McDOWELL.

Are the Frequencies in the Series of X-rays the Highest Frequencies Characteristic of Chemical Elements? WILLIAM DUANE and TAKEO SHIMIZU.

The Spectral Photoelectric Sensitivity of Silver Sulphide. W. W. COBLENTZ and H. KAHLER. (Read by title.)

Some Photoelectric Lecture Experiments. JACOB KUNZ. (Read by title.)

The Crystal Structure of Carborundum. A. W. HULL.

A Possible Relation between the Resonance Potential and Specific Inductive Capacity of a Metallic Vapor and its Properties in the Solid State. K. T. COMPTON.

Collisions of Slow Moving Electrons with Nitrogen Molecules. S. KARRER. (Read by title.)

The Ionizing Potentials of Argon, Neon and Helium. H. C. RENTSCHLER.

Note on the Use of the Hot Cathode Vacuum Tube for the Measurement of Voltage. LYNDE P. WHEELER.

An Interferential Method for Measuring the Expansion of very small Samples. C. G. PETERS and IRWIN G. PRIEST.

On the Measurement of the Detection Coefficient of Thermionic Vacuum Tubes. H. J. VAN DER BIJL.

The Properties of the Electron as Derived from the Chemical Properties of the Elements. IRVING LANGMUIR.

The Law of Absorption of High Frequency Radiation. ARTHUR H. COMPTON.

Some Physical Properties of Dental Materials. C. G. PETERS and W. H. SOUDER.

The Origin of the General Radiation Spectrum of X-Rays. DAVID L. WEBSTER.

Note on Bomb Trajectories. EDWIN B. WILSON.

On the Relation Between the *K* Series and the *L* Series of X-Rays. WILLIAM DUANE and TAKEO SHIMIZU.

A New Formula for the Spectral Distribution of Energy from a Complete Radiator. IRWIN G. PRIEST.

Transmission of a Quartz Grating Replica. R. S. WETZEL and WALTER SCHWANHAUSER.

On the Measurement of the True Barometric Pressure in a Rapidly Moving Current of Air. J. G. COFFIN.

DAYTON C. MILLER,  
*Secretary.*



THERMIONIC AMPLIFIER.<sup>1</sup>

BY H. J. VAN DER BIJL.

THE current voltage characteristic of the vacuum-tube amplifier manufactured by the Western Electric Company can be represented by the equation

$$I = \alpha(\gamma E_B + E_C + \epsilon)^2, \quad (1)$$

where  $I$  is the current in the filament-plate circuit,  $E_B$  the potential difference between filament and plate,  $E_C$  the potential difference between filament and grid, and  $\epsilon$  a small quantity which depends on the contact potential difference between filament and grid, the potential drop in the filament, etc.

The equation is obtained as follows: Suppose the grid and filament be electrically connected and a potential difference  $E_B$  be applied between them and the plate, then the field at a point near the filament is the same as if a potential difference  $E_B + \epsilon$  be applied between the filament and a plane coincident with the grid. If a potential difference  $E_C$  be also applied between the filament and the grid the effective voltage is the sum, namely  $\gamma E_B + E_C + \epsilon$ . The exponent 2 has been found empirically.

With the help of this equation the amplification equations of the tube can be obtained. It shows that the relation between plate current and grid voltage is of the same form as the relation between plate current and plate voltage. If an alternating voltage  $e \sin pt$  be applied between grid and filament the current is

$$I = \alpha(\gamma E_B + E_C + \epsilon + e \sin pt)^2. \quad (2)$$

The admittance of the tube  $K$  is given by

$$\frac{1}{2K} \int_0^{2\pi} \frac{\partial I}{\partial E_B} dt = 2\alpha\gamma(\gamma E_B + E_C + \epsilon).$$

The impedance is  $1/K$ , and this can be expressed as

$$R_0 = \frac{E_B + \mu_0(E_C + \epsilon)}{2I}, \quad (3)$$

where  $\mu_0 = 1/\gamma$  and is the maximum voltage amplification obtainable from the tube.

If a resistance  $R$  be placed in the filament plate circuit, the voltage  $E_B$  between filament and plate does not remain constant, but is given by

$$E_B = E - RI, \quad (4)$$

where  $E$  is the constant voltage of the battery in the plate circuit. The characteristic equation then becomes

$$I = \alpha[\gamma(E - RI) + E_C + \epsilon + e \sin pt]^2. \quad (5)$$

It is seen from this that the external resistance has the effect of straightening out the characteristic. For a perfect amplifier the characteristic must be

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, April 27, 1918.

straight, otherwise distortion is produced, because in virtue of the curvature of the characteristic an increase in the input voltage (filament grid voltage) produces a greater increase in current than the current decrease produced by an equivalent decrease in the input voltage. It has been found that if the external resistance is equal to or greater than the impedance of the tube itself, the characteristic is sufficiently straight to reduce distortion to a negligible quantity.

Considering now the case in which the circuit contains an external resistance  $R$ , it is seen that a variation in the current in  $R$  due to a variation in the grid voltage causes the plate voltage also to be variable, that is,

$$I = f(E_B, E_C)$$

and hence

$$\frac{dI}{dE_C} = \frac{\partial I}{\partial E_B} \frac{dE_B}{dE_C} + \frac{\partial I}{\partial E_C} \quad (6)$$

Referring to equations (1) and (4) this gives

$$\frac{dI}{dE_C} = 2\alpha (\gamma E_B + E_C + \epsilon) \left( \gamma \frac{d(E - RI)}{dE_C} \right),$$

i. e.,

$$\frac{dI}{dE_C} = \frac{2\alpha (\gamma E_B + E_C + \epsilon)}{1 + 2d\gamma R(\gamma E_B + E_C + \epsilon)}.$$

Multiplying by  $R$  and putting  $\gamma = 1/\mu_0$  we obtain

$$R \frac{dI}{dE_C} = \frac{\mu_0 K}{R + \frac{E_B + \mu_0(E_C + \epsilon)}{2I}} \quad (7)$$

Now  $RdI$  is the voltage change set up in the resistance  $R$  and  $dE_C$  is the change in the input voltage, so that equation (7) gives the voltage amplification  $\mu$  which with the help of (3) can be written

$$\mu = \frac{\mu_0 R}{R + R_0} \quad (8)$$

From this it readily follows that if  $R_i$  be the input impedance of the tube, the power amplification is given by

$$\eta = \frac{\mu^2 R R_i}{(R + R_0)^2} \quad (9)$$

and the current amplification by

$$\xi = \frac{\mu_0 R_i}{R + R_0} \quad (10)$$

These equations have been tested and found to hold with sufficient accuracy to serve for the computation of circuits involving thermionic amplifiers.

It must be remarked that the characteristic equation used here is not sufficiently accurate to apply in the case in which the tube is used as a radio-detector. The detecting action of the tube depends on the second derivative of the characteristic, so that when using the tube as a detector a more accurate equation for the characteristic is used.

RESEARCH LABORATORY,  
WESTERN ELECTRIC CO.,  
NEW YORK.

WATER VAPOR PRESSURE IN UNITS OF FORCE.<sup>1</sup>

BY ALEXANDER MCADIE.

THERE are many instructors in physics and chemistry who are not at the present time, for good and sufficient reasons, fully conversant with the units used by aërographers.

Probably no one feature of the war, not even the use of gas, nor the success of the submersible, stands out so sharply as the development of the airship. Whether we be advocates of the plane or the blimp, representing in general heavier-than-air and lighter-than-air craft, we all recognize that a new and vastly important field of engineering has been opened up in aëronautics, and a new and promising field of investigation for the physicist in studies of the structure of the atmosphere, more briefly called aërography.

With the first use of the sounding balloon it became apparent that the Fahrenheit scale would have to be scrapped and that the Centigrade scale was likewise unserviceable. The reasons are self-evident. And so the temperatures in all upper air work are given on the Absolute scale. The form used however is not the most convenient, and we have been using with success at Blue Hill for several years a new form which has been called the Kelvin kilograde and which has been described elsewhere. The zero is the absolute zero and the melting point of ice under standard pressure is marked 1000. No degree signs are used. The values used later are in these units.

In upper air work it also became evident that inches and millimeters as units for the expression of atmospheric pressure were, like the temperature units, out-of-date; and so the aërographic services of many countries, especially Great Britain, France and Italy have been expressing pressure in units of force. This usage is spreading rapidly.

So far as I can ascertain credit for the suggestion and first use of an absolute unit to express pressure must be given to Professor Theodore Richards. In the classic paper on "New Method of Determining Compressibility" (Carnegie Inst., 1903), the *bar* is clearly defined as the pressure of a dyne per square centimeter. The pressure of an atmosphere is expressed as one million bars or megabar. For convenience this is written 1,000 kilobars. The pressure of the atmosphere at a height of 1 km. is approximately 900 kb., at an elevation of 10 km. the pressure is about 300 kb., and at the top of the atmosphere or as high as we may hope to explore for some time to come, say 50 km., the pressure is as low as 5 kb.

The use of these units of force has been extended at Blue Hill to measurements of water vapor pressure. There appears to be no difficulty in using the new units and the formulæ below.

European aërographic services use the following formulæ when the cover of the wet bulb is not frozen, and it may be noted here that in high level observations the temperature is generally so low that the wet bulb is covered with ice. No satisfactory solution has yet been reached.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

With wind velocity 0 to 0.5 m/s (meter per second)

$$p_s = p_e - .0012 P(t - f_1) (1 + f'/610)$$

with wind velocity 1 to 1.5 m/s

$$p_s = p_e - .0008 P(t - t_1) (1 + t'/610)$$

with wind velocity exceeding 2.5 m/s

$$p_s = p_e - .000656 P(t - t_1) (1 + t'/610)$$

When the water is frozen, the numerical coefficients .001060, .000706 .000579, are used, and 610 in the last factor changed to 689.

We prefer to express the relation in the following form

$$p_s = p_e - PK(t - t')$$

in which  $p_s$  is the pressure of the water vapor at the temperature of the dew-point or saturation,  $p_e$  the pressure at the temperature of evaporation or wet-bulb,  $P$  the pressure of the atmosphere,  $K$  a constant which like most constants varies somewhat (with ventilation of the bulb and probably other conditions). All pressures are given in the same unit, kilobar, and all temperatures in same unit, Kelvin kilograde. The value of  $K$  within ordinary limits is .00179. For quick determination of vapor pressure, absolute humidity and relative humidity, the value of  $PK$  may be taken as .18.

The wind effect is an uncertain quantity. All the tables of relative humidity so laboriously compiled by various meteorological services are invalidated by the omission of statement of temperature and wind values. Reference is made to papers by the author: "New Units in Aërology," PHYSICAL REVIEW, N.S., Vol. VI., No. 6, December, 1915; "Comparative Accuracy of Whirled Psychrometer," etc., PHYS. REV., February, 1918.

#### THE THERMAL EXPANSION OF LIVING TREE TRUNKS.<sup>1</sup>

BY C. C. TROWBRIDGE (DECEASED) AND MABEL WEIL.

THE transverse coefficient of expansion of living tree trunks has been found to be extremely large below 0° C. and very much smaller, *i. e.*, of about the order of the coefficient of expansion of dead wood, above that temperature. In the longitudinal direction a small *expansion* with *rise* of temperature was noted above the freezing point while an equally small *expansion* with *fall* of temperature was measured below that point. When, however, the temperature fell to about -18° C., a slight *contraction* with *fall* of temperature was observed. This behavior indicates the presence of two processes superposed, one a thermal coefficient of expansion, the other a physiological or cellular expansion. A lag of the transverse expansions and contractions behind temperature changes, also points to the same conclusion.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

PYROMETER ABSORPTION GLASSES.<sup>1</sup>

BY PAUL D. FOOTE, C. O. FAIRCHILD AND F. L. MOHLER.

THE proper methods of using absorption glasses and the choice of a suitable absorption glass for extending the high temperature scale are questions of considerable importance. The calibration of an absorption glass is given by the formula

$$1/\theta - 1/S = A,$$

where  $\theta$  is the absolute temperature of a black body,  $S$  the absolute observed temperature with the absorption glass in place and  $A$  is in general a function of  $\theta$ .

A glass is described for which  $A$  is a constant. Various methods of determining  $A$  are given. The use of two absorption glasses together is discussed and several theorems are developed in regard to effective wave-lengths of colored glasses, effective wave-length when sighted on non-black radiators, etc. A synopsis of the paper will appear in the Journal of the Optical Society and the complete paper will be published in the Bulletin of the Bureau of Standards.

THE MEASUREMENT OF SMALL ALTERNATING CURRENTS BY MEANS OF A RAYLEIGH RESONATOR.<sup>1</sup>

BY F. R. WATSON.

THE resonator consisted of a brass cylinder closed at one end by a telephone receiver and open at the other end through a small tube in which a Rayleigh disc was suspended by a quartz fiber. Alternating currents sent through the telephone receiver set up vibrations of the air in the cylinder that caused the disc to rotate, the amount of rotation being noted by the deflection of a beam of light reflected from the disc. A 110-volt, 60-cycle alternating current gave readable deflections for currents of the magnitude of  $1 \times 10^{-5}$  amperes. Later experiments indicated a greater sensitivity.

Various modifications of the apparatus were tested. A double resonator was made of two resonators similar to the one described connected by a tube with the Rayleigh disc in the center of the connecting tube. This arrangement allowed the use of two telephones and, being closed from the outside air, eliminated almost entirely the effect of extraneous sounds. An ordinary Bell telephone arrangement of a transmitter, dry cell, induction coil and receiver gave deflections for speech and musical sounds. Sensitiveness was increased by tuning the resonators and the electrical circuit to the source of sound. The frequencies of the currents ranged from 60 to 1,200 per second.

UNIVERSITY OF ILLINOIS,  
December 1, 1918.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

AN ADDITION TO THE THEORY OF THE QUADRANT ELECTROMETER.<sup>1</sup>

BY A. H. COMPTON AND K. T. COMPTON.

THE ordinary theory of the quadrant electrometer is extended to take account of a tilt of the needle about its long axis, a vertical displacement of one pair of quadrants with respect to the other by an amount  $\delta$  and a vertical displacement of the needle from its position of symmetry by an amount  $p$ . Under these conditions, if the slope of the tilted needle is  $s$ , the torque due to electrical forces is

$$T = K_1 V(v_2 - v_1) + V^2 \left( K_3 \frac{Rs\delta}{h^2} \theta + K_4 + K_5 \frac{\delta p}{h^2} \right), \quad (1)$$

neglecting terms of second and higher order of small quantities. Here  $V$  is the potential of the needle,  $v_1$  and  $v_2$  are the potentials of the two pairs of quadrants,  $h$  is the height of the quadrants,  $R$  is the radius of the needle and  $\theta$  is the deflection.

The terms

$$\left( K_4 + K_5 \frac{\delta p}{h^2} \right) V^2$$

cause the deflection usually observed when the needle is charged. By suitably adjusting  $\delta$  or  $p$  these terms may be made to cancel. Then the remaining terms are balanced against the torque  $K_2\theta$  from the suspension, giving

$$K_1 V(gv_2 - v_1) = \left( K_2 - K_3 \frac{Rs\delta}{h^2} V^2 \right) \theta; \quad (2)$$

whence the sensitiveness is

$$S = \frac{\theta}{v_2 - v_1} = \frac{K_1 V}{K_2 - K_3 \frac{Rs\delta}{h^2} V^2}. \quad (3)$$

By suitably adjusting  $s$ ,  $\delta$  and  $V$ , the magnitude and sign of the second term in the denominator may be varied within wide limits permitting a range of sensitiveness, theoretically from zero to infinity, to be obtained with any given suspension.

A model of an electrometer designed to take advantage of these considerations is demonstrated.

PERMANENT—CONTACT CRYSTAL DETECTORS.<sup>1</sup>

BY LOUISE S. MCDOWELL.

THE following investigation of possible methods of securing a permanent-contact crystal detector was initiated by a request from the Signal Corps to the Bureau of Standards to investigate a method proposed by Second Lieutenant Henri Lauer. His proposal to secure a permanent multiple-contact crystal detector by alternating-current electrolysis was based upon the hypothesis that, when immersed in an electrolyte, the rectifying action of the crystal would permit current to flow in only one direction through the sensitive areas and that

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

upon these areas alone there should be a deposit of the metal since the flow of current in both directions through the insensitive areas would prevent deposition. To the deposits upon the sensitive areas permanent contact could then be made by direct-current electrolysis or otherwise.

Audibility tests made upon two or more detectors in parallel and upon single crystals with contacts in parallel to two or more sensitive areas indicated that no gain in loudness of signal was to be expected from the use of multiple contacts. Crystals were further divided into three classes according to the direction of flow of the rectified current. Class 1, in which the direction of the current was usually from point to crystal, included molybdenite, zincite, chalcopyrites, iron pyrites, and bornite. Class 2, in which the direction of current flow varied from one sensitive point to another on the same crystal, included carborundum, approximately fifty per cent. of the specimens of silicon tested and one or two specimens of galena and molybdenite. Class 3, in which the direction of the current was usually from crystal to point included galena and fifty per cent. of the silicon tested.

Alternating-current electrolysis, with currents of 60 cycles and of radio frequency, was tried with crystals of class 1. A few adherent deposits of copper were obtained, but results indicated that these were due to accidental causes, not to rectification by the crystal since (1) only a few of the sensitive spots were coated, (2) there were many insensitive deposits, (3) lack of perfect adhesion between deposit and crystal increased the sensitiveness when contact was made to the deposit.

With galena, of class 3, an attempt was made to secure a metallic deposit upon the sensitive spots by cathode sputtering, using an alternating discharge and making the crystal one electrode. Results indicated that under these conditions no rectification occurs of a nature to permit deposition upon the sensitive areas of the crystal.

Further attempts to obtain a fixed, sensitive contact by the use of direct-current electrolysis, by soldering, and by welding proved unsuccessful with the more sensitive crystals such as galena and silicon although partially successful with molybdenite and carborundum. The results point to the necessity of a high resistance at the contact and suggest that with the more sensitive crystals the only satisfactory permanent-contact detector will be one with a mechanically maintained, high resistance contact.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.

#### ARE THE FREQUENCIES IN THE *K* SERIES OF X-RAYS THE HIGHEST FREQUENCIES CHARACTERISTIC OF CHEMICAL ELEMENTS?<sup>1</sup>

BY WILLIAM DUANE AND TAKEO SHIMIZU.

A NUMBER of investigators have found experimental evidence, which they interpreted as indicating the existence of radiation or absorption

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

characteristic of a chemical element, and of higher frequency than those of its *K* series of X-rays. On the other hand curves have been published showing the emission of X-rays by chemical elements such as rhodium and molybdenum, which include rays in the general X-radiation up to frequencies 50 per cent. and 75 per cent. greater than those of the *K* radiation and these curves furnish no evidence of the existence of characteristic rays of higher frequency than the *K* absorption frequency.

In this paper we wish to report to the society the results of experiments on the X-rays emitted by an aluminium target. The wave-lengths of the X-rays that we have examined lie between  $.1820 \times 10^{-8}$  cm. and  $1.259 \times 10^{-8}$  cm. This is the portion of the X-ray spectrum usually obtained from an ordinary X-ray tube excited by a difference of potential of 71,200 volts. Such a difference of potential does not produce X-rays shorter than  $.1820 \times 10^{-8}$  cm. and rays longer than about  $1.26 \times 10^{-8}$  cm. are almost completely absorbed by the glass walls of the tube. In order to investigate such long waves, specially constructed tubes must be employed.

As the shortest wave-length in the *K* series of aluminium is  $7.98 \times 10^{-8}$  cm., the range of frequencies in our experiments extends from 6.34 to 43.8 times the highest frequency in this *K* series.

In making the measurements we used the apparatus described in the PHYSICAL REVIEW for December, 1917, p. 624. This consisted of an X-ray spectrometer with a calcite crystal, the beams of X-ray being defined by two slits in lead blocks, which lay between the X-ray tube and the spectrometer. The current through the tube came from a high tension transformer connected to a system of kenotrons and condensers, which produced approximately a constant voltage. During part of the work the difference of potential amounted to 53,200 volts and during the rest of it, to 71,200 volts. The X-ray tube (kindly furnished us by Dr. W. D. Collidge) was similar to the ordinary Coolidge tube, except that an aluminium target replaced the usual tungsten target.

The curves obtained by plating the ionization currents against the grazing angles of incidence are, within the errors of measurement, smooth curves without such prominences as indicate characteristic rays, except at the following points:

(a) Near the angles corresponding to  $\lambda = .622 \times 10^{-8}$  cm. and  $\lambda = .709 \times 10^{-8}$  appears a pair of well defined prominences, indicating characteristic radiation amounting to 2 per cent. and 5 per cent. of the general X-radiation. These are undoubtedly due to the characteristic *K* radiation of molybdenum, the wave-lengths of the  $\beta$  and  $\alpha$  lines of which are  $.641 \times 10^{-8}$  cm. and  $.716 \times 10^{-8}$  cm. respectively, as given by Wooton.<sup>1</sup>

(b) There are also prominences near the angles corresponding to  $\lambda = .975 \times 10^{-8}$  cm.  $\lambda = 1.18 \times 10^{-8}$  cm. These lie very close to the  $\beta$  and  $\alpha$  lines in the *L* series of lead. According to Siegbahn and Friman's tables the wave-lengths of the strong  $\beta$  and  $\alpha$  lines in the *L* series of lead are  $\lambda = .981 \times 10^{-8}$  cm., and  $\lambda = 1.17 \times 10^{-8}$  cm.

<sup>1</sup> PHYSICAL REVIEW, January, 1918.



Part of the Coolidge cathode in the X-ray tube consists of metallic molybdenum so that the presence of the molybdenum lines may be accounted for either as coming from a thin layer of metallic molybdenum deposited on the target during the construction of the tube, or else as the result of secondary radiation from the molybdenum in the cathode producing tertiary radiation from the target.

The lead lines undoubtedly are due to secondary radiation from the lead blocks containing the slits through which the X-rays passed before reaching the spectrometer.

If the voltage applied to an X-ray tube lies considerably above that required to produce the characteristic rays of its target (as is the case here with molybdenum and lead), the characteristic radiation is many times more intense than the general radiation in its neighborhood. Further molybdenum and lead are far more efficient radiators than aluminium is (their atomic numbers being much higher, 47 and 82 respectively instead of 13 for aluminium).

In addition to these prominences there appears a sharp break in the ionization curve at an angle corresponding to  $\lambda = .3727 \times 10^{-8}$  cm. which is the characteristic ionization wave-length of iodine. This break is due to the fact that methyl-iodide was used as the gas in the ionization chamber. To make sure that no radiation characteristic of aluminium occurs near this wave-length, we removed the methyl-iodide, and refilled the ionization chamber with ethyl-bromide. Under these conditions no prominence nor break appears in the curve near  $\lambda = .3727 \times 10^{-8}$  cm.

We conclude, therefore, that aluminium possesses no characteristic radiation within the range of wave-lengths examined.

#### THE SPECTRAL PHOTO-ELECTRIC SENSITIVITY OF SILVER SULPHIDE.<sup>1</sup>

BY W. W. COBLENTZ AND H. KAHLER.

**B**OTH the natural mineral acanthite and a laboratory preparation by Mr. G. W. Vinal, were examined. The latter material which was hammered into a thin plate, was found insensitive photo-electrically, at room temperature, but at  $-157^{\circ}$  C. a sharp maximum of photo-electrical sensitivity was observed for radiations of wave-length  $\lambda = 1.05 \mu$ .

At room temperatures the natural crystalline mineral differs from other photoelectrically sensitive substances studied in that the photo-electric response becomes fatigued and after an exposure of 2 to 3 seconds to light, the positive resistance change (galvanometer deflection) begins to be effective. For unlimited exposure, the galvanometer deflection returns to about one fourth of its

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

maximum deflection. On removing the light stimulus, the galvanometer gives a negative deflection, which in the course of a few minutes returns to the original zero scale reading. In other words, the change in resistance of the crystal when exposed to radiation, is first negative then positive, the resultant change being negative and roughly one fourth the original change. At low temperatures,  $-158^{\circ}$  C., this polarization phenomenon disappears, and the response to radiation is the same as that of other substances, *e. g.*, selenium and molybdenite.

At room temperature, acanthite has a maximum of photo-electrical sensitivity for radiations of wave-length  $\lambda = 1.35 \mu$  with a region of high sensitivity at  $0.8$  to  $1 \mu$ .

At  $-158^{\circ}$  C. the sensitivity curve is quite symmetrical with a maximum at  $1.2 \mu$ . Mechanical working of the crystal (hammering it into a thin plate) has a marked effect upon this maximum which, in one sample, occurred at  $1.13 \mu$  to  $1.23 \mu$ , depending upon the amount of hammering the crystal had undergone.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.,  
February 5, 1919.

#### THE CRYSTAL STRUCTURE OF CARBORUNDUM.<sup>1</sup>

BY A. W. HULL.

IN the December number of the Journal of the American Chemical Society Burdick and Owen have described experiments on the crystal structure of carborundum. They conclude that the structure is like that of diamond, with half of the carbon atoms replaced by silicon, with this exception, that the carbon atoms are not in the centers of the tetrahedra formed by the silicon atoms, but are displaced toward the apices of the tetrahedra, corresponding to a lack of symmetry in the carbon atom for which we have no other evidence. Their conclusions are based entirely upon the relative intensities of the X-ray reflections, and their calculations are made in exactly the same manner as those of the Braggs on calcite and similar crystals. It is assumed that the scattering electrons of each atom are all concentrated at the center of the atom, and that some undefined cause produces a "normal" fall of intensity in successive orders represented by the numbers 100, 20, 7 and 4. The variations from this normal fall of intensity are the basis of the conclusions drawn regarding the positions of the atoms.

Inasmuch as the assumptions on which this analysis is based are certainly not valid, it seemed worth while to investigate what variations in intensity might be expected from a *perfect* diamond lattice, composed of atoms having

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

stationary electrons in cubical arrangement, such as have been found to account for the intensities of the reflection observed in diamond and crystal silicon respectively. The only assumptions made are that the electrons are fairly stationary in the positions specified, and that they scatter X-rays independently according to classical electromagnetic theory.

In order to make the conclusions as definite and free from arbitrary assumptions as possible, the C atom is assumed to have *exactly the same size and shape* as in diamond, namely, two electrons very near the nucleus, and four electrons at the opposite (tetrahedral) corners of a cube at distances of .77 aengstroms from the nucleus; and the silicon atom to be *exactly as in crystal silicon*, except that its four outer electrons are drawn in about seven per cent. It has two electrons very near the nucleus, eight at the corners of a cube at distances of .15 aengstroms from the nucleus, and the remaining four at opposite (tetrahedral) corners of a larger cube at distances of 1.12 aengstroms from the center. The atoms are assumed to be all similarly oriented, with the axes of their cubes parallel to the crystallographic axis of the carborundum crystal (assumed cubic). The calculations are made in a manner exactly analogous to that of the Braggs, applying Darwin's formula to *single electrons*.

The following table gives the intensities calculated in this way for the different reflections observed by Burdick and Owen.

The agreement with the experimental values is as good as the data warrants, and is fully as good as in Burdick and Owen's calculations. When more exact experimental data is available it will probably be found that the arrangement of electrons in exact cubes as assumed above is not correct, but that these cubes are distorted, at least slightly, by the forces of attraction holding the atoms together.

The facts which it is desired to emphasize here are (1), that the assumption of simple cubical atoms, that is, atoms composed of a nucleus surrounded by stationary electrons in cubical arrangement, accounts completely for the observed variation in the intensity of the reflections of different orders in the case of "normal" planes. (2) that in the case of carborundum this "shape factor" is capable of explaining completely the observed *variations* from the "normal" intensity, without assuming any lack of symmetry in the arrangement of the atoms.

The structure of carborundum may therefore be considered, as far as our present information goes, to be a lattice *exactly* like that of diamond, in which half the carbon atoms, that is, those belonging to one of the two face-centered cubic lattices, are replaced by silicon. Each carbon atom is surrounded by two electrons very near its center, plus 8 equidistant electrons, four of its own and one each from the four surrounding silicon atoms along the four "chemical bonds" joining these silicon atoms to the carbon. These eight electrons form an exact cube of side .89 Å. around the carbon atom. Each silicon atom has two electrons very near its center, surrounded by eight electrons in a cube of side .17 aengstroms, surrounded in turn by the four above mentioned electrons

Reflecting Plane.	Order of Reflection.	Intensity of Lines.			
		Observed.		Calculated.	
		Crystal I.	Crystal II.	Hull.	Burdick and Owen.
100	1	100	100	100	100
	2	126	470	47	72
	3	57	196	130	17
$\bar{1}10$	1	100	100	100	100
	2	61	38	77	20
	3	22	7	29.2	7
111	1	100	100	100	100
	2	35	55	18	31
	3	19	38	16	19
	4	13	23	3.2	4
	5	3	—	8	2
	6	1	—	3.5	2
$11\bar{1}$	1	100	100	100	100
	2	35	37	18	7
	3	44	57	16	3
	4	18	14	3.2	5
$2\bar{1}1$	1	100	100	100	100
	2	23	27	38	20
$11\bar{3}$	1	100	100	100	100
	2	12	12	18	14
$31\bar{1}$	1	100	100	100	100
	2	21	21	18	31
	3	—	8	10	19
210	1	100	100	100	100
	2	10	11	14	13

in tetrahedral arrangement, on the lines joining it to the four nearest carbon atoms. The distance of these outer four electrons from the center of the silicon atom is 1.12 aengstroms. This is slightly less than their distance, 1.20 aengstroms, from the centers of silicon atoms in crystal silicon. In other words, the atoms of carbon and silicon are about 4 per cent. closer to each other in carborundum than would be obtained by packing together carbon and silicon atoms of the exact size and shape as found in diamond and crystal silicon. This closer approach should correspond to the chemical stability of carborundum as compared with diamond and silicon. For simplicity, it has been assumed in the above calculations that the carbon atom maintained its

exact size, the shrinkage being attributed entirely to the silicon electrons as being the more mobile.

February 14, 1919.

A POSSIBLE RELATION BETWEEN THE RESONANCE POTENTIAL AND SPECIFIC  
INDUCTIVE CAPACITY OF A METALLIC VAPOR AND ITS PROPERTIES IN THE  
SOLID STATE.<sup>1</sup>

By K. T. COMPTON.

RECENT discoveries have shown that the long wave-length limit of photoelectric sensitiveness of a metal is greater in the solid than in the vapor state. In other words, the energy required to liberate an electron from an atom of the vapor is greater than that required to liberate an electron from a metal. However the work required to extract an electron from the metal is strikingly similar to that required to produce the "single line" spectrum of the vapor. A table is given showing the degree to which these quantities are identical, from which it appears that in the case of a majority of the metals the discrepancies fall within the limits of experimental error. Theoretical considerations indicate that exact coincidence would not be expected even if the relation suggested were true.

If this relation is true, it is suggestive of interesting speculations regarding the condition of the electrons in a metal. It would appear that the atoms retain practically the same structure as in the vapor state as far as the equilibrium orbits lying within that orbit involved in the production of the single lined spectrum are concerned, but that an electron outside this orbit becomes a "free electron" in that it is not subject to appreciable attractions from the parent atom. Thus the work required to extract an electron from a metal is very nearly that required to withdraw it from the "single line" orbit to the one next outside. These considerations are highly speculative at present, though it would not be surprising if an attempt to apply the Bohr theory of the atom to solids would yield fruitful results.

Attention is also called to the fact the argument used by the writer to derive the ionizing potential of a gas in terms of the specific inductive capacity of the gas<sup>2</sup> applies equally well to the resonance potential, applying to whichever has the lower value. Recent published and unpublished data on ionizing and resonance potentials and absorption spectra are shown to support the validity of the argument as a first approximation.

PRINCETON UNIVERSITY.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

<sup>2</sup> PHYS. REV., 8, p. 412, 1916.

THE LAW OF ABSORPTION OF HIGH FREQUENCY RADIATION.<sup>1</sup>

BY ARTHUR H. COMPTON.

OWENS' law for the absorption of X-rays, that

$$\mu = k_1 \lambda^3,$$

where  $\mu$  is the mass-absorption coefficient,  $\lambda$  is the wave-length and  $k$  a constant, has been modified by Barkla to account for the scattering which becomes important at very high frequencies. In his expression,

$$\mu = k_2 \lambda^3 + \sigma_0$$

the mass-scattering coefficient  $\sigma_0$  was originally considered as a constant, having the value 0.2, calculated by J. J. Thomson on the usual electron theory. Barkla and White have recently shown, however, that this quantity must diminish at short wave-lengths, but were unable to suggest any reason for the decrease.

If the electron is taken as a ring of electricity with a radius comparable with the wave-length of very hard X-rays and V-rays, the mass-scattering coefficient should vary according to the expression recently proposed by the writer,

$$\sigma = \sigma_0 \left( 1 - 29.61 \frac{a^2}{\lambda^2} + 524.2 \frac{a^4}{\lambda^4} - 5396 \frac{a^6}{\lambda^6} + \dots \right), \quad (1)$$

where  $\sigma_0$  is the mass scattering coefficient as calculated by Thomson, and  $a$  is the radius of the electronic ring. At the same time the true or fluorescent absorption will depend upon the magnitude of the action of the incident radiation upon the electron. This will diminish for short wave-lengths, since in this case different parts of a wave may work against each other in trying to pry an electron loose from its fixed position. The energy absorbed in displacing an electron may be shown to be proportional to  $\sqrt{\sigma}$ . The mass absorption coefficient may therefore be represented by

$$\mu = k_3 \sqrt{\sigma} \lambda^3 + \sigma, \quad (2)$$

where  $\sigma$  is the function of  $a/\lambda$  represented by equation (1).

This expression is compared with the experimental values for the absorption coefficient of aluminium as given by Hull and Rice, Williams, and Bragg and Pierce, and is found to be satisfactory, especially for the short wave-lengths where the formulas of Owen and Barkla fail.

The value of the radius of the electron necessary to give best agreement with the experimental values is  $1.85 \times 10^{-10}$  cm. Unless there is some consistent error in the measurements of Hull and Rice, this value is determined within a probable error of about 5 per cent.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

COLLISIONS OF SLOW MOVING ELECTRONS WITH NITROGEN MOLECULES.<sup>1</sup>

By S. KARRER.

THE collisions of slow-moving electrons with nitrogen molecules has been studied by projecting electrons having known velocities into a retarding electric field between two parallel plate electrodes. The variation of the current between the electrodes with the distance between them indicates that the collisions are inelastic, if the velocity of the electrons is below the resonance velocity and the number of collisions per second is not too great. The nature of the collisions seems to be somewhat influenced by the rapidity with which they follow each other.

No traces of ionization of nitrogen were observed with electron velocities as high as ten volts, a result in accord with the recent report of Davis and Goucher.

A plane equipotential source of electrons was used.

PHYSICAL LABORATORY,  
UNIVERSITY OF ILLINOIS.

THE IONIZING POTENTIALS OF ARGON, NEON AND HELIUM.<sup>1</sup>

By H. C. RENTSCHLER.

THE object of the investigation was to measure the ionizing potentials of these gases and to determine whether they show the two types of inelastic impact between an electron and an atom of the gas, as found in the case of the metallic vapors. The first type of inelastic impact occurs when the colliding electron produces a displacement of an electron of the atom without detaching the electron from the atom known as the "resonance potential." The second type of inelastic impact occurs when the colliding electron produces ionization.

The test for "resonance potential" was made by the method first used by Tate in his work on mercury vapor.

The ionizing potentials were measured by Lenard's method, and also by measuring the current between a hot tungsten filament and a surrounding cylinder as a function of the potential between the cylinder and filament.

Argon was found to have a resonance potential of about 12 volts and an ionizing potential of about 17 volts.

Neon shows no resonance potential. The ionizing potential was found to be about 21 volts instead of 16 as determined by Franck and Hertz. This difference is probably due to the greater purity of the neon used in the present experiment.

Helium shows no "resonance potential." The ionizing potential was found to be about 27 volts instead of 20 volts as found by Franck and Hertz, Powlow, and Bazzoni. This value is in close agreement with Bohr's theory and also with the value calculated by the quantum theory from the limit of the helium spectrum measured by Richardson and Bazzoni.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Experiments are described showing the probable cause for the difference between the present and former values.

RESEARCH LABORATORY,  
WESTINGHOUSE LAMP COMPANY.

NOTE ON THE USE OF THE HOT CATHODE VACUUM TUBE FOR THE MEASUREMENT OF VOLTAGE.<sup>1</sup>

BY LYNDE P. WHEELER.

1. *Methods Employed.*—Two general schemes of connection have been tried. In one the voltage to be measured is applied between the grid and the filament of a three element tube and the change thus produced in the plate current observed. In the other, the voltage is introduced between the plate and filament of a two element tube and the resulting change in plate current measured. A third method (for higher voltages) in which the voltage is applied between the plate and filament of a three element tube and the change in grid current measured, is still under investigation.

2. *Results Obtained.*—Using the first of the above methods it has been found possible to devise a compact portable apparatus which serves satisfactorily for the measurement of low voltages (up to 50 volts) in A.C. circuits of any frequency. Such voltmeters are in every day use in the high frequency laboratory of the Sloane Laboratory at Yale University. By the second method somewhat higher voltages can be measured, and it is hoped by means of the third to extend the range still further. Only the results obtained by the first method are discussed in this note. These instruments have been found to hold their calibration well, are not too complicated in their manipulation to be put into the hands of electrical engineering seniors, and if properly used their indications can be made independent of wave form as well as frequency.

3. *Details of Operation.*—Only the smaller sizes of the high vacuum tube used for sending and receiving in radio have been tried. The best plate and grid potentials to use vary with the characteristic of the particular tube. They are determined (a) by the condition that the grid current must be kept very small in order not to draw appreciable energy from the circuit in which the voltage is being measured; (b) by the sensibility of the instrument used to measure the plate current; (c) by whether it is desired to make the scale of the instrument uniform when calibrated in volts. The first condition necessitates in general a negative, zero or at most a small positive grid potential. A considerable range in the choice of plate potential in no way affects the usefulness of the contrivance (provided the plate current does not surpass that at the upper bend of the characteristic) and enables the second and third conditions to be met. Any device for flattening out the characteristic which does not introduce inductance in the plate circuit will aid in meeting the third condition.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.



If care is taken to keep the variations of plate current within the limits of the straight portion of the characteristic of the tube, the calibrated scale is linear. Also since under these conditions there is no distortion, the impressed voltage wave form will be faithfully reproduced.

The calibration must be carried out on a low frequency A.C. circuit. For measurements up to five volts, using tubes such as the V T-1 or V T-11, it has been found satisfactory to work with the grid at zero and the plate at one and a half volts in connection with a microammeter having a full scale reading of 120. With these values of the grid and plate potentials the scale is not quite uniform, nor are the readings independent of wave form. The compensating advantages are simplicity and that owing to the rectifying action of the tube when so used no thermocouple is needed.

4. *Limitations.*—These are of two kinds. The first is that imposed by the necessity of using two auxiliary batteries. This restricts the portability of the contrivance and of course it is essential that each battery should yield a strictly constant current. The second lies in the variability of the tubes commercially available. Each tube requires individual adjustment, although once a good tube has been selected and the working potentials properly adjusted its indications seem to be reliable for the best part of the life of the filament. Until the manufacture of tubes has been better standardized however, this variability is a distinct handicap to the use of these methods of measuring voltage.

#### AN INTERFERENTIAL METHOD FOR MEASURING THE EXPANSION OF VERY SMALL SAMPLES.<sup>1</sup>

BY C. G. PETERS AND IRWIN G. PRIEST.

THIS paper describes an interferential method for measuring accurately the thermal expansion of samples even much smaller than can be used in the Fizeau interferential method, which requires *three* similar pins, usually about ten millimeters long or a homogeneous ring about forty millimeters in diameter and usually ten millimeters long in the direction of the measured expansion. Data are presented relating to *single* samples about five millimeters in cross section and ten millimeters in the direction of the measured expansion. Even smaller samples may be used if necessary.

The essential novelty of the method lies in the fact that measurements are made on the change in width instead of the displacement of interference fringes. For a fuller explanation of this, the complete paper must be consulted.

The new method yields results of the same accuracy as the Fizeau method while it has the following advantages over the latter:

1. Only one small pin is needed for a sample.
2. Contact errors are decreased and the error due to creeping of the cover plates is eliminated.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

3. The computations are simplified by having the standard and sample nearly equal in length.
4. The trouble of counting the fringes while the temperature is changing is eliminated entirely.
5. The correction for change of refractive index with the density of the air is eliminated without evacuating the container.
6. The method is especially advantageous in determining small differential expansions, for the difference in expansion of small samples may be obtained directly by carrying them together through the same temperature cycle.

The complete paper will be published as a Scientific Paper of the Bureau of Standards.

BUREAU OF STANDARDS,  
WASHINGTON, D. C.,  
February 14, 1919.

THE PROPERTIES OF THE ELECTRON AS DERIVED FROM THE CHEMICAL  
PROPERTIES OF THE ELEMENTS.<sup>1</sup>

BY IRVING LANGMUIR.

IN a paper soon to appear in the Journal of the American Chemical Society a theory of the arrangement of electrons in atoms and molecules will be given. This theory explains the periodic properties of all the elements including those of the eighth group and the rare earths. It also meets with success in explaining the magnetic properties of the elements. It leads to a simple theory of chemical valence applying equally well to polar and to non polar substances. In the case of organic compounds the results are identical with those of the ordinary valence theory, but with oxygen, nitrogen, chlorine, sulphur and phosphorus compounds the new theory applies as well as to organic compounds, while the ordinary valence theory fails completely. The so-called physical properties such as boiling points, freezing points, electric conductivity, etc., are accounted for as easily as the chemical properties. The arrangement of electrons even in molecules of nitrogen, carbon monoxide, nitric oxide and cyanogen is worked out and explains the properties of these remarkable substances.

The theory is an extension of Lewis' theory of the "cubical atom."<sup>2</sup>

The very satisfactory agreement between this theory and the enormous number of experimental data of organic and inorganic chemistry gives great importance to the postulates underlying the theory for these represent the conditions which must apparently be fulfilled if the properties of matter are to be explained. Thus far the physicist has studied the electron from a rather narrow field of observation. The chemical data now gives much additional information about the electron, some of which seems inconsistent with conclusions resulting from a study of the problem by physical methods.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

<sup>2</sup> Jour. Amer. Chem. Soc. 38, 762, 1916.

The theory is based on the following postulates, given here in rather incomplete form.

1. The electrons in atoms are either stationary or rotate, revolve or oscillate about certain definite positions in the atom. In the most stable atoms, namely those of the inert gases, the electrons are arranged in pairs symmetrically placed with respect to a plane called the equatorial plane. No electrons lie in this plane. There is an axis of symmetry (polar axis), perpendicular to this plane, through which four secondary planes of symmetry pass forming angles of  $45^\circ$  with each other. The atoms as a whole thus have the symmetry of a tetragonal crystal.

2. The electrons in any given atom are distributed through a series of concentric (nearly) spherical shells, all of equal thickness. Thus the mean radii of the shells form an arithmetic series 1, 2, 3, 4, and the effective areas are in the ratios  $1:2^2:3^2:4^2$ .

3. Each shell is divided into cellular spaces or cells occupying equal areas in their respective shells and distributed over the surface, of the shells according to the symmetry required by postulate 1. The first shell thus contains 2 cells, the second 8, the third 18, and the fourth 32.

4. Each of the cells in the first shell can contain only one electron, but each other cell can contain either one or two electrons. All the inner shells must have their full quotas of electrons before the outside shell can contain any. No cell in the outside layer can contain two electrons until all the other cells in this layer contain at least one.

The inert gases are those in which all the cells in the outside shell have equal numbers of electrons. Thus according to the first four postulates helium has two electrons, neon has ten, argon 18, krypton 36, xenon 54, and niton 86. All atoms with an atomic number greater than that of helium (2) have as their first shell a pair of electrons close to the nucleus. The line connecting the two electrons establishes the polar axis for the atom. Neon has in its second shell eight electrons, four in each hemisphere (*i. e.*, above and below the equatorial plane), arranged symmetrically about the polar axis. The eight electrons are thus nearly at the corners of a cube. In argon there are eight more electrons in the second shell. In all elements of higher atomic number the second shell is like that in argon. Krypton has in its third shell nine electrons in each hemisphere symmetrically placed with respect to the polar axis and to the four electrons in the second shell. The ninth electron in each hemisphere goes into the polar axis. Xenon is like krypton except that it has twice as many electrons in its third shell. Niton has sixteen electrons in each hemisphere of its fourth shell. These are easily placed symmetrically with respect to the polar axis and the eight underlying electrons.

5. Two electrons in the same cell do not repel nor attract one another with strong forces. This probably means that there is a magnetic attraction (Parson's Magneton Theory) which nearly counteracts the electrostatic repulsion.

6. When the number of electrons in the outside layer is small, the arrange-

ment of the electrons is determined by the (magnetic?) attraction of the underlying electrons. But when the number of electrons increases, especially when the layer is nearly complete, the electrostatic repulsion of the underlying electrons and of those in the outside shell becomes predominant.

7. This postulate refers to the properties of atoms and need not be considered here.

8. The stable and symmetrical arrangements of electrons corresponding to the inert gases are characterized by strong internal and weak external fields of force. The smaller the atomic number, the weaker the external field.

9. The most stable arrangement of electrons is that of the pair in the helium atom. A stable pair may also be held by: (a) a single hydrogen nucleus; (b) two hydrogen nuclei; (c) a hydrogen nucleus and the kernel of another atom; (d) two atomic kernels (very rare).

10. The next most stable arrangement of electrons is the *octet*, that is, a group of eight electrons like that in the second shell of the neon atom. Any atom with atomic number less than eighteen, and which has more than three electrons in its outside layer tends to take up enough electrons to complete its *octet*.

11. Two octets may hold one, two, or sometimes three pairs of electrons in common. One octet may share one, two, three or four pairs of its electrons with one, two, three or four other octets. One or more pairs of electrons in an octet may be shared by the corresponding number of hydrogen nuclei. No electron can be shared by more than two octets.

The results obtained by the use of these postulates are so striking that one may safely reason that the results establish the fundamental correctness of the postulates.

How can these results be reconciled with Bohr's theory and with our usual conception of the electron? It is too early to answer. Bohr's stationary states and the cellular structure postulated above have many points of similarity. It seems that the electron must be regarded as a complex structure which undergoes a series of discontinuous changes while it is being bound by the nucleus or kernel of an atom. There seems to be strong evidence that an electron can exert magnetic attractions on other electrons in the atom even when not revolving about the nucleus of the atom.

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#### SOME PHYSICAL PROPERTIES OF DENTAL MATERIALS.<sup>1</sup>

BY C. G. PETERS AND W. H. SOUDER.

**I**N carrying out an investigation of dental amalgams, the following properties are being considered: thermal expansivity, dimensional changes, and crushing strengths.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

*Thermal Expansion Coefficients.*—In determining the thermal expansion coefficients, a differential interference method, was adopted as the most accurate and convenient.

The thermal expansion of different sections of teeth, immersed in a solution, determined over the range 20 to 70 degrees centigrade were approximately  $6.5 \times 10^{-6}$  per degree centigrade for the root and  $10 \times 10^{-6}$  for the enamel.

The coefficient of a specimen of synthetic porcelain tested under similar conditions was about  $8 \times 10^{-6}$ .

Similar tests on amalgams showed coefficients from  $24$  to  $28 \times 10^{-6}$ . The critical point near  $75^\circ$  centigrade, discovered by other observers while making different tests, was confirmed.

This differential expansion between tooth structure and amalgam, amounting to .2 micron per centimeter per degree centigrade, is thought to be worthy of serious consideration, by dental clinicians.

*Dimensional Changes During Amalgamation.*—Samples of different amalgams were packed in a mold of definite shape such that these would fit the interferometer and their changes measured exactly as in the previous case. These samples were placed in position in a few minutes after amalgamation and observations taken over a period of one to three days.

Curves plotted to show change in length with time show an initial contraction which lasts about 20 minutes and is thought to be due to the specimens returning to the equilibrium temperature. This contraction is followed by an expansion of from 4 to 8 microns per centimeter at the end of 3 hours. Additional changes with time are relatively small.

*Flow Under Continuous Pressure.*—Subjecting specimens of amalgam to a constant pressure of about 3,000 pounds per square inch (beginning 48 hours after amalgamation) caused a flow in all cases. In most instances this was less than 2 per cent. at the end of five days. In some cases it caused a change of 10 per cent. or caused a rupture of the material.

It is thought that the rate of application of pressure modifies the crushing strength quite materially.

## THE ORIGIN OF THE GENERAL RADIATION SPECTRUM OF X-RAYS.<sup>1</sup>

BY DAVID L. WEBSTER.

AT a meeting of this society, four years ago, Duane and Hunt<sup>2</sup> announced the experimental proof that the spectrum of X-rays contained no frequencies above the value whose quantum is the energy of a cathode ray; and I pointed out<sup>3</sup> the fact that this law meant that the old pulse theory could not hold, but that the general radiation, just as much as the characteristic, must consist of long trains of continuous waves. Since then the evidence for the

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

<sup>2</sup> W. Duane and F. H. Hunt, *PHYS. REV.*, 6, 166, 1915.

<sup>3</sup> D. L. Webster, *PHYS. REV.*, 6, 56, 1915.

wave train theory has been strengthened by a later analysis showing that for an infinitely thin anticathode, not only the derivatives of the intensity, but the intensity itself, would be discontinuous at the limit of the spectrum.

The question suggested by this is, What is the nature of the oscillators producing these trains of waves? Are they cathode rays or are they something fixed in the atom? If they are cathode rays, we may expect them to show the Doppler effect, and with it also, according to the electromagnetic theory, we may expect an unsymmetrical distribution of intensity of X-rays as though the X-rays were thrown forward by the cathode rays. Such a combination of effects was indeed found by Kaye,<sup>1</sup> in 1909, using very thin targets of Al, Cu, Au and Pt. In every case he found more rays thrown forward than back, and in most cases the forward rays were more penetrating, showing the higher frequency that one would expect from the Doppler effect. With thick targets of carbon, the same sort of asymmetry of intensity was found by Stark,<sup>2</sup> and Kirschbaum,<sup>3</sup> but they could not be sure of any difference in frequency. And in thick targets of Pb investigated by Ham,<sup>4</sup> silver by Miller<sup>5</sup> and Pt by Kirschbaum,<sup>6</sup> no asymmetry could be detected. From this we should infer that the asymmetry was a property of light elements or thin layers of heavier ones, but not of thick layers of heavy ones. Before drawing any conclusions we should therefore try to see why the effect is limited in this way.

One obvious reason for expecting less asymmetry from heavy elements is the presence of characteristic rays, which, for various reasons, are likely to be uniformly distributed. But this is not enough. Another significant fact is that none of the experiments on the limit of the spectrum have shown any Doppler effect there, although with the high speeds of cathode rays such an effect might be expected even at angles not much different from a right angle from the cathode rays. This indicates that the highest frequencies emitted are emitted by electrons that have already lost all their velocity before emitting, although they were necessarily the fastest ones present at all when they struck the atoms that caused the emission. If so, the electrons that emit while in motion are therefore those that emit lower frequencies than the highest they are capable of, or in other words the lower frequencies will only show asymmetry when produced by electrons whose energy is initially much greater than the quanta for those frequencies. With aluminum, which gives no errors from characteristic rays, exactly this effect was found by Kaye, since the softer components of a hard beam showed more asymmetry than the harder ones. This explains at once why thick targets show less asymmetry than thin ones. They contain cathode rays of low velocity, and a large proportion of their low frequency X-rays are due to these cathode rays. And it also explains why

<sup>1</sup> G. W. C. Kaye, *Proc. Camb. Phil. Soc.*, 15, 269, 1909.

<sup>2</sup> J. Stark, *Phys. Zeitschr.*, 10, 902, 1909.

<sup>3</sup> H. Kirschbaum, *Ann. der Phys.*, 46, 85, 1915.

<sup>4</sup> W. R. Ham, *Phys. Rev.*, 30, 96, 1910.

<sup>5</sup> F. C. Miller, *Franklin Inst. Journal*, 171, 457, 1910.

<sup>6</sup> *Loc. cit.*

heavy elements show less asymmetry than light ones if we may assume that their atomic electrons are bound more firmly and are less easy for the cathode rays to dislodge and move in their own direction, and their more powerful nuclei will cause greater scattering of the moving electrons.

This brings the experiments into line with each other, but leaves us with two very startling conclusions. First, that electrons can radiate continuous wave trains while in rapid forward motion; and second, that when an electron radiates all its energy, it stops first and radiates after stopping. If the electron is in motion, and moves the diameter of an atom while radiating a wave train, where does it get the elastic force under which it vibrates? How can it carry its elastic restoring force and whole vibrating mechanism with it for such a distance? And if an electron does radiate all its energy, as at the high frequency limit of the spectrum, and must stop first, before radiating, where is its energy after its forward motion has stopped but before it radiates? These questions seem very hard to answer on the hypothesis that the only properties of the electron are those of a small uniformly charged sphere.

But they are easily answered by the theory of heat radiation that I developed some time ago on the basis of a modified form of the Parson magneton.<sup>1</sup> For in this theory the electron is a thin conducting ring and the emitting oscillation is that of the electricity on the ring, and this oscillation can occur without the help of elastic forces from outside, as it must when the electron is moving. The magneton also provides for the disposal of the energy between the stoppage of the cathode ray and the radiation of the X-rays, transforming it into energy of the oscillation. And it explains why, if all the energy of translation of the cathode ray has gone into the electric oscillation it will be radiated from a stationary center, but if only part of it goes into the oscillation, the translational motion will continue during the oscillation and cause the observed asymmetry.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,  
CAMBRIDGE, MASS.,  
March 1, 1919.

#### NOTE ON BOMB TRAJECTORIES.<sup>2</sup>

BY EDWIN B. WILSON.

THE trajectory in vacuo would be a parabola of which the equation is  $y = gx^2/2u_0$ , if  $u_0$  be the initial velocity (supposed horizontal) of the bomb. It is possible to make a simple approximation to the path when the resistance of the air is taken into account.

Namely:

$$y = \frac{gx^2}{2u_0^2} + \frac{g^2x^3}{3u_0^2U^2},$$

<sup>1</sup> A. L. Parson, Smithsonian Miscellaneous Collections, 65, 11, 1, 1915. D. L. Webster, Proc. Amer. Acad., 50, 131, 1915.

<sup>2</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.



where  $U$  is the limiting velocity of the bomb and where it must be assumed that the length of the path of the trajectory is not over one or two thousand feet. The equation may be solved for  $x$  in terms of  $y$ , or rather in terms of the square root of  $y$ , with the result

$$x = \sqrt{\frac{2y}{g}} u_0 - \frac{2}{3} \frac{u_0^2 y}{U^2}.$$

The first term in this expression represents the horizontal travel of the bomb in vacuo during a drop of amount  $y$  and the second term represents the distance that the bomb is retarded relative to the parabolic path by the action of the air supposed to resist according to the ordinary law of the square of the velocity. The approximation is good only when  $\sqrt{2yg}/3 \cdot u_0/U^2$  is small. If the limiting velocity of the bomb is 900 feet per second and the initial velocity is 100 feet per second, the correction remains small for very considerable lengths of drop  $y$ —in fact, for lengths much greater than appear valid for approximations previously introduced in the work.

#### ON THE RELATION BETWEEN THE $K$ SERIES AND THE $L$ SERIES OF X-RAYS.

BY WILLIAM DUANE AND TAKEO SHIMIZU.

SOME years ago Kossel called attention to the fact that the difference between the frequencies of the  $\beta$  and  $\alpha$  lines in the  $K$  series of X-rays characteristic of a chemical element very nearly equals the frequency of one of the strong lines in the  $L$  series of that element. Since then the relation between the two series of X-rays has been the subject of considerable discussion, especially with regard to its bearing on the modern theories of the structure of the atom and of the mechanism that produces characteristic line spectra. Kossel's equation cannot represent a general relation between the lines in the  $K$  series and those in the  $L$  series, however, for the  $L$  series contains many more lines than the  $K$  series contains.

In the researches described in this paper we have endeavored to measure the absorption and emission frequencies characteristic of tungsten in both the  $K$  and the  $L$  series, using the same calcite X-ray spectrometer throughout the work.

In the experiments on emission lines the X-rays came from the tungsten target of an ordinary Coolidge tube, but in most of the experiments on absorption we replaced this tube with one containing a molybdenum target, and inserted a thin layer of potassium tungstate between the tube and the spectrometer.

In each experiment the current exciting the X-ray tube came from a high tension transformer with a system of condensers and kenotrons attached to it for producing approximately a constant difference of potential.

In order to avoid errors due to the penetration of the X-rays into the crystal

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.



we placed the two narrow slits that define the X-ray beam between the X-ray tube and the spectrometer.

The curves shown in the lantern slides represent the ionization currents as functions of the readings of one of the verniers attached to the crystal table.

To avoid making an accurate determination of the zero point directly readings were taken on both sides of the zero line of the spectrometer.

The curves on the first slide represent the *K* series of lines in both the first and the second order spectra. To produce the *K* series in tungsten requires a difference of potential greater than 69,200 volts applied to the X-ray tube, and in order that the peaks on the curves that represent characteristic lines should be well marked we employed a difference of potential of 100,000 volts.

The curves show the following characteristics:

(a) At the angles corresponding to  $\lambda = .1785 \times 10^{-8}$  cm. (calculated from the grazing angle of incidence  $\theta$  by the formula

$$n\lambda = 6.056 \times 10^{-8} \times \sin \theta$$

appear sharp drops in the curves. This, then, is the characteristic absorption wave-length of tungsten in the *K* series.

(b) At the angles corresponding to  $\lambda = .2087 \times 10^{-8}$  cm. and  $\lambda = .2134 \times 10^{-8}$  cm. occur two sharp peaks completely separated from each other. These are the wave lengths of the two emission lines  $\alpha_1$  and  $\alpha_2$  in the *K* series. The peak corresponding to  $\alpha_2$  does not appear to be quite symmetrical. It is slightly broader on the long-wave length side than on the short, suggesting that there may be a weak line  $\alpha_3$  very close to  $\alpha_2$ , with a wave-length of about  $.214 \times 10^{-8}$  cm. There is also a well defined peak at the angle corresponding to  $\lambda = .1842 \times 10^{-8}$  cm. (the  $\beta$  line in the *K* series), and a slight elevation close to the absorption drop (a  $\gamma$  line in the *K* series).

In the neighborhood of the peaks readings were taken  $15''$  of arc apart; and, since the double grazing angle is about  $4^\circ$ , an error of  $15''$  in estimating the point of a peak would produce an error of 1/10 per cent. in the value of  $\lambda$ . If the same error were made in corresponding peaks on the two sides, an error 1/5 per cent. would result.

The curves representing experiments with a molybdenum target and a tungsten absorbing screen show marked breaks at  $\lambda = .1785 \times 10^{-8}$  cm.,  $\lambda = 1.230 \times 10^{-8}$  cm. (strong) and  $\lambda = 1.08 \times 10^{-8}$  cm. (medium) and a less well defined break at  $\lambda = 1.025 \times 10^{-8}$  cm. The first is the characteristic *K* absorption wave-length, and the other three are the characteristic *L* absorption wave-lengths of tungsten.

The following table contains the frequencies of vibration calculated from the wave-lengths obtained from the curves.

*Emission and Absorption Frequencies of Tungsten  $\times 10^{-19}$ . (Probable error  $\pm 0.003$ .)*

*K Absorption Frequencies.*

First order tungsten target.....	1.680
First order molybdenum target.....	1.679
Second order molybdenum target.....	1.681
Average $\nu$ .....	= 1.680

*L Absorption Frequencies.*Strong  $\nu_1 = .2438$ Medium  $\nu_2 = .2773$ Weak  $\nu_3 = .2922$ *K Emission Frequencies.*

	$\beta$ Medium	$\alpha_1$ Strong	$\alpha_2$ Medium	$\alpha_3$ Weak
1st order.....	1.628	1.435	1.405	1.40?
2d order.....		1.437	1.405	1.40?
Average.....		1.436	1.405	1.40?

*Relation between K Series and L Series.*

Absorption	Emission
$\nu - \nu_1 = 1.436$	$\nu_{\alpha_1} = 1.436$
$\nu - \nu_2 = 1.403$	$\nu_{\alpha_2} = 1.405$
$\nu - \nu_3 = 1.388$	$\nu_{\alpha_3} = 1.40?$

It appears from the last five lines in the table that the difference between the *K* absorption frequency and one of the *L* absorption frequencies equals one of the  $\alpha$  emission frequencies in the *K* series to within the limits of error of the measurements. Further, if we subtract the strong *L* absorption frequency  $\nu_1$  from the *K* absorption frequency  $\nu$  we get the frequency of the strong  $\alpha_1$  line, if we subtract the medium *L* absorption frequency  $\nu_2$  from  $\nu$  we get the frequency of the medium *K* emission line  $\alpha_2$  and, if we use the weak *L* absorption frequency we get the third  $\alpha$  line in the *K* series,  $\alpha_3$ .

The curve representing experiments with a tungsten target shows about 12 lines in the *L* series between  $\lambda = 1.2 \times 10^{-8}$  cm. and  $\lambda = 1.50 \times 10^{-8}$  cm. Six of these lines can be accounted for by an extension of the above law, that an emission frequency is the difference between two absorption frequencies, if we assume the existence of two *M* absorption wave-lengths between  $\lambda = 6 \times 10^{-8}$  cm. and  $\lambda = 7.5 \times 10^{-8}$  cm. Such *M* absorption wave-lengths would account, also, for the presence of the  $\beta$  line in the *K* emission series.

If the above law proves to be of general applicability, characteristic absorption frequencies similar to those of X-rays should be sought for in other parts of the spectrum (the visible and ultra-violet, etc.). Characteristic X-ray absorption frequencies are quite different from the absorption lines that obey Kirchhoff's law.

It is unnecessary to lay further stress upon the importance of this absorption-emission relation in speculations on the structure of matter and on the mechanism of the absorption and emission of radiation.

TEMPERATURE COEFFICIENT OF RESISTENCE OF MOLYBDENUM.<sup>1</sup>

By E. C. BLOM.

THESE experiments were conducted to determine the temperature coefficient of resistance of molybdenum over a greater range of temperature than had previously been recorded.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

Some 10 mil (0.254 mm.) drawn molybdenum wire was made into lamps. The filament length of one group of these was 150 mm. while that of the other was 120 mm. With everything else the same, this gave a filament length difference of 30 mm., which was used for all resistance calculations.

For the temperature from 0° C. to 30° C. the lamps were immersed in a water-bath until constant conditions were reached. The resistance was then measured with a Wheatstone Bridge. The temperature was measured with a calibrated thermometer. Over the range 800° C. to 2000° C. the temperature

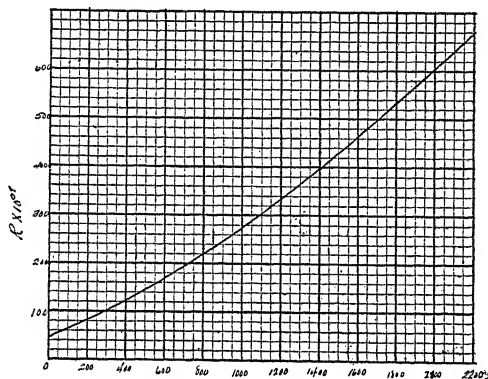


Fig. 1.

was measured with both brightness and color match optical pyrometers. Direct current was used to light the lamps and the resistance was determined by measuring volts and amperes with laboratory standard instruments. For the interval 30° C. to 300° C., the bulbs were removed, and the filaments immersed in a paraffine bath where again the temperature was measured with a calibrated thermometer and the resistance measured with the bridge. The interval 300° C. to 800° C. was not covered but the results up to 300° C. and beyond 800° C. ran so evenly that it seems plausible to assume that the curve which fits those temperatures already covered will fit the others also.

The accompanying curve gives a summary of the results obtained. The following empirical equation was found to yield the correct values:

$$R \cdot 10^7 = 44 + 0.177t + 0.000053t^2,$$

where  $t$  is in °C and  $R$  in ohms per cm.<sup>3</sup>

PHYSICAL LABORATORY,  
WESTINGHOUSE LAMP CO.,  
BLOOMFIELD, N. J.,  
February 11, 1919.

## SOME PHOTOELECTRIC LECTURE EXPERIMENTS

BY JACOB KUNZ.

IN a short article on the photoelectric relay in the *Electrical World*, Vol. 66, p. 934, October, 1915, the writer has indicated some simple experiments with the photoelectric cell and the three electrode photoelectric relay. Three more experiments will here be indicated by which some properties of the photoelectric cell can be demonstrated.

The tube *A* represents a photoelectric cell containing potassium sensitized with hydrogen and argon. The argon is used at such a pressure that the

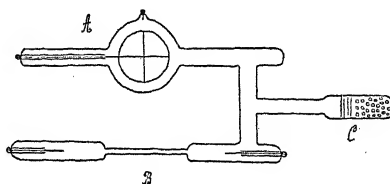


Fig. 1.

maximum sensitiveness of the cell for 130 volts, for instance, is surpassed, *B* represents a spectroscopic tube and *C* a bulb containing charcoal. Now for a given tube the photoelectric current depends essentially on three independent variables: the intensity of light, the potential difference, and the gas pressure. Let us start with a certain light intensity, for instance, with an incandescent lamp of 16 candlepower at a distance of two meters, with a certain pressure and a potential difference of 80 volts. We shall find in the galvanometer a certain deflection which will increase with increasing potential difference, first slowly, later very rapidly until the gas begins to glow. This point should not be reached with the ordinary cells, because the sensitiveness of potassium cells as a rule decreases when the glow occurs.

In a second experiment let us keep the potential difference, say 110 volts, and the gas pressure constant, and increase the intensity of light. The current is proportional to the illumination. Many photoelectric cells have been constructed in which this proportionality does not hold. The current may either increase more slowly or more quickly than the illumination. If the current approaches a saturation value with increasing illumination, it is apparently due to volume charges; and if it increases more rapidly than the illumination it is due to the fact that the gas approaches the glowing stage, which depends on the pressure, potential difference and illumination, like the photoelectric current itself.

In a third experiment let us keep the illumination and the potential difference constant and vary the pressure by putting the charcoal into liquid air. As the argon is absorbed the photoelectric current will at first increase, reach a maximum, then decrease and become very small when all the argon is absorbed.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

The absorption of the argon can be seen in the changes of the light in the tube *B*. When the liquid air is withdrawn, argon is emitted, the photoelectric current passes through the same maximum as before and falls off to the initial small value (Stoletow's law).

The point at which the gas of the photoelectric cell becomes luminous depends on the potential difference, the intensity of illumination and the gas pressure. If for a certain pressure and potential difference we approach the source of light, suddenly the gas begins to glow, and of course continues to glow when the light is withdrawn. This beautiful experiment is interesting insofar as it shows that an increase of the number of electrons emitted from the metal leads to such an increase in ionization that luminous discharge occurs in which the current rises enormously over the value attained without glow. The photoelectric cell becomes a fairly good source of cold light. In such experiments the sensitive layer changes considerably and it is better to use two different tubes for this set of experiments and for the previous set, which does not lead to glow discharge.

If we place a rotating sector disc between the source of light and the photoelectric current, amplify the current by means of an audion as shown by E. Pike in a recent article in the *PHYSICAL REVIEW*, and use a telephone instead of a galvanometer in the secondary of the audion, then we can hear the source of light, even a very weak source, quite distinctly. This principle may be used in an instrument by which a blind man may hear the letters of a printed page.

UNIVERSITY OF ILLINOIS,  
URBANA, ILLINOIS.  
January 11, 1919.

#### ON THE MEASUREMENT OF THE DETECTION COEFFICIENT OF THERMIONIC VACUUM TUBES.<sup>1</sup>

BY H. J. VAN DER BIJL.

THE relation between current and voltage in thermionic tubes of the audion type, can be represented by

$$(1) \quad I = f \left( \frac{1}{\mu} \sum E_B + \sum E_c \right),$$

where  $\sum E_B$  and  $\sum E_c$  are the potentials of the anode and grid with respect to the filament. The function  $f$  is non-linear and hence the device can be used as a radio detector. If, for example, a varying e.m.f.  $e$ , be impressed between filament and grid, the varying current  $i$  in the circuit connecting anode and filament can be represented by a simple convergent power series:

$$(2) \quad i = a_1 e + a_2 e^2 + a_3 e^3 + \dots$$

In order to properly design tubes it is necessary to know the relation between

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

the coefficients  $a_1, a_2 \dots$  and the structural parameters of the tube. This requires a knowledge of the form of the function  $f$ . When using the tube as an amplifier  $f$  can be given a simple quadratic form.<sup>1</sup> This is, however, not sufficiently accurate for purposes of radio detection which is determined by the second derivative of the characteristic, *i. e.*, by the coefficient  $a_2$  of the above series. The problem we want to consider here is the direct determination of  $a_2$ , which may be called the detection coefficient. The difficulty of this deter-

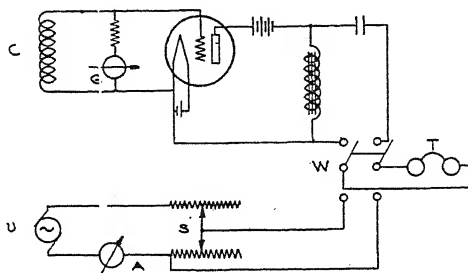


Fig. 1.

mination lies in the necessity for measuring extremely small alternating currents,—ranging from  $10^{-8}$  to  $10^{-6}$  amp. The use of hot wire meters is therefore entirely out of the question. Hence the method shown in the accompanying diagram was used.

The input voltage impressed on the tube at  $C$  ranged from a few hundredths to a few tenths of a volt, and was measured with the Duddell galvanometer  $G$ . It was of the form

$$(3) \quad e = A \sin pt (1 + \sin qt)$$

*i. e.*, it was a voltage of high frequency  $p/2\pi$  completely modulated by a voltage of frequency  $q/2\pi$  lying within the audible range. If (3) be inserted in the second term of (2) the resulting current will be seen to have an audible component of frequency  $q/2\pi$  which can be heard in the telephone receiver  $T$ . In order to measure this current the generator  $U$  is used to give a current of the same frequency as that in the receiver and which is strong enough to be measured with a thermo-couple and microammeter  $A$ . In fact, the same generator supplied the current used to modulate the high frequency input. This current is then attenuated by means of the shunt  $S$  until the intensity of the note in the receiver is the same for both positions of the switch  $W$ . Knowing the value of the shunt  $S$  the current  $i$  in the receiver can be computed from  $i_1$  as measured by  $A$ . This shunt is so designed that for all its adjustments the impedance into which the generator  $U$  works is constant, so that  $i_1$  remains constant.

For convenience in representing the results we can put

<sup>1</sup> H. J. Van der Bijl, *Phys. Rev.*, 12, p. 171, 1918.

$$(4) \quad d = \log \frac{i_1}{i}, \quad \circ$$

where  $d$  can be previously determined for any combination of shunt and series resistances of the shunt  $S$ . Since the audible current  $i$  in the output of the detector is given by

$$(5) \quad i = a_2 e^2$$

we get

$$(6) \quad d = -2 \log e + (\log i_1 - \log a_2).$$

The accuracy with which this relation holds is shown in the diagram, the circles and crosses indicating measurements made by two different observers.

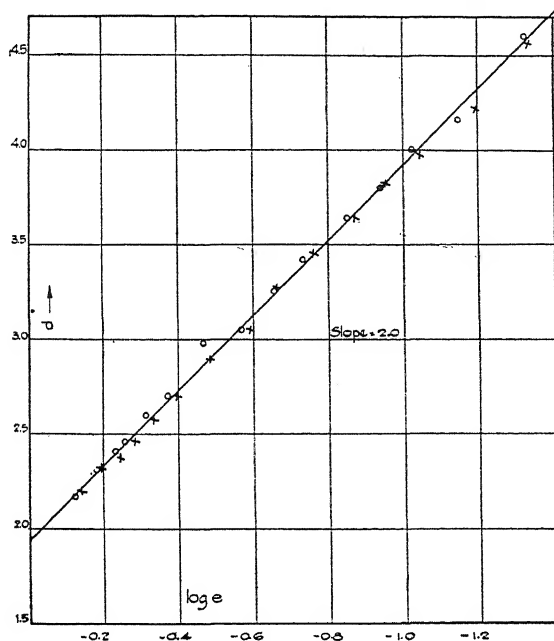


Fig. 2.

Since  $i_1$  is large enough to be measured with the thermo-couple and microammeter  $A$  the detection coefficient  $a_2$  can be obtained from the intercept of the line.

The detector used in these experiments was a standard Western Electric tube of the type that was designed specially for aëroplane radio telephone service. Its detecting efficiency had not changed to any noticeable extent over a period of more than seven months, during which time it was in frequent use. In these experiments the operating voltages were not adjusted to give the best efficiency.

From these results the detection coefficient was found to be  $36 \times 10^{-6}$  amp./ $(\text{volt})^2$ , the resistance of the receiver being 6,400 ohms. This corresponds to a power dissipation in the receiver of  $8.3 \times 10^{-6}$  watt/ $(\text{volt})^4$ . An idea of this

amount of power in the receiver can be obtained by noting that the minimum power dissipation necessary in this receiver to give a sound that just barely enables one to distinguish signals is about  $3 \times 10^{-12}$  watt.

RESEARCH LABORATORY OF THE WESTERN ELECTRIC CO., INC.,  
NEW YORK CITY.

February, 1919.

# A NEW FORMULA FOR THE SPECTRAL DISTRIBUTION OF ENERGY FROM A COMPLETE RADIATOR.<sup>1</sup>

BY IRWIN G. PRIEST.

THE radiant energy from a complete radiator ("black body") may be expressed as a function of temperature and wavelength by the following empiric formula which does not appear to have been noticed heretofore:

$$E_{\lambda} = D_1 T^5 e^{-D_2 [A^{-1/3} - (\lambda T)^{-1/3}]^2} \quad (1)$$

where

$E_{\lambda}$   $\equiv$  energy of wave-length  $\lambda$  in the same sense as in the Planck equation.

$D_1$  and  $D_2$  are empiric constants.

$T$   $\equiv$  absolute temperature in degrees Kelvin.

$A$   $\equiv$  Wien displacement constant in micron degrees. ( $A \equiv \lambda_m T$ , where  $\lambda_m$   $\equiv$  wave-length of maximum energy.)

$\lambda$   $\equiv$  wave-length in microns.

From equation 1, we have for relative energy at any temperature

$$E_{\lambda r} \equiv \frac{E_{\lambda}}{E_m} = e^{-D_2 [A^{-1/3} - (\lambda T)^{-1/3}]^2} \quad (2)$$

where  $E_m$   $\equiv$  maximum energy, of wave-length  $\lambda_m$ .

This formula was originally obtained as an empiric approximation to the Planck formula by graphic transformations of the latter. It was later found, for  $D_2 = 4481$  and  $A = 2940$ , that this formula represents some of the best of Coblentz' experimental distribution data *better* than the Planck formula can represent them for any value of  $A$  and the corresponding value of  $C_2$  ( $C_2 = 4.965A$ ). This conclusion may be verified by reference to the accompanying table.

The proposed equation includes all of the following previously recognized radiation laws, each of which may be derived from it:

1. The Wien displacement law.

The special case of Wien's displacement law ( $\lambda_{\max} T = A$ ) may be inferred from inspection of the equation (1 or 2) for condition of maximum  $E_{\lambda}$ , viz:  $A^{-1/3} - (\lambda T)^{-1/3} = \text{zero}$ .

Further, the following equation may be derived from equation 2:

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.



Priest: A New Formula for Spectral Distribution, etc.

Values of $E_m$ as a Function of $\lambda T$						Residuals					
$\lambda T$	Computed by Formulas					Exp. Data <sup>*</sup> Coblentz B.S. Sci. Pap. 284, Fig. 2 T=15965 K	Planck for A=2890 minus Exp. Data	Proposed Equation for A=2890 minus Exp. Data	Planck for A=2940 minus Exp. Data	Proposed Equation for A=2940 minus Exp. Data	
	For A=2890			For A=2940							
	Wien $C_2=14450$	Planck $C_2=14350$	Proposed Equation <sup>*</sup> Priest, Dec. 1908	Planck Equation <sup>*</sup> $C_2=14597$	Proposed Equation <sup>*</sup> Priest, Dec. 1908						
Column No.	(I)	(II)	(III)	(IV)	(V)	(VI)	(II)-(VI)	(III)-(VI)	(IV)-(VI)	(V)-(VI)	
500	$0.27 \times 10^{-6}$	$0.32 \times 10^{-6}$	$0.87 \times 10^{-6}$	Note $\rightarrow$	$0.72 \times 10^{-6}$						
800	$.13 \times 10^{-2}$	$.14 \times 10^{-2}$	$.18 \times 10^{-2}$		$.16 \times 10^{-2}$						
942		$.94 \times 10^{-2}$	$.11 \times 10^{-1}$	$0.78 \times 10^{-2}$	$0.9 \times 10^{-2}$	$0.15 \times 10^{-1}$	-0.006	-0.004	-0.007	-0.005	
1000	$.16 \times 10^{-1}$	$.17 \times 10^{-1}$	$.19 \times 10^{-1}$								
1102		$.39 \times 10^{-1}$	$.42 \times 10^{-1}$	$.34 \times 10^{-1}$	$.38 \times 10^{-1}$	$.35 \times 10^{-1}$	+0.004	+0.007	-0.001	+0.003	
1200		$.80 \times 10^{-1}$	$.82 \times 10^{-1}$	$.71 \times 10^{-1}$	$.77 \times 10^{-1}$	$.58 \times 10^{-1}$	+0.022	+0.024	+0.013	+0.019	
1300		$.39$	$.42$	$.1254$	$.1330$	$.110$	+0.029	+0.032	+0.015	+0.023	
1400		$.2525$	$.2559$	$.2334$	$.2406$	$.233$	+0.019	+0.023	+0.000	+0.008	
1500	2.582	.2646	.2678								
1600		$.4151$	$.4166$	$.3904$	$.3973$	$.395$	+0.020	+0.022	-0.005	+0.002	
1700		$.5135$	$.5150$	$.4877$	$.4929$	$.495$	+0.019	+0.020	-0.002	+0.002	
1800		$.6079$	$.6096$	$.5827$	$.5863$	$.581$	+0.027	+0.029	+0.002	+0.005	
2000	6.810	.6871	.6859								
2200		$.7183$	$.7179$	$.6959$	$.6963$	$.700$	+0.018	+0.018	-0.004	-0.004	
2400		$.7924$	$.7926$	$.7700$	$.7722$	$.797$	-0.005	-0.004	-0.027	-0.025	
2600		$.9238$	$.9240$	$.9111$	$.9094$	$.929$	-0.005	-0.005	-0.018	-0.020	
2714											
2800		$.9903$	$.9900$	$.9858$	$.9846$	$.994$	-0.004	-0.004	-0.008	-0.009	
3000		$.9964$	$.9966$	$1.000$	$.9995$	$.998$	-0.001	-0.000	+0.002	+0.002	
3200		$.9665$	$.9675$	$.9769$	$.9753$	$.963$	+0.003	+0.005	+0.014	+0.012	
3512		$.9180$	$.9167$	$.9322$	$.9296$	$.940$	-0.022	-0.023	-0.008	-0.010	
3600		$.8624$	$.8600$	$.8788$	$.8758$	$.884$	-0.019	-0.021	-0.003	-0.006	
3800	7.887	$.8332$	$.8332$	$.8553$	$.8509$		-0.027	-0.031	-0.009	-0.013	
4000		$.7967$	$.7933$								
4071		$.7798$	$.7730$	$.7976$	$.7929$	$.771$	+0.008	+0.002	+0.027	+0.022	
4200		$.7535$	$.7508$	$.7717$	$.7659$	$.751$	-0.002	-0.016	+0.015	+0.003	
4400	5.323	$.7390$	$.7366$								
4600	3.465	$.7312$	$.7356$								
6003		$.3712$	$.3570$	$.3868$	$.3751$	$.374$	-0.003	-0.017	+0.013	+0.001	
6434		$.3113$	$.2979$	$.3274$	$.3146$	$.313$	+0.000	-0.015	+0.014	+0.002	
7089		$.2439$	$.2288$	$.2558$	$.2437$	$.244$	-0.000	-0.015	+0.012	-0.000	
7408		$.2165$	$.2021$	$.2275$	$.2155$	$.218$	-0.002	-0.016	+0.010	-0.002	
7695		$.1949$	$.1807$	$.2047$	$.1926$	$.197$	-0.002	-0.016	+0.008	-0.004	
7800		$.1786$	$.1615$	$.1847$	$.1731$	$.179$	-0.003	-0.017	+0.006	-0.006	
8000		$.1746$	$.1607$								
8238	1.501	$.1606$	$.1466$	$.1690$	$.1574$	$.160$	+0.001	-0.013	+0.009	-0.003	
8493		$.1469$	$.1340$	$.1546$	$.1437$	$.147$	-0.000	-0.013	+0.008	-0.003	
8765		$.1340$	$.1212$	$.1412$	$.1305$	$.132$	+0.002	-0.011	+0.009	-0.002	
9930		$.92 \times 10^{-1}$	$.81 \times 10^{-1}$	$.97 \times 10^{-1}$	$.88 \times 10^{-1}$	$.86 \times 10^{-1}$	+0.006	-0.005	+0.011	+0.002	
10000	$.71 \times 10^{-1}$	$.90 \times 10^{-1}$	$.80 \times 10^{-1}$								
20000	$.45 \times 10^{-2}$	$.86 \times 10^{-2}$	$.69 \times 10^{-2}$	Note Agreement $\leftarrow$ Col. s. II, III, V, cf. Low values in I.	$.77 \times 10^{-2}$						
30000	$.76 \times 10^{-3}$	$.19 \times 10^{-2}$	$.15 \times 10^{-2}$								
40000	$.20 \times 10^{-3}$	$.65 \times 10^{-3}$	$.54 \times 10^{-3}$			$.63 \times 10^{-3}$					
50000	$.72 \times 10^{-4}$	$.28 \times 10^{-3}$	$.25 \times 10^{-3}$								
60000	$.30 \times 10^{-4}$	$.14 \times 10^{-3}$	$.13 \times 10^{-3}$		$.15 \times 10^{-3}$						
70000	$.15 \times 10^{-4}$	$.75 \times 10^{-4}$	$.74 \times 10^{-4}$								
80000	$.76 \times 10^{-5}$	$.45 \times 10^{-4}$	$.51 \times 10^{-4}$		$.60 \times 10^{-4}$						
90000	$.43 \times 10^{-5}$	$.28 \times 10^{-4}$	$.32 \times 10^{-4}$								
100000	$.26 \times 10^{-5}$	$.19 \times 10^{-4}$	$.25 \times 10^{-4}$		$.29 \times 10^{-4}$						
Notes to Column Headings III, V, VI.											
*Proposed Equation $E_m = e^{-4481/A^2(\lambda T)^2}$											
*Omitting points affected by atmospheric absorption.											
Means											
for Arith.						+0.015	+0.016	+0.009	+0.010		
for $\lambda < \lambda_m$ Algeb.						+0.012	+0.013	-0.004	-0.0004		
for Arith.						+0.006	+0.014	+0.011	+0.005		
for $\lambda > \lambda_m$ Algeb.						-0.004	-0.013	+0.008	-0.0003		
for all values of $\lambda$						+0.010	+0.015	+0.010	+0.0075		
						+0.003	-0.002	+0.003	-0.0003		

Fig. 1.

$$\lambda T = \left[ \frac{1}{A^{-\frac{1}{3}} \pm \sqrt{\frac{\log_e E_{\lambda r}}{-D_2}}} \right]^3 \quad (3)$$

which is in accord with the general case of Wien's law.

2. The law that the maximum energy is proportional to the fifth power of the absolute temperature.

This may be seen by inspection of the equation for the condition

$$\lambda = \lambda_m = \frac{A}{T}.$$

3. The empiric Stefan law that total energy (integral with respect to wave-length) is proportional to the fourth power of absolute temperature.

This follows from the fact that the definite integral between any wave-length limits for equal values of  $E_{\lambda r} > 0.33 \times 10^{-9}$  is proportional to the fourth power of temperature.

From the experimental point of view, therefore, the proposed equation accords with the fourth power law just as well as the Planck equation does.

However, the theoretical Stefan-Boltzmann law that the definite integral with respect to wave-length between *zero* and *infinity* is proportional to the fourth power of absolute temperature can *not* be derived from this equation, because  $E_{\lambda}/E_m$  has a finite although very small value ( $0.33 \times 10^{-9}$ ) for  $\lambda T = \text{infinity}$ .

Another interesting property of the proposed equation is the fact that it has precisely the form of the well known equation of the "probability curve." If  $E_{\lambda r}$  (equation 2) be plotted as ordinate against  $\sqrt[3]{I/\lambda}$  or  $\sqrt[3]{300/\lambda}$  (cube root of frequency) as abscissa, the resulting curve is strictly symmetrical about the maximum ordinate. It is possible that this is the mathematical essence of what may be sometime recognized as a theoretical law of radiation.

If the general validity of this equation as representative of experimental data be admitted, the law of symmetry just mentioned provides a very simple method of determining the wave-length of maximum energy from an isothermal curve.

Graphically, the intersection of the axis of symmetry with the abscissa scale gives  $\sqrt[3]{I/\lambda_m}$  from which  $\lambda_m$  may be found directly. Or, by algebraic solution,

$$\lambda_m = \frac{8}{\left( \sqrt[3]{\frac{I}{\lambda_1}} + \sqrt[3]{\frac{I}{\lambda_2}} \right)^3}$$

where  $\lambda_1$  and  $\lambda_2$  are any wave-lengths for equal relative energy.

The departures of the proposed formula from the corresponding formulas of Wien and Planck for various values of  $\lambda T$  may be seen in the accompanying table. It will be noticed:

1. Throughout the region best known by experimental investigation, it does not depart much from the Planck formula, although it approximates closer to the experimental data.

2. The curve of the proposed equation intersects the Planck curve at a point in the "Reststrahlung" region ( $\lambda T =$  about 60,000).

For numerical evaluation this formula is more convenient than the Planck formula.

The author has no theoretical basis for proposing this equation; but recalling that the Planck equation was first an *empiric* relation to which theory was later *forced* to conform, he has thought that it might be worthy of notice and perhaps of some theoretical consideration by others.

P.S. April 18, 1919.—A second abstract giving a revised value of the constant  $D^2$  on the basis of isochromatic data, will appear in the Proceedings of the Physical Society, Washington Meeting, April 25-26, 1919.

NATIONAL BUREAU OF STANDARDS,  
February 25, 1919.

# ON THE MEASUREMENT OF THE TRUE BAROMETRIC PRESSURE IN A RAPIDLY MOVING CURRENT OF AIR.<sup>1</sup>

By J. G. COFFIN.

THE accuracy of the measurement of barometric pressure in rapidly moving airplanes is open to serious question.

If a body of any shape is moving rapidly through the air, there are regions on the surface of it which receive an aërodynamic pressure greater than the normal static or barometric pressure and there are also regions under smaller pressure than normal.

For a sphere, for example, the pressure variation is shown in Fig. 1, where radial distances outside the circle represent pressures in excess of normal and radial distances inside represent pressures less than normal. The radius, therefore, represents true barometric pressure, but not necessarily to scale.

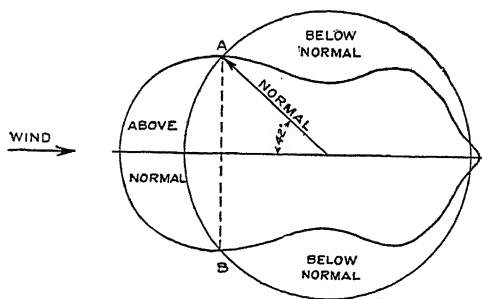


Fig 1

It is seen that there is an excess pressure on the nose where the air strikes the sphere directly and that at points on a circle of which  $AB$  is the trace there exists exactly normal pressure. At points aft of  $AB$  the pressure is below

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

normal and on the sides reaches values as much or more under normal than the values over normal at the nose. Any form of barometric instrument must necessarily be open to the air whose pressure it is to measure, hence the position of the opening or openings is of importance.

For example, at 60 miles per hour the total possible error may be as much as corresponds to an altitude of 240 feet and at 120 miles per hour, a usual speed at the present time, it can be nearly 1,000 feet!

This difficulty cannot be met by placing the barometer in any other receptacle or in particular, inside the fusilage; as the outside container must also be open to the air and as before the instrument will measure the total pressure obtaining at this opening.

In order to overcome the difficulty Dr. Zahm proposed, some years ago, to surround the barometer with a streamline body with a small opening at a point corresponding to a point on the circle *AB* on the sphere; that is at the point of zero aërodynamic pressure.

As is seen from Fig. 1, the rate change of aërodynamic pressure due to slight displacements along the surface is a maximum and any slight change in the air flow would cause large variations in the registered pressure. There could also be a large constant error due to initial uncertainty in the knowledge of the actual direction of air flow where the body is attached.

It occurred to the writer that these objections could be overcome by placing the barograph inside of a small sphere which is fully exposed to the airstream and connecting its hermetically tight case to a small opening in the sphere.<sup>1</sup> If this sphere be now rotated by clockwork or other means at a uniform angular speed the opening will be carried around in a horizontal great circle and the

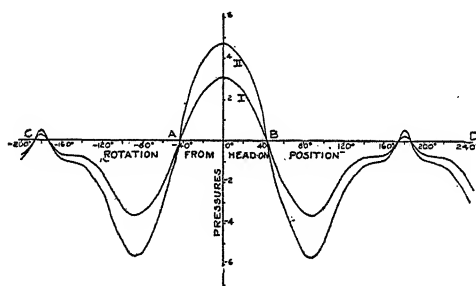


Fig. II

barograph will register all the pressures, of course, passing through normal pressure two (or more) times per revolution.

The trace registered will be similar to Fig. 2.

From experiments made in the Curtiss Wind Tunnels at Garden City it is found that the zero points *A* and *B* remain fixed at all speeds experimented with. On the other hand, the change in speed produces an enlargement of the

<sup>1</sup> The barograph can, of course, also be connected to this opening by means of an air tight tube and be placed in any convenient location for observation.

pressure scale and the pressures follow exactly the same law as if registered with a Pitot tube of standard design.

It is a simple matter to calibrate such an instrument and draw the line *CD*, representing true barometric pressure, so that this troublesome problem is completely solved.

As a by-product of this idea we obtain the following advantages: If a barograph of this type is placed on an airplane and the trace examined on its return from a flight, this trace will show the true barometric pressure at every instant during the flight, the amplitude of the variations in the trace, will show at every instant the speed of the airplane and as equal periods are marked off on the graph by the rotations of the sphere, the slope of the graph will show the rate of climb at every instant so that this single trace contains a complete record of the performance of the airplane.

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#### THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.<sup>1</sup>

BY R. V. L. HARTLEY.

THE location of a source emitting a pure tone is a binaural process depending upon differences in the intensity and phase of the sounds at the two ears. Considerable experimental work has been done in which the intensities were kept equal and only the phases varied. A good review of this has been given by Stewart.<sup>1</sup>

If the observed location effects are the result of the listener's subconsciously perceiving the phase difference and associating it with past experiences in which he has encountered the same phase difference when the position of the source was known; it should be possible to trace a correlation between these experimental sound images and the positions of sources which would cause the phase differences used in producing the images.

Using a method due to Stokes,<sup>2</sup> Stewart<sup>3</sup> and, more recently, T. C. Fry of the Western Electric Co. have made calculations, from which, on the assumption that the head is a rigid sphere with the ears on a horizontal great circle, it is possible to plot the phase difference, *P*, against the lateral angular displacement of the source, for various distances and frequencies of the source. We have then to compare the direction corresponding to a particular phase difference, as given by the curves, with the direction of the image obtained experimentally with that phase difference.

<sup>1</sup> Abstract of a paper presented at the New York meeting of the American Physical Society, March 1, 1919.

<sup>2</sup> G. W. Stewart, *PHYS. REV.*, June, 1917, p. 502.

<sup>3</sup> Lord Rayleigh, *Theory of Sound*, Ch. 17.

<sup>4</sup> G. W. Stewart, *PHYS. REV.*, 1914, 4, p. 252.

The agreement revealed by this comparison is striking. The curves account for the rotation of the image, with continuously varying  $P$ , that is observed at frequencies below about 600 cycles, including its motion toward the ear, followed by a range of more or less uncertainty, just before  $P$  reaches  $180^\circ$ . They also indicate that the value of  $P$  for maximum lateral displacement increases with the frequency as Bowlker<sup>1</sup> has observed. This value reaches  $180^\circ$  for about 600 cycles. Above that frequency, the curves indicate the appearance of a second image on the opposite side. At first this is present only for values of  $P$  near  $180^\circ$ , but with increasing frequency, it extends to smaller values, finally covering the whole cycle. A third image then appears near  $P = 0$ , and so on for still higher frequencies. This accounts for the generally observed falling off in the definiteness of the image with increasing frequency above about 600 cycles. Bowlker however was able to distinguish as many as three simultaneous images, and his description of them shows an agreement with those deduced from the theoretical curves which is remarkable considering the assumptions involved.

This general correlation constitutes strong evidence in favor of the theory of direct perception of phase in sound location.

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WESTERN ELECTRIC COMPANY, INC.

<sup>1</sup> T. J. Bowlker, *Phil. Mag.*, (6), 15, p. 318, 1908.

# THE PHYSICAL REVIEW.

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## AN OPTICAL LEVER MANOMETER.

By J. E. SHRADER AND H. M. RYDER.

**SYNOPSIS.**—To meet the demand for a gauge which will measure vapor pressures in the range between that which can be measured by the Knudsen Gage (.001 mm.) and the ordinary mercury manometer, a new type of gage has been devised. This consists of a rather large mercury manometer in one arm of which a small optical lever is so adjusted that it measures small changes in level due to differences of pressure. The range of the instrument for accurate work is from .001 mm. to 3 or 4 mm. The instrument can also be used as a differential gage with the same accuracy. In use the changes in pressure can be observed by the deflection of a beam of light and may be recorded by a photographic device. Curves are given from a photographic record of the action of the gage during the release of CO<sub>2</sub> and water vapor which had been frozen out in a trap with liquid air. Attention is called to the rapidity with which this gage responds to changes of pressure.

THE McLeod gage has long served as a means of measuring pressures of several millimeters or mercury down to the order of  $10^{-5}$  mm. There are two fundamental cases, however, to which this gage is not applicable in this range, that is, when vapors are to be measured, and where there is a rapidly changing pressure to be recorded. In the first of these cases the McLeod gage becomes useless, due to the fact that Boyle's Law is not applicable to vapors. In the second case the McLeod gage, in its most favorable form, may be much too slow moving, and has no recording feature.

At the present time, when many pumps are on the market which enable more or less leaky systems to be maintained at low pressures, it has become desirable to determine accurately vapor pressure of many substances at pressures below those usually recorded. One of the writers<sup>1</sup> has reported a method of obtaining low vapor pressures accurately, which in a simplified and refined form is capable of operation down to the lower limits of the McLeod gage, but which is applicable to only a certain class of substances, namely those which can be distilled within range of tem-

<sup>1</sup> Ryder, Journal Franklin Institute, Vol. 186, No. 1, July, 1918.

peratures available in the laboratory for such work, and which is not suitable for rapid determinations. The ability to observe rapidly changing pressures, or to record them, would mean, too, the facilitating of a number of operations in the laboratory or shop.

One of the writers<sup>1</sup> has given a preliminary report of a gage designed to provide for the particular requirements suggested in the foregoing. It is the object of this paper to describe the essential details of this gage, and to give a few examples of its actual performance.

The principle of this gage is shown in Fig. 1. A mercury U-tube manometer is formed in the usual manner, except that the surfaces of the mercury are so arranged as to be of relatively large area. Above one

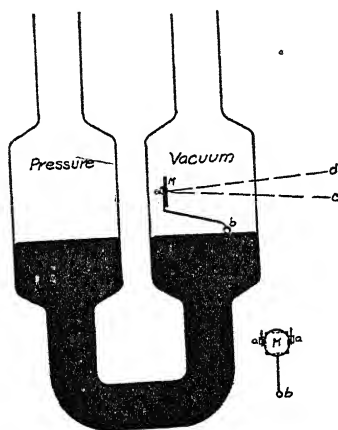


Fig. 1.

Optical Lever Manometer.

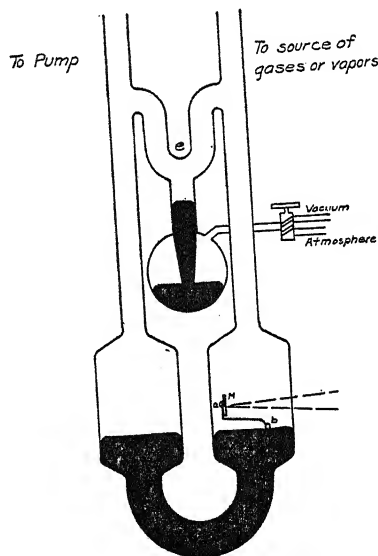


Fig. 2.

System for Manometer.

of the surfaces, within the tube, is arranged an optical lever, as shown in the figure. This lever is supported by two knife edges, *a-a*, which rest on loops of wire, which in turn are sealed into the glass walls of the tube, a glass bead *b*, fused to the end of the lever arm acts as a float on the mercury surface, and in this way transmits the motion of the mercury surface to the lever arm. A mirror *M* attached at the position shown acts in the usual manner to reflect a beam of light from the lamp *c* to the scale *d*, if the gage is to be arranged as an indicating instrument. If the gage is to be used for recording variations in pressure, the scale may be replaced by a photographic device such as is used in oscillographic work.

<sup>1</sup> Shrader, Pittsburgh Meeting, American Physical Society, December, 1917.



Fig. 2 shows a convenient method for connecting this gage in a system. The cross connection *e* provides an easy means of evacuating the whole system with one pump located as shown. With this stop-cock or mercury cutoff open, a zero reading can be easily obtained, after which this connection may be closed and the gases or vapors introduced for measurement. This system provides also for the measurement of small variations in pressure, with an original pressure of any desired value, this value in no way affecting the absolute sensibility of the gage.

The gage as shown is a primary instrument, since the pressure corresponding to any deflection on the scale may be calculated from dimensions which may be readily determined. The lowest pressure which may

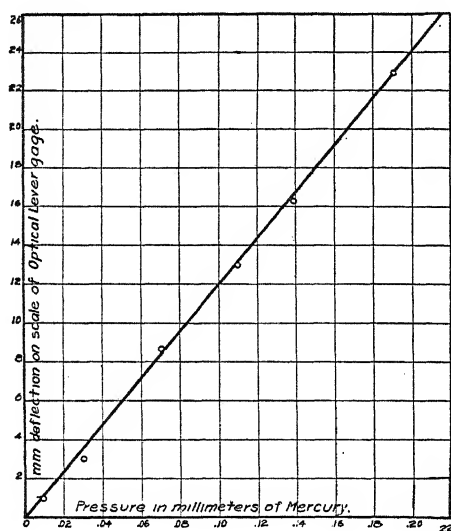


Fig. 3.

Calibration curve for an Optical Lever Gage.

be observed with the gage is that which gives the minimum appreciable movement of the mercury, and has been determined by experience to be approximately one thousandth of a millimeter of mercury in a properly designed gage. The upper limit, is, of course, determined by the dimensions of the gage, and is easily made to overlap the lower accurate readings of an ordinary U-tube manometer. The only source of error which need be considered is that due to the adhesion of the mercury to the walls of the tube, the deflection being therefore less than the actual value. If the mercury is very clean, and the glass in good condition a large mercury surface reduces this error to a negligible quantity. Mercury surfaces of from 5 cm. to 10 cm. diameter are in use. Some gages

have had a window of plane glass at the point of transmission through the tube of the beams of light, to prevent any possibility of a refraction of the beams due to the curvature of the glass.

Fig. 3 shows a check curve plotted between scale readings of this "optical lever" gage as ordinate and the actual pressures, as measured by a McLeod gage (hydrogen being the gas used). It will be seen that with this gage, up to the limit of this curve, .2 mm., a straight line relation exists. Above this point, the line is slightly curved, due to the flat scale used, and to the angular movement of the lever arm. This curve shows, too, the variations from the true value which may be expected from consecutive readings. The relative variation of this curve from its calculated position depends principally on the accuracy of the calculation. A convenient method of obtaining accurately the lever arm ratio consists in mounting the lever in a vessel of known cross sectional area

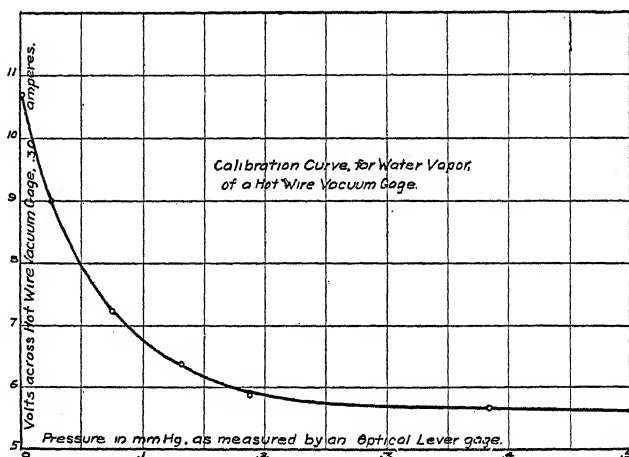


Fig. 4.

Use of Optical Lever Gage as a Primary Instrument.

with the float resting on mercury, in its normal position. When small known amounts of mercury are added, the movement of the beam of light on a properly placed scale may be observed, and the lever arm ratio calculated.

By means of this gage as a primary instrument the use of the hot wire gage<sup>1</sup> may be extended to the measurement of vapor pressures. Fig. 4 is a calibration curve for the hot wire gage for water vapor, the pressure being determined by the "optical lever" gage.

Fig. 5 is a reproduction of a graphic chart obtained by using the

<sup>1</sup> Pirani, Verh. I. Deutsch Phys. Gesell., 24, p. 686, 1906.

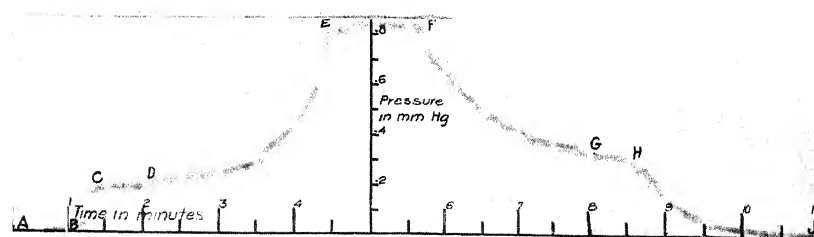


Fig. 5.

J. E. SHRADER AND H. M. RYDER.



"Optical lever gage." The apparatus used was that shown in Fig. 2 with the addition of a tube arranged for immersing in liquid air, connected to the right leg of the U-tube. At the point *A* on the chart, all gases have been removed from the system except some water vapor and  $\text{CO}_2$  which are held frozen in the tube immersed in liquid air. With the cross connection closed, the liquid air is removed. At *B* active vaporizing starts. At *C* the  $\text{CO}_2$  has been released, at *D* water vapor is coming off, and at *E* this vaporization is complete. At *F* solid  $\text{CO}_2$  is applied and the water starts to freeze out again, this freezing out becoming complete at *G*. At *H* the solid  $\text{CO}_2$  is replaced by liquid air and freezing out of the  $\text{CO}_2$  begins. This is completed at *J*, the gage being thus brought back to zero reading. This chart is shown merely to give an idea of the possibilities of the "optical lever" gage in this direction. Many problems of diffusion, gas evolution, etc., immediately present themselves for solution with the aid of this device.

Some idea of the maximum rapidity of pressure variation which the optical lever gage will record may be gained from Fig. 6 which shows a

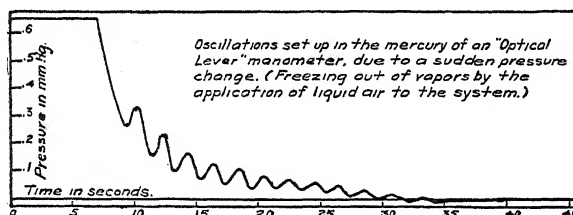


Fig. 6.

record of the freezing out of water vapor by means of liquid air. The oscillations are due to the original sudden pressure drop, and have a period of slightly more than two seconds (approximately 1,000 grams of mercury in the gage). The design of the gage may be such that this period may be materially reduced.

To summarize, a primary vacuum gage, consisting of the optical lever principle applied to a mercury U-tube manometer has been devised for indicating or recording pressures, including pressure variations, of vapors as well as gases, the pressure range being from several millimeters of mercury to approximately one thousandth of a millimeter. All errors of any consequence may be eliminated. By the use of this gage with the addition of a standard photographic device, charts may be obtained showing accurately pressure changes due to such phenomena as vaporization, freezing, diffusion, etc.

WESTINGHOUSE RESEARCH LABORATORY,  
EAST PITTSBURGH, PA.,  
January, 1919.

THE LOGARITHMIC LAW CONNECTING ATOMIC NUMBER  
AND FREQUENCY DIFFERENCES IN SPECTRAL SERIES.

BY GLADYS A. ANSLOW.

SYNOPSIS.—In continuation of a previous note by Anslow and Howell graphic and algebraic relations are derived between the logarithms of the atomic numbers and of the  $\nu$ ,  $\nu_1 + \nu_2$ ,  $\nu_1$  and  $\nu_2$  constant frequency differences occurring in the spectral series of alternate members of the same chemical family. Two systems of four approximately straight lines approaching parallelism resulted when a family possessed both doublet and triplet series. The equations derived are of the form  $\nu = (N/mnk)^A$ , where  $N$  is the atomic number of the element;  $A$  is a constant depending upon the system;  $n$  is an integer, the same for both systems in the same family;  $k$  is a constant, 0.080; and  $m$  takes on successive integral values for the  $\nu$ ,  $\nu_1 + \nu_2$ , and  $\nu_1$  equations. The agreement between the calculated and observed values of frequency differences indicates that the logarithmic lines should curve upwards for elements of high atomic number. The equations derived and the fact that an approximate linear relation seems to exist between the convergence frequencies of members of the same system of series and their atomic numbers do not seem to follow from the Bohr-Sommerfeld theory of atomic structure as so far developed and indicate that some adjustments to the theory are necessary if it is to be extended to predict optical series.

IN an earlier note Mrs. Janet Howell Clark and the author<sup>1</sup> called attention to an extension of the modified law of Runge and Precht<sup>2</sup> concerning the frequency differences of the doublet and triplet spectral series in a given family of the Mendeleeff chemical series. Ives and Stuhlmann<sup>3</sup> had found by a graphical method that a linear relation exists between the logarithms of the atomic numbers of the elements of one family and the logarithms of their doublet separations. Plotting the logarithms of the atomic numbers of the members of the second chemical family against the logarithms of the sum of the constant frequency differences of the triplets in their spectra, we found that linear relations exist between alternate members of the family, the points for magnesium, zinc, cadmium and mercury lying on one line, and those for calcium, strontium, barium and radium on another.

Bell<sup>4</sup> has recently published a treatise on the relation of atomic number to the doublet and triplet separations in the different chemical families. Testing the relation  $\nu = A(N - N_0)^2$ , where  $\nu$  stands for the frequency

<sup>1</sup> Anslow and Howell, Proceedings of the National Academy of Sciences; III., p. 409, 1917.

<sup>2</sup> Runge and Precht, Philosophical Magazine; V., p. 476, 1903.

<sup>3</sup> Ives and Stuhlmann, PHYS. REV., V., p. 368, 1915.

<sup>4</sup> Bell, Philosophical Magazine, XXXVI., p. 337, 1918.

difference,  $N$  for the atomic number, and  $A$  and  $N_0$  are constants, he finds that such an equation approximately predicts the doublet and triplet separations of the different elements if we assume two such

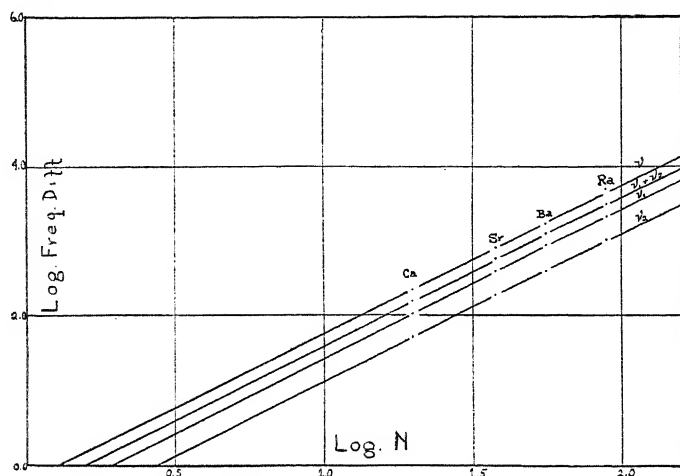


Fig. 1.

equations for each family which branch in the case of the first and second families at potassium and magnesium respectively. He also derives an equation for the logarithmic relation between the same quantities, putting

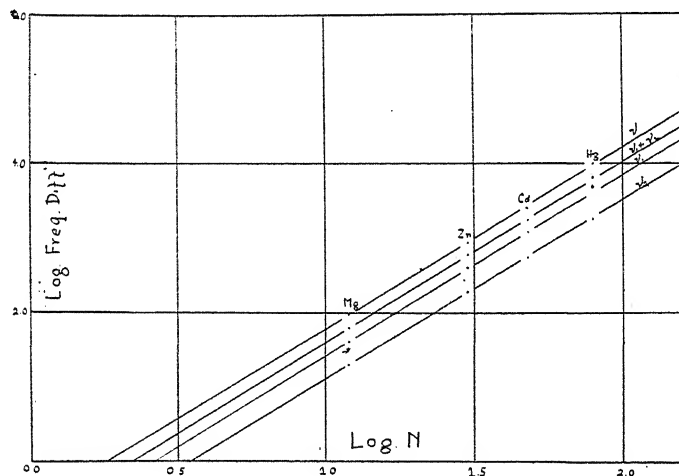


Fig. 2.

the equation in the form,  $\log \nu = p \log N + q$ , where  $p$  and  $q$  are constants, but discards such a relation as not an essential improvement

upon the relation  $\nu = A(N - N_0)^2$ . He also states that "the logarithmic method fails to show graphically the branching relation of the columns."

However, the plotting of the logarithmic equations does show the existence of two equations for each family as is evident from the diagrams in the article by Mrs. Clark and myself, where we attempted to make this fact clear. It happens that I had calculated the equations of the logarithmic lines for the different families where series are known and since the form in which I stated my final results is slightly different from Mr. Bell's, it shows some interesting relationships.

Since the spectral series of the elements in the second chemical family have been the most fully worked out for any family, I will consider it first. I have plotted the logarithms of the doublet separation,  $\nu$ , and of the triplet separations  $\nu_1$ ,  $\nu_2$ , and  $\nu_1 + \nu_2$  against the logarithms of the

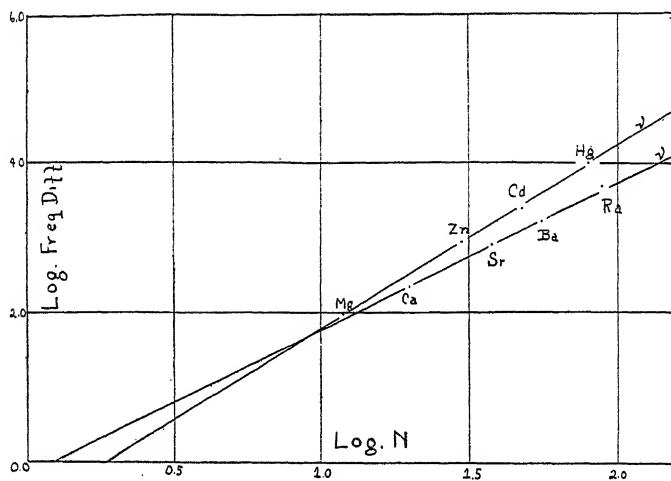


Fig. 3.

atomic numbers of the elements and have obtained lines which pass through Ca, Sr, Ba and Ra (Fig. 1), and through Mg, Zn, Cd and Hg (Fig. 2). The striking feature of the result is that the lines for each system are approximately parallel. If the two systems are plotted on the same diagram, corresponding lines will intersect at points slightly below those for magnesium. This fact is shown for the  $\nu$  lines in Fig. 3. The equations of the lines were first put into the form,

$$\log \nu = A(\log N - C)$$

and this equation then solved into  $\nu = (N/B)^A$ , where  $B = \text{antilog. } C$ . The constants  $A$  and  $B$  which appear in the mean equations of the lines are tabulated below.



The values for the frequency differences are taken, for the sake of comparison, the same as those taken by Bell and given by Dunz.<sup>1</sup> The values for the radium triplets are those given in our earlier paper. It should be noticed that the triplet differences 2050 and 832 found by Hicks<sup>2</sup> fit almost as well into the equations derived. Values marked \* were not used in the calculation of the formulæ.

TABLE I.

Element.	N.	$\nu$ Obs.	$\nu$ Calc.	$\nu$ Calc. - $\nu$ Obs.	$\nu_1 + \nu_2$ Obs.	$\nu_1 + \nu_2$ Calc.	$\nu_1 + \nu_2$ Calc. - $\nu_1 + \nu_2$ Obs.
Ca.....	20	222.9	225.6	+ 2.7	158.10	159.8	+ 1.7
Sr.....	38	801.3	795.8	- 5.5	581.49	578.0	+ 3.5
Ba.....	56	1690.5	1704.6	+ 14.1	1248.7	1255.6	+ 6.9
Ra.....	88	4858.0*	4140.9	-717.1	3052.7*	3103.9	+ 51.2
$A = 1.964$ $B = 1.267 = 4 \times .317$					$A = 2.002$ $B = 1.586 = 5 \times .317$		
Element.	N.	$\nu_1$ Obs.	$\nu_1$ Calc.	$\nu_1$ Calc. - $\nu_1$ Obs.	$\nu_2$ Obs.	$\nu_2$ Calc.	$\nu_2$ Calc. - $\nu_2$ Obs.
Ca.....	20	105.99	105.5	- 0.5	52.11	52.1	+ 0.0
Sr.....	38	394.44	394.7	+ 0.3	187.05	187.0	+ 0.0
Ba.....	56	878.4	875.6	- 2.8	370.3*	408.4	+ 38.1
Ra.....	88	2016.6*	2216.6	+200.0	1036.1*	1005.8	- 30.3
$A = 2.055$ $B = 2.072 = 6 \times .345$					$A = 1.994$ $B = 2.747$		
Element.	N.	$\nu$ Obs.	$\nu$ Calc.	$\nu$ Calc. - $\nu$ Obs.	$\nu_1 + \nu_2$ Obs.	$\nu_1 + \nu_2$ Calc.	$\nu_1 + \nu_2$ Calc. - $\nu_1 + \nu_2$ Obs.
Mg.....	12	92.0	91.4	- 0.6	60.86	61.0	+ 0.1
Zn.....	30	872.4	872.6	+ 0.2	578.79	578.1	- 0.8
Cd.....	48	2484.1*	2776.4	+312.3	1712.91*	1831.7	+118.8
Hg.....	80	9835.06	9766.2	- 68.9	6397.50	6412.2	+ 14.7
$A = 2.462$ $B = 1.917 = 6 \times .319$					$A = 2.454$ $B = 2.247 = 7 \times .321$		
Element.	N.	$\nu_1$ Obs.	$\nu_1$ Calc.	$\nu_1$ Calc. - $\nu_1$ Obs.	$\nu_2$ Obs.	$\nu_2$ Calc.	$\nu_2$ Calc. - $\nu_2$ Obs.
Mg.....	12	40.95	40.9	+ 0.0	19.89	19.9	+ 0.0
Zn.....	30	388.91	388.9	+ 0.0	189.78	189.8	+ 0.0
Cd.....	48	1171.05*	1234.1	+ 63.0	541.86*	604.2	+ 63.3
Hg.....	80	4630.31*	4328.4	-301.9	1767.19*	2127.0	+359.8
$A = 2.457$ $B = 2.648 = 8 \times .331$					$A = 2.462$ $B = 3.560$		

The equations derived show that  $A$  is approximately constant for each system, as is to be expected since the plotted lines approach parallelism. Moreover, the constants  $B$  for the  $\nu$ ,  $\nu_1 + \nu_2$  and  $\nu_1$  lines of

<sup>1</sup> Dunz, *Bearbeitung unserer Kenntnisse von der Serien*, Tübingen, 1911.

<sup>2</sup> Hicks, *Philosophical Transactions, A*, CCXIL., 33.

both systems are successive integral multiples of the same factor, 0.32. The agreement between the observed and calculated values for the separations is fairly good for elements of low atomic number, but poor for those elements which lie beyond the rare earths in the chemical series. The differences between the values found for cadmium seem to indicate that either the series used are not related to those with which they are commonly supposed to be similar to in other elements, or the structure of the atom of this element is not exactly similar to that of the other elements in the same group, so that it acts as if its atomic number were less than that which has been assigned to it. Attention is also called to the fact that much better values could be calculated for the  $\nu_1$  separations of radium and mercury if the values of  $B$  were more nearly equal to  $6 \times .320$  and  $8 \times .320$ , respectively.

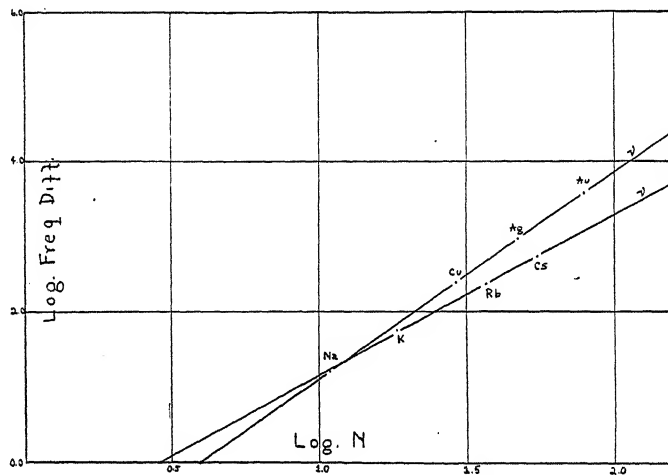


Fig. 4.

Since alternate elements are related in this way, it should be possible to predict frequency differences in elements where they are not yet known. The lines through Ca, Sr, Ba and Ra predict the separations  $\nu = 9.56$ ;  $\nu_1 + \nu_2 = 6.95$ ;  $\nu_1 = 3.91$ ; and  $\nu_2 = 2.17$  for beryllium. The spectrum shows the differences 8.9, 6.8, 4.1 and 2.5 between lines which occur in pairs, but not in triplets. A magnetic study of the lines in question has been made by Popow<sup>1</sup> proving that they belong to doublet and triplet series. Possibly the simplicity of the atom prevents the appearance of the three lines in the spectrum at once, but permits doublets with the separations corresponding to the triplet separations.

The results for Group I are plotted in Fig. 4 and the equations derived

<sup>1</sup> Popow, Verhandlungen der Schweizerischen Naturforschenden Gesellschaft, Sept., 1913.

are shown in Table II. In this group only the doublet series have been discovered with the exception of copper.

TABLE II.

Element.	N.	$\nu$ Obs.	$\nu$ Calc.	$\nu$ Calc. - $\nu$ Obs.	Element.	N.	$\nu$ Obs.	$\nu$ Calc.	$\nu$ Calc. - $\nu$ Obs.
Na.....	11	17.21	17.21	+0.0	Li.....	3	0.34*	1.09	+0.7
Cu.....	29	248.13*	245.4	-2.7	K.....	19	57.90	57.1	-0.8
Ag.....	48	920.56	922.0	+1.4	Rb.....	37	237.71	239.0	+1.3
Au.....	79	3817.20	3826.7	+9.5	Cs.....	55	564.10	559.7	-4.4
$A = 2.741$ $B = 3.895 = 8 \times .486$					$A = 2.147$ $B = 2.887 = 6 \times .481$				

As in Group 2 two values appear for  $A$ , and  $B$  is seen to be an integral multiple of the constant, 0.483. If the behavior of the light emitting electrons of the two families is similar, we should expect the appearance of triplet series in this group, and the equations which should predict the values of  $\nu_1 + \nu_2$  and  $\nu_1$  are for Na, Cu, Ag and Au

$$\nu_1 + \nu_2 = \left( \frac{N}{9 \times .483} \right)^{2.741}$$

and

$$\nu_1 = \left( \frac{N}{10 \times .483} \right)^{2.741};$$

and for Li, K, Rb and Cs

$$\nu_1 + \nu_2 = \left( \frac{N}{7 \times .483} \right)^{2.147}$$

and

$$\nu_1 = \left( \frac{N}{8 \times .483} \right)^{2.147}.$$

Two systems of triplets have been discovered in the spectrum of copper by Rydberg<sup>1</sup> with the separations  $\nu_1 + \nu_2 = 129.50 + 50.58 = 180.08$ ; and  $680.10 + 212.21 = 892.41$ . Assuming that  $A$  should have the same value for these triplets as for the doublets the values derived for  $B$  are

$$B^{\nu_1 + \nu_2}, \quad 4.324 = 9 \times .481; \quad 2.432 = 10 \times .243$$

$$B^{\nu_1}, \quad 4.918 = 10 \times .491; \quad 2.685 = 11 \times .244$$

$$B^{\nu_2}, \quad 6.728 \qquad \qquad \qquad 4.102$$

The agreement between the predicted values for  $B$  and those derived from the triplets with the smaller frequency differences is striking, the other system of triplets being, apparently, a secondary one with  $B$  a

<sup>1</sup> Rydberg, Astrophysical Journal, VI., p. 239, 1897.

multiple of a constant half as large as for the other. These results support the hypothesis that the graphs for the  $\nu$ ,  $\nu_1 + \nu_2$ ,  $\nu_1$ , and  $\nu_2$  separations in Figs. 1 and 2 are parallel straight lines.

If the latter three lines are parallel, the following equations can be stated for the triplets in any element

$$\nu_1 + \nu_2 = \left( \frac{N}{B_{\nu_1 + \nu_2}} \right)^A \quad (1)$$

$$\nu_1 = \left( \frac{N}{B_{\nu_1}} \right)^A \quad (2)$$

$$\nu_2 = \left( \frac{N}{B_{\nu_2}} \right)^A \quad (3)$$

and subtracting (2) from (1)

$$\nu_2 = (N)^A \left[ \left( \frac{1}{B_{\nu_1 + \nu_2}} \right)^A - \left( \frac{1}{B_{\nu_1}} \right)^A \right] \quad (4)$$

which leads to the following relation between the constants,

$$\left[ \left( \frac{1}{B_{\nu_1 + \nu_2}} \right)^A - \left( \frac{1}{B_{\nu_1}} \right)^A \right]^{1/A} = \frac{1}{B_{\nu_2}}. \quad (5)$$

I have tested this result for the triplets considered. The  $\nu_1$  value for  $A$  for Ca, Sr, Ba, and Ra is so different from those for  $\nu_1 + \nu_2$  and  $\nu_1$  that I have calculated second values for  $B$  from the Ca, Sr and Ba  $\nu_1$  separations, and from the Ca and Sr  $\nu_2$  separations, assuming  $A = 2.002$ , which is that derived for the  $\nu_1 + \nu_2$  separations. The results are placed below those taken from Table I.

TABLE III.

System.	$B_{\nu_1 + \nu_2}$ .	$B_{\nu_1}$ .	$[(B_{\nu_1 + \nu_2}^{-1})^A - (B_{\nu_1}^{-1})^A]^{-1/A}$ .	$B_{\nu_2}$ .	$d(B_{\nu_2})$ .
Ca, Sr, Ba, Ra....	1.586	2.054	2.494	2.746	0.252
Ca, Sr, Ba, Ra....		1.922	2.800	2.780	0.020
Mg, Zn, Cd, Hg...	2.247	2.648	3.543	3.560	0.017
Cu.....	4.324	4.918	6.728	6.930	0.202
Cu.....	2.432	2.685	4.102	4.108	0.006

The values in the last column give the differences between the two calculated values of  $B_{\nu_2}$ , and are so small that the hypothesis of parallelism between the series separations is strengthened.

Triplets appear again in Group VI., and have been discovered in the spectra of O, S and Se (Fig. 5). If the alternate members in this group are spectroscopically related those in O should not be related to those in

S and Se, but to the undiscovered triplets in Cr, Mo, W and U. From the separations  $\nu_1 + \nu_2 = 18.15 + 11.13 = 29.16$  in S, and  $\nu_1 + \nu_2 = 103.7 + 44.07 = 148.48$  in Se, the following equation is derived.

$$\nu_1 + \nu_2 = \left( \frac{N}{3.356} \right)^{2.160} = \left( \frac{N}{7 \times .479} \right)^{2.160}.$$

The similarity between this equation and the one predicted for the undiscovered triplets of Li, K, Rb and Cs is apparent. If they are identical

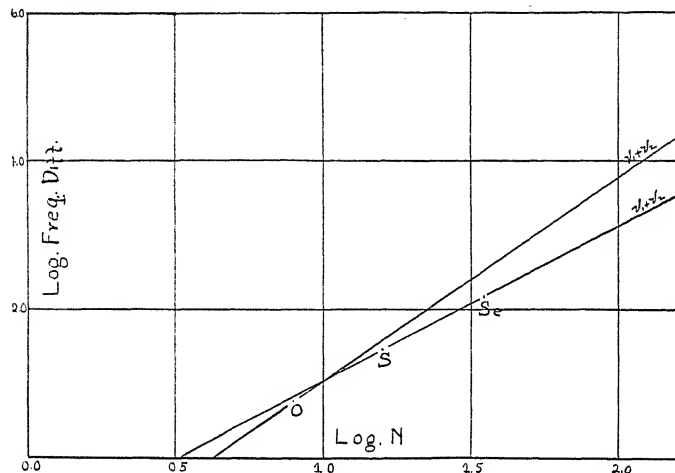


Fig. 5.

$\nu_1$  and  $\nu_2$  should be given by parallel equations with  $B = 8 \times .479$ , and

$$B = \left[ \left\{ \left( \frac{1}{7} \right)^{2.160} - \left( \frac{1}{8} \right)^{2.160} \right\} \left( \frac{1}{.479} \right)^{2.160} \right]^{-1/2.160}.$$

These values give

$$\nu_1 + \nu_2 = 21.12 + 7.27 = 28.39 \text{ for S,}$$

and  $111.63 + 37.04 = 148.67$  for Se. Better agreement would have been obtained if the values for  $B$  predicted for Li, K, Rb and Cs had been used.

The analogy suggests that the triplets in O may be given by the equations derived from the triplets in copper. Using these equations we obtain  $\nu_1 + \nu_2 = 5.43$ ;  $\nu_1 = 3.80$ ; and  $\nu_2 = 1.61$ . The observed differences as given by Runge and Paschen<sup>1</sup> are  $3.70 + 2.08 = 5.78$ .

The only other group where series of the first type have been worked out, with the exception of those in manganese, is in the third, where

<sup>1</sup> Runge and Paschen, *Annalen der Physik*, LXI, p. 641, 1897; *Astrophysical Journal*, VIII., p. 70, 1898.

doublets occur. The graph is shown in Fig. 6. The values of the constants derived from the frequency differences which are known are shown in Table IV.

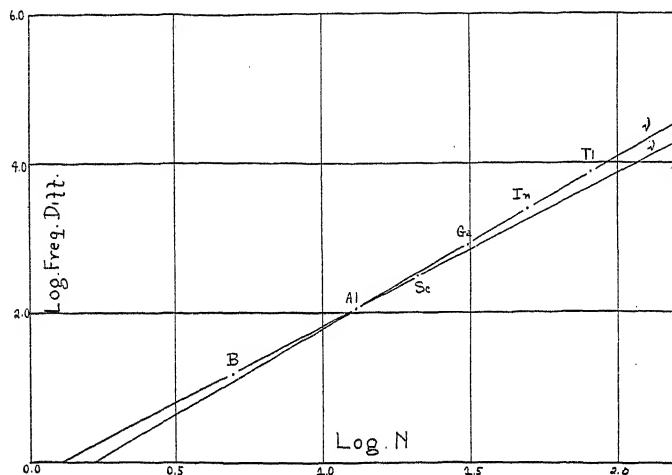


Fig. 6.

TABLE IV.

Element.	N.	$\nu$ Obs.	$\nu$ Calc.	$\nu$ Calc. - $\nu$ Obs.
Al.....	13	112.07	110.9	- 1.2
Ga.....	31	823.6	829.5	+ 5.9
In.....	49	2212.63	2397.1	+184.5
Tl.....	81	7792.45	7753.5	- 39.0
B.....	5	15.4	12.09	- 3.4

$$A = 2.318$$

$$B = 1.706 = 7 \times .246$$

It is seen that the value of the doublet separation in boron lies above the line connecting the points determined by the other elements which are alternate members in the family. If another line should determine the frequency differences for the other elements *B* should lie on it, and it should intersect the known line in the neighborhood of the point determined by Al. Hicks<sup>1</sup> has found a doublet difference for scandium of 320. Combining this value with that known for boron we find that  $A = 2.166$ , and  $B = 1.433 = 6 \times .239$ .

#### DISCUSSION.

The results stated show that the logarithmic method of obtaining the relation between atomic number and frequency differences in spectral

<sup>1</sup> Hicks, Philosophical Transactions, A, CCXIII., p. 408.

series give comparatively good agreement between the observed and calculated values for the constant frequency differences for elements of low atomic number, although the agreement is not within the limit of probable error of the mean of the observed differences, the probable error of the observed value being usually not more than  $0.5 \text{ cm}^{-1}$ . For elements of high atomic weight the observed values are greater than the calculated, as should be expected if the mass of the electron varies with its velocity. The logarithmic line should, therefore, curve upwards for elements of high atomic weight. Attention is also called to the fact that the calculated values are all too high for the elements which just precede the rare earths in the chemical table.

The importance of the logarithmic line does not lie so much in an attempt to obtain good agreement between observed and calculated values as in the connections which are brought out between different types of series in the same and different chemical families. The equations derived show that the values of  $B$  are successive integral multiples of the same constant for the  $\nu$ ,  $\nu_1 + \nu_2$ , and  $\nu_1$  lines in an element. This seems to mean that the frequency differences between the longer wavelength line in triplets and each of the other two are more important than the differences between adjacent lines. The equations give the same value for  $A^1$  for all separations in the same system, and the constants  $B$  seem to be integral multiples of the same constant,  $0.080$ , in all systems. The variation of the value of  $A$  seems to indicate that spectral series are of several types, just as there are several types of X-ray spectra. In general, the types vary with the valency of the element, but it is possible for the same type to occur in elements with different valencies as is done in groups I. and VI.

The form of the equation derived does not seem to be predicted from the equation for the frequency of lines as developed by Sommerfeld<sup>2</sup> for X-rays, from which we should expect an equation of the type of Bell's to be approximately true. Sommerfeld's equation, moreover, indicates that the convergence frequency of a series should be given by

$$n_0 = \left( \frac{N-k}{p} \right)^2 \left[ 1 + \frac{a}{4} \left( \frac{N-k}{p} \right)^2 + \frac{a}{8} \left( \frac{N-k}{p} \right)^4 \dots \right]$$

where  $a$ ,  $k$  and  $p$  are constants,  $a$  being a small quantity. Therefore, the convergence frequencies of series of the same type should be approxi-

<sup>1</sup> An inspection of the values for  $A$  shows common differences of approximately  $0.15$ , the value  $2.60$  being the only absent member in such an arithmetic series. It is a curious coincidence that the frequency differences between the  $\alpha$  and  $\beta$  lines in the  $K$  system of X-ray series may be predicted by the equation  $\nu = (N/0.065)^{2.58}$ .

<sup>2</sup> Sommerfeld, *Annalen der Physik*, LI., 1916.

mately proportional to  $(N - N_0)^2$ . That this is not the case is seen from an inspection of Fig. 7, where the convergence frequencies of the series with constant frequency differences have been plotted. The values used were obtained from Dunz, and may also be found in Kayser's *Handbuch der Spectroscopie*, Vol. II. The graph shows that an approximate linear relation exists between the convergence frequencies and

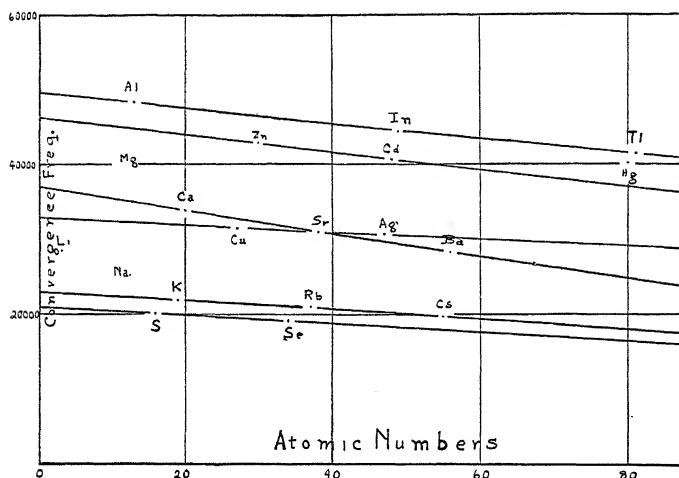


Fig. 7.

atomic numbers of K, Rb and Cs; Ca, Sr and Ba; and Al, In and Tl, which elements I have shown to be related in spectral type. Moreover, a line through the points for S and Se is approximately parallel to that through K, Rb and Cs, a fact which is in support of the hypothesis that the series in these elements is predicted by the same equation.

It would seem that an adequate atomic theory should explain the relation just stated between atomic number and convergence frequency, and the physical reason for the approximate relation

$$\nu = \left( \frac{N}{mnk} \right)^4,$$

where  $\nu$  represents the frequency difference,  $N$  the atomic number,  $A$  is a constant,  $n$  an integer, both of which vary with the system,  $m$  successive integers for the  $\nu$ ,  $\nu_1 + \nu_2$ , and  $\nu_1$  members of the system, and  $k$  a constant for all the systems considered.

SMITH COLLEGE,  
NORTHAMPTON, MASS.,  
February, 1919.



## ON THE CHARACTERISTICS OF ELECTRICALLY OPERATED TUNING FORKS.

BY H. M. DADOURIAN.

SYNOPSIS.—A series of experiments was performed to determine the conditions affecting the period of an electrically operated tuning fork with the following results:

(1) The more massive the base of the fork and the table upon which it is placed the smaller is the period. This effect, however, is not greater than one part in 10,000. (2) A change in the constants of the electrical circuit containing the electromagnet affects the period. The effect is less than one part in 10,000 for moderate changes necessary for keeping the amplitude of vibration constant. (3) The period increases linearly with the increase in the length of the gaps between the contact springs and contact points. For the forks used this increase was about one part in 500 for a change of 0.1 mm. in the length of the gaps. (4) For a given fork there is an amplitude at which the period has a stationary value. This may be a maximum or a minimum depending upon the arrangement of the mounting of the contact springs. At small amplitudes the change in the period due to a variation in the amplitude may be considerable. (5) The temperature effect increases from  $1.04 \times 10^{-4}$  at  $-25^{\circ}$  C. to  $1.43 \times 10^{-4}$  at  $56^{\circ}$  C. The values corresponding to temperatures above  $0^{\circ}$  C. are from 20 to 40 per cent. greater than those obtained by other observers. There are no other published data giving the temperature effect below  $0^{\circ}$  C. (6) By keeping the temperature, the length of the gap and the amplitude constant, a well-made fork can be relied upon to give a constancy of rate accurate to one part in 50,000. (7) The theoretical expression for the period of vibration of a bar holds good for a tuning fork. (8) The velocity of sound in the steel of which the forks used were made is  $5.49 \times 10^5$  cm. per sec. (9) The coefficient of modulus of elasticity of the steel is  $19.10 \times 10^{11}$ . (10) The temperature coefficient of the modulus of elasticity increases from  $-2.2 \times 10^{-4}$  at  $-25^{\circ}$  C. to  $-2.96 \times 10^{-4}$  at  $56^{\circ}$  C. These values were computed from the relation  $\epsilon = -(2\theta + \alpha)$ , where  $\epsilon$  is the temperature coefficient of modulus of elasticity,  $\theta$  the temperature effect upon the period of the fork, and  $\alpha$  the coefficient of linear expansion of steel.

### I. INTRODUCTION.

THE determination of the frequency of tuning forks and the study of the causes affecting the frequency have been the subjects of investigation by a large number of physicists during the last hundred years or more. Consequently the author would not have thought of carrying out the researches described in the following pages had not circumstances led him to them. While working on problems of sound ranging with the Engineer Detachment of the United States Army in Princeton, it became part of the author's work to adjust and to determine the periods of vibration of a number of tuning forks. As these forks were slightly different in shape and in details of mounting from those used

by previous investigators it was found desirable to know the extent to which the results obtained by them held good for these forks. A brief investigation was therefore planned with this end in view. But as the work progressed, interesting and important results were obtained which led the author to widen the scope of inquiry until the problem had been carefully studied in all its phases.

## II. DESCRIPTION OF THE FORKS.

Most of the experiments described in the following pages were carried out with forks made by the Western Electric Company, of the type shown in Fig. 1. The fork proper is milled out of a solid piece of soft machine steel and is electro-galvanized to prevent rusting. The prongs of the fork are about 39.5 cm. long, 0.95 cm. thick, 1.90 cm. deep, and 2.90 cm. apart. The outer curves at the shoulders as well as the inner curve are circular. Consequently the prongs are thinner at the shoulders than the prongs of standard Koenig forks of the same general dimensions, and therefore vibrate about their fixed ends as axes more like rigid bars than do the prongs of the Koenig type of fork.

The fork is rigidly attached to the back of a brass casting which has the form of a rectangular trough. The electromagnet is provided with movable pole-pieces placed on the outside of the prongs. This feature had been introduced by the Western Electric Company to satisfy the special conditions for which the fork was originally designed and is not at all necessary for its general serviceability. In fact the common type of electromagnet with pole-pieces between the prongs is preferable for a fork for which constancy of frequency is the principal desideratum.

A steel contact spring, 0.28 mm. thick and 3.92 mm. wide, is attached to each of the prongs by means of a steel clamp screwed to the prong at a point about midway between its ends. The contact spring is reinforced with two shorter springs placed one on each side and clamped together. When the fork is adjusted the free end of each contact spring comes midway between two platinum contact points soldered to the ends of two brass screws. The latter are set in brass bars provided with set screws for the purpose of making the contact screws fast after adjustment. These bars are screwed to a piece of micanite which forms the top piece of the bridge over the prongs. In order to secure good electrical contact a circular piece of platinum foil is soldered to each side of the free end of each contact spring.

The scheme of electrical connections is shown in Fig. 2, where the arrowheads marked *A*, *B*, *C* and *D* indicate the four contact points at the ends of the horizontal brass screws shown in Fig. 1. The condensers

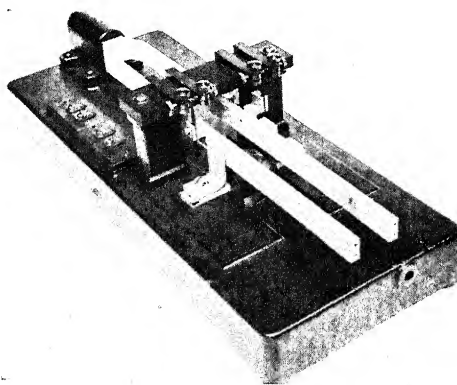


Fig. 1.

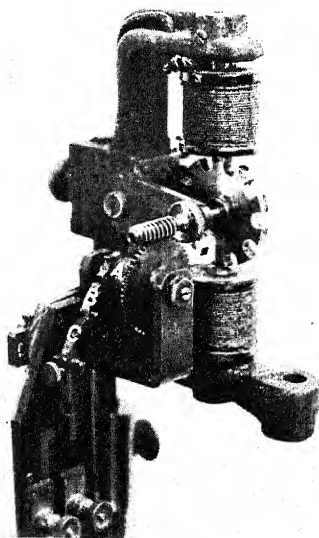


Fig. 4.

H. M. DADOURIAN.



marked *c* have each a capacity of 0.5 M.F., the resistance coils marked *r* have each a resistance of 150 ohms, and the field coil, *R*, of the electro-magnet has a resistance of 225 ohms and about 6,300 turns of wire. The condensers and the resistance coils are attached to the under side of the brass base. The contact points *C* and *D* do not form parts of the electrical circuits, but they are necessary for the symmetry of the mechanical action of the contact points upon the contact springs. The contact point *D* is provided with the necessary connections, so that it can be made use of electrically in case it is desired to energize the electro-magnet of the phonic wheel every half period instead of every full period,

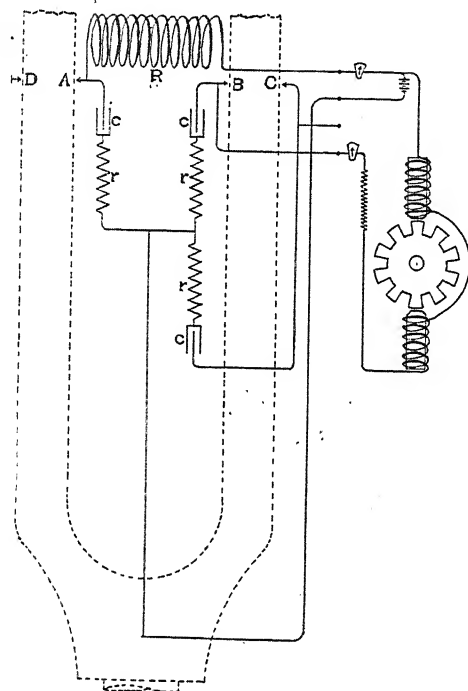


Fig. 2.

for instance. It may be stated here parenthetically that the wheel runs with greater smoothness between two than between four pole-pieces as the fork functions under normal running conditions remarkably free of sparking at the contact points, so that the pieces of platinum foil soldered at the ends of the contact springs last indefinitely.

In addition to the W. E. (Western Electric) forks three others were used. One of these was a Leeds and Northrup fork kindly loaned by the Company. Another was a fork made by Pirard and Coeurdevache of

Paris. The third was an old 50 V. D. Koenig fork mounted upon the base of one of the W.E. forks. In order to adapt the Koenig fork to the W.E. base, contact springs were clamped to the sides of the prongs at points about one-fourth of the length from the free ends. The way in which this was done is indicated in Fig. 3, where *A* is a piece of steel

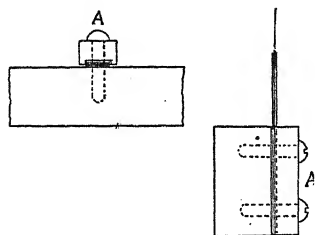


Fig. 3.

channeled to a depth slightly less than the total thickness of the three springs between it and the prong. This piece was attached to the prong by means of two screws which pass through holes made in the springs. The arrangement is an improvement over that of the original W.E. forks, first because it is simpler and second because it does not unnecessarily modify the uniformity of the effective thickness of the prong in the neighborhood of the clamp, as the spring in contact with the prong has only one-third the width of the clamp in contact with the prong in the W.E. fork.

### III. METHOD OF DETERMINATION OF THE PERIOD.

The period of vibration of the fork was determined by a method devised by Captain H. B. Williams. Chronographic records of every fiftieth complete vibration were compared with the records of the mean-time clock of the Princeton University Observatory. In order to record the vibration the phonic wheel and contact making device shown in Fig. 4 were used as intermediary between the fork and the chronograph. When the fork is set in vibration the circuit, of which the field coils of the phonic wheel form a part, is made and broken at the contact point *A*, Fig. 2, thus periodically exciting the electromagnet of the wheel. If the armature of the wheel is given a motion of rotation so that one of its teeth comes nearly in front of each of the pole-pieces of the magnet it falls in step with the magnetization of the electromagnet and rotates synchronously with the vibration of the fork. The rotation of the armature causes the toothed wheel *A*, Fig. 4, to rotate and to push against the rod *B* every time one of its teeth passes by the end of the latter. The rod *B* then comes into contact with the rod *C*, thus closing the circuit which operates the pen of the chronograph. The gearing intervening between the phonic wheel and the contact maker is such that this occurs once in every fifty complete vibrations of the fork. The short flat springs at the lower ends of the rods *B* and *C* hold them in position and connect them to the binding post at the lower left hand corner of Fig. 4 to which are attached the leads of the chronographic

circuit. The time of contact may be made as short as desired by adjusting the position of the contact maker relative to the wheel *A*, and of the rod *C* relative to the rod *B*. The clock records were obtained by the intermediary of a quick contact making device, attached to the scope wheel of the observatory clock, of which the contact maker shown in Fig. 4 is a simplified copy.

The phonic wheel was invented by La Cour and Rayleigh, independently. They used it, however, to compare the frequencies of two forks by the stroposcopic method and not to obtain chronographic records of a fork. The method described in the preceding paragraph which may, properly, be called the *chronographic method*, is more convenient and is capable of yielding a greater accuracy than any other method hitherto used for determining the period of a tuning fork, as will be observed from the following statement.

First, in the chronographic method a record of two hours can be obtained without any more care on the part of the experimenter than that of winding up the clock-work of the chronograph. The record obtained can be read with great precision, and being permanent can be read at one's convenience and kept for future reference. The clock does not have to be adapted to the fork nor the fork to the clock; consequently the frequency of any fork can be obtained by comparing its record with that of the clock, the rate of which is practically constant and known very accurately. The rate of the mean time clock used in these experiments was known to one part in 200,000.

Second, in this method the recording arrangement does not affect the motion of the fork as in the case of the vibrographic method in which the variable pressure of the drum against the recording style must have an appreciable effect upon the period of the fork. In this connection it may be stated that an experiment was performed in which the period of W.E. fork No. 496 was observed while the mean current through the field coils of the phonic wheel was changed from 60 to 180 milliamperes, but no appreciable change in the period was observed.

Third, the method gives a direct comparison of the fork with a primary standard like a clock, and not with a secondary standard in the form of another fork.

The precision of the chronographic method depends upon (*a*) the accuracy with which the rate of the clock is known, (*b*) the accuracy of the positions of the fork signals relative to the positions of the clock signals on the chronographic record, (*c*) the length of the record, and (*d*) the precision with which it is read. The error due to the rate of the clock may be made negligible by using a clock the rate of which is accurately

determined. The error due to the irregularities of the positions of the signals need not be more than .01 sec. in a well-constructed instrument. The chronographic record can be read with great accuracy by means of a differential scale often used by astronomers. So that even if an error of .05 sec. is made, due to the imperfections of the recording apparatus, the effect of this comparatively large error can be made negligible by taking a fairly long record. If the record is an hour long, for instance, an error of .05 sec. due to the end signals will introduce in the determination an error of one part in 72,000 only.

#### IV. EFFECT OF BASE.

It follows from theoretical considerations that the base upon which the fork is mounted should have no mass at all or it should have an infinite mass in order that it absorb no energy from the fork. It is to be expected therefore that the base of a fork will have some effect upon the period. In order to study this effect the following experiments were performed.

The W.E. fork No. 496 was placed upon a brick pier with a marble top and its period determined. A lead weight of 10 kg. was then placed upon the standard by means of which the fork proper is attached to its base and the period was again determined. The result of increasing the mass of the base by this means was to decrease the period by one part in 30,000.

A similar experiment was made with the Pirard and Coeurdevache fork and a decrease of one part in 1,000 observed. The reason for the comparatively large change in the period of this fork can be accounted for by the fact that the P. & C. fork weighs about 4 kg. while the W.E. forks weigh about 12 kg.

Next the effect of the table upon which the W.E. fork was placed was determined by observing its period while it was (a) upon the pier, (b) upon a wooden table and (c) upon a shaky stool. The result was a progressive increase in the period, the period on the stool being one part in 10,000 greater than on the pier.

The relatively large weight of the base and mounting of the W.E. forks is therefore a desirable feature, which makes the effect on the period of the mass of the table upon which it is placed relatively small.

#### V. EFFECT OF CHANGES IN THE CONSTANT OF THE ELECTRICAL CIRCUIT.

In Fig. 5 let  $oo'$  indicate the position of rest of the contact spring which is in series with the field coil of the electromagnet,  $bb'$  and  $cc'$  indicate the positions of extreme displacement, and  $aa'$  its position at the



instant it comes into contact with  $A$ , Fig. 2, and closes the circuit of the electromagnet. On account of the self induction of the circuit the driving current does not attain its maximum value instantaneously but increases according to the well-known exponential law during the excursion of the spring from  $aa'$  to  $bb'$  and back to  $aa'$ . At  $aa'$  the spring leaves the contact point thus opening the circuit and permitting the current to die down. The values of the current at different parts of a complete vibration are roughly indicated by the curve  $ab'a'ca$ . The motion of the prongs is opposed by the magnetic field, while the spring moves from  $aa'$  to  $bb'$ , is helped while moving from  $bb'$  to  $cc'$  and is opposed again while moving from  $cc'$  to  $aa'$ . Thus the motion of the prongs is opposed during the time the prongs come together and is helped the rest of the time. Therefore a periodic but non-harmonic force is impressed upon the prongs in addition to the forces of restitution and of damping. When a steady state of vibration is reached the energy supplied per cycle by this force equals the energy dissipated by the damping forces. This energy is a function of the area enclosed by the broken curve  $ab'a'ca$ . The area as well as the position of its center can be altered

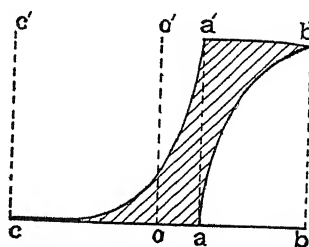


Fig. 5.

while the maximum value of the current is kept constant by changing the constants of the exponential curves and the position of the line  $aa'$ . In other words the energy applied per cycle and the effective phase of its application may be altered by changing the time constant of the circuit and the length of the gap between the spring and the contact point.

It will be seen in Part VII. that for constancy of frequency it is necessary to keep the amplitude constant. But to do this it may be necessary to regulate the driving current and hence it is of importance to know to what extent, if at all, a change in the time constant due to a change in the resistance of the circuit affects the period while the amplitude is kept constant. This point was investigated with the W.E. fork No. 488 in two experiments. In one of these the effect of an additional non-inductive resistance was observed, and in the other the effect of adding a self induction to the circuit was determined, the amplitude being kept constant in both cases. The results of the first experiment are shown by the curves of Fig. 6, where the abscissas denote the external non-inductive resistance added to the circuit of the electromagnet, and the ordinates denote the percentage increase in the period of the fork in ten-thousandths parts of the period corresponding to zero external resistance.

Curve I. was obtained with the length of the gaps between the contact springs and the contact points equal to .05 mm., while curve II. was obtained with the length equal to 0.3 mm. It will be observed that the effect is greater for the longer gap. But the effect is negligible for both, so long as the change in the resistance is small. If a change of 100 ohms is made in the resistance of the circuit to keep the amplitude constant,

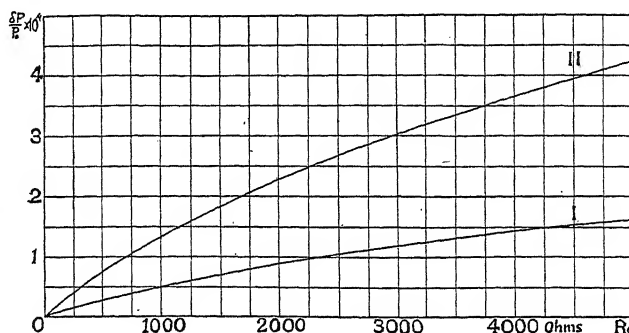


Fig. 6.

the change in the period is under one part in 100,000 for curve I. and under two parts for curve II.

The effect of an increase in the self induction of the circuit was next observed by placing alternately an inductive resistance and a non-inductive resistance of equal value in series with the coil of the electromagnet and the change in the period was found to be less than one part in 10,000.

#### VI. EFFECT OF THE LENGTH OF THE GAP.

It is evident from Fig. 5 that a change in the position of the line  $aa'$  would result in a change in the area of the cycle  $ab'a'ca$  and the position of the center of this area. Consequently changing the gap between the contact spring and the contact point introduces a change in the amount of energy applied to the fork per period and in the phase of application of this energy. Furthermore the circumstances of the mechanical action of the contact points upon the springs, and through them upon the prongs, is changed when the gaps are changed. Therefore an alteration in the length of the gaps is bound to affect the period of vibration of the fork.

This effect was studied by determining the period of vibration for gaps of different lengths. The results of an experiment with the W.E. fork No. 488 are shown by curve I of Fig. 7, where the abscissas denote the lengths of the gaps between the springs and the contact points in fractions of one millimeter, while the ordinates denote percentage increases in the period in ten-thousandths of the period corresponding to zero gap.

All four of the gaps were made of the same length, changing them from .04 mm. to .25 mm. in ten steps. It will be observed that the effect is very nearly linear and that it is comparatively large, being one fifth of one per cent. per one tenth of a millimeter change in the length of the gaps.

Curve *II* was obtained with the Koenig fork. In spite of the fact that the springs were less stiff than those of fork No. 488 the gap-effect of the Koenig fork is greater than that of No. 488. This can be accounted for

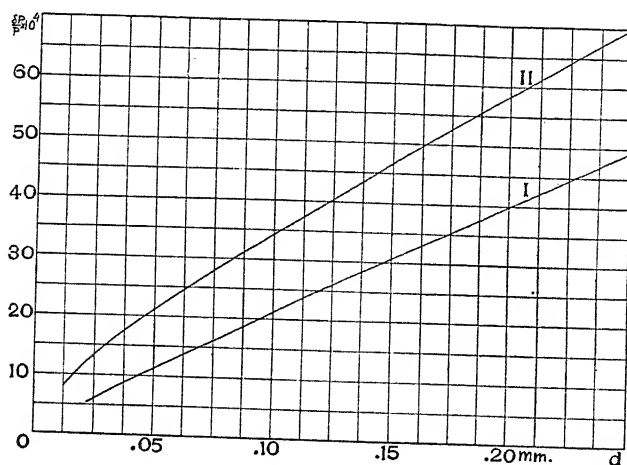


Fig. 7.

by the fact that the springs were nearer the free ends of the prongs in the case of the Koenig fork and consequently the moment of the forces acting upon the springs was greater in proportion, and that the forks were relatively thinner and consequently their motion was more easily affected by this moment.

These results show that the gap-effect is considerable and that the gaps must be kept constant in length if constancy of frequency is desired. This fact does not seem to have been recognized by students and makers of tuning forks, as is evident from the form of contact springs of standard types of forks. Most of these forks have twisted platinum wires for contact springs which change their shape under the action of the contact points, making the length of the gaps variable. The springs of the W.E. forks can be relied upon to keep their shape and position so that the length of the gaps do not vary more than .01 mm.

It was found that the positions of the springs relative to the contact points were slightly changed when the position of the fork was changed. This was due to a redistribution of the weight of the fork among the four

rubber feet of the base and the consequent strain in the frame of the base. In order to avoid possible errors from this source, the gaps were examined whenever the forks had to be moved and one of the feet raised by placing a few thicknesses of paper, thereby bringing the springs to positions midway between the contact points. This precaution can be made unnecessary by making the frame of the base more rigid and providing it with three instead of four feet.

The effect upon the period of fork No. 488 of eliminating the outer contact points was determined by observing the period alternately with the outer contact points at 0.1 mm. from the springs and at distances beyond the reach of the springs. The average results of four separate determinations was to change the time of 50 periods from 1.00096 sec. with the outer contact points at 0.1 mm. to 1.00369 sec. with the outer contact points beyond the reach of the springs, the inner contact points being kept at 0.1 mm. from the springs. In other words the mechanical action of the outer contact points upon the springs decreases the period by a fraction of about 27 parts in 10,000.

#### VII. EFFECTS OF CHANGES IN AMPLITUDE.

Mercadier<sup>1</sup> and Ettinghausen<sup>2</sup> have observed that the frequency of a fork increases with decreasing amplitude and approaches asymptotically the frequency corresponding to zero amplitude. Poske<sup>3</sup> and Heerwagen<sup>4</sup> on the other hand have found that the frequency increases linearly with decreasing amplitude. Heerwagen has expressed his results by the equation

$$n = n_0 - p\alpha$$

where  $n$  and  $n_0$  denote the frequency,  $\alpha$  the amplitude, and  $p$  a constant. Hartmann-Kempf<sup>5</sup> has observed, however, that the frequency increases more rapidly than Heerwagen's linear equation would imply and has expressed his results by the empirical relation

$$n = n_0 - (p + \Delta\alpha)\alpha.$$

The differences in the conclusions attained by these investigators can be accounted for in the light of the results obtained from the following experiments upon the effect of changes of amplitude upon the period. Throughout these experiments the temperature and the spark gaps were kept constant. The amplitude was measured by means of a traveling

<sup>1</sup> Mercadier, C. R., 83, p. 800, 1876; Journ. de Phys., 5, p. 201, 1876.

<sup>2</sup> A. Ettinghausen, Pogg. Ann., 156, p. 337, 1875.

<sup>3</sup> H. Poske, Pogg. Ann., 152, p. 448, 1874.

<sup>4</sup> F. Heerwagen, Diss. Dorpat, 1890.

<sup>5</sup> R. Hartmann-Kempf, Ann. d. Phys., 13, p. 124, 1904.

microscope provided with a vernier to read twentieths of one millimeter. The microscope was focused upon a bright spot at the end of one of the prongs and the double amplitude measured; the double amplitude of the other prong was then measured and one half of the average of the two observations taken as the amplitude of the fork.

Curves *I* and *II* in Fig. 8 show the results obtained with the W.E.

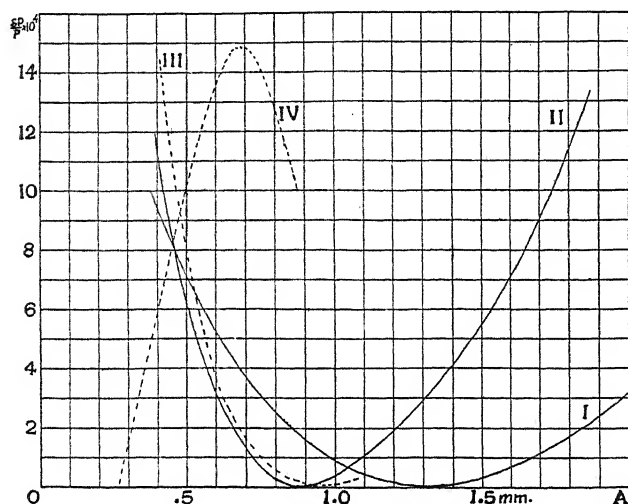


Fig. 8.

fork No. 496. The abscissas denote single amplitudes in millimeters and the ordinates denote percentage increases in the period in ten-thousandths of the minimum period observed. The changes of amplitude of curve *I* were made by increasing the current in the field coils of the electromagnet while the pole-pieces were kept fixed. On the other hand, the changes in the amplitude of curve *II* were obtained by moving the pole-pieces relative to the prongs while the current was kept constant. The curves represent the averages of two sets of curves, one obtained by increasing the amplitude and the other by decreasing it. The two component curves did not quite coincide; each formed, however, a very smooth curve representing the experimental data.

These curves bring out the interesting fact that there is an amplitude for which the period has a minimum value. In other words there is an amplitude in the neighborhood of which the period has a stationary value and consequently is not affected by small changes of amplitude. The value of this amplitude is different for the two curves because of the difference in the manner in which the changes were brought about. We shall see later that the amplitude corresponding to the stationary

value of the period depends also upon the stiffness of the contact springs and upon the length of the gaps.

Curve *III* was obtained with the Leeds & Northrup fork. It will be observed that it has the general shape of curves *I* and *II*. This was to be expected because the Leeds & Northrup contact arrangements are somewhat similar to those of the W.E. forks. Curve *III* and curve *IV* could not be extended further to the right because the amplitude could not be increased beyond certain limits on account of the proximity of the prongs to the pole-pieces of the electromagnet.

Curve *IV* was obtained with a Pirard & Coeurdevache fork. In this case the period has a maximum instead of a minimum value because in the P. & C. fork the contact springs as well as the contact points are attached to the base and consequently the prongs are not in contact with the springs during a fraction of the period which forms a greater and greater portion of the period as the amplitude is increased. This explanation is sustained by the experiment described in the last paragraph of Part VI. where it was shown that relieving the prongs from the mechanical action of the outer contact points resulted in a considerable increase in the period.

Amplitude-period curves were also obtained for the Koenig fork and

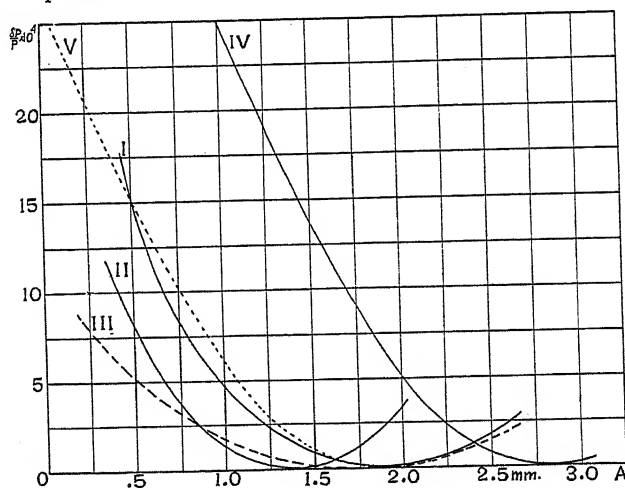


Fig. 9.

for the W.E. fork No. 488. Curve *III* of Fig. 9 was obtained with the Koenig fork, the changes in the amplitude being obtained by changing the driving current, while curves *I* and *II* were obtained with the W.E. fork No. 488. In the experiments corresponding to curve *I* the pole-pieces of the magnet were moved; in the experiment corresponding to

curve *II* the driving current was changed. It will be observed that as in the case of fork No. 496 the amplitude corresponding to the minimum period is smaller for curve *I*.

Curve *IV* was obtained with the 488 fork under the same conditions as curve *I* except that the gaps were changed from .05 mm. in the case of curve *I*, to 0.30 mm. in the case of curve *IV*. In order to compare these two curves, curve *V* similar to curve *IV* was drawn so that its minimum point coincides with that of curve *I*. It will be observed that the rate of decrease of the period is greater for *I* than for *V*. This can be accounted for on the ground that for a given magnitude of the mechanical action of the contact points upon the springs, the energy of the prongs was greater in the experiments corresponding to curve *V*. This explanation is in accord with the fact that the curves are steeper to the left of the minimum point, where the amplitude is smaller and consequently the energy of the prongs is smaller.

From the foregoing considerations it will be expected that for a given length of gap and at a given amplitude less than the amplitude which corresponds to the stationary value of the period, the rate of decrease

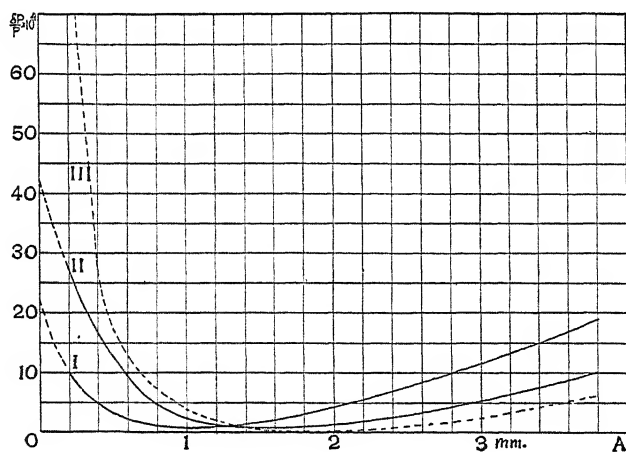


Fig. 10.

of the period will be smaller the smaller the stiffness of the springs. Certain experiments, the results of which are indicated by the curves of Fig. 10, sustain this conclusion. All three of these curves were obtained with fork No. 488 with gaps equal to .05 mm., the amplitude being altered by changing the driving current. In the experiment represented by curve *I* the fork had the same contact springs as in the experiment of curve *I* of Fig. 9, namely, springs with cross-sections of .41 mm.  $\times$  4.64 mm.

In the experiments represented by curves *II* and *III* the fork had springs with cross sections  $0.28 \text{ mm.} \times 3.92 \text{ mm.}$ , and  $.48 \text{ mm.} \times 7.86 \text{ mm.}$ , respectively. A glance at these curves shows that the stiffer the springs the steeper are the curves on both sides of the minimum point, and the greater is the amplitude corresponding to the minimum period.

The results of other experimenters can be made to agree among themselves and with the results obtained by the author if it is supposed that the former have to do with that part of the period-amplitude curve which lies to the right of the minimum point and that the region investigated by Mercadier and Ettinghauser comes nearer the point of minimum period than the region studied by Poske, Heerwagen and Hartmann-Kempf.

The more important results of the experiments described in Part VIII. may be summarized in the following terms.

*First*, the period-amplitude curve passes through a stationary value and consequently there is an amplitude at which the fork can be operated with maximum constancy of period.

*Second*, the amplitude which corresponds to the stationary value of the period depends upon the stiffness of the contact springs and the length of the gaps.

From the results of the experiments described in Parts V. and VI. certain inferences may be drawn with regard to the best position for the contact springs. For a given contact pressure the action of the contact points upon the springs, and through them upon the prongs, is greater the nearer the springs are placed to the free ends of the prongs. Furthermore, for a given change in the amplitude the change in the contact pressure is greater the nearer the springs are placed to the free ends. On the other hand there is a limit to the extent to which the position of the springs can be approached to the shoulders of the fork. Hence there must be a position which is more favorable than any other. The determination of this position did not form a part of the investigations described here, but considerations based upon the mechanical action of the contact points upon the prongs, the position of center of percussion of the prongs, and the nodal points of the first harmonic of the vibration have led the author to the conclusion that the most favorable position for the contact springs must lie between the middle of the prong and one third of the distance from the free ends.

#### VIII. TEMPERATURE EFFECT.

A score or more physicists have determined the effect of temperature upon the frequency of tuning forks and have obtained concordant results.



These men, however, do not appear to have appreciated the importance of controlling certain factors which we have found to affect the frequency. It was deemed worth while, therefore, to redetermine the temperature effect under the favorable conditions which could be secured in the light of the experiments described in the preceding pages. Another reason which induced the author to carry his investigations into this phase of the tuning fork problem was the fact that he had at his disposal facilities to extend the investigation on the temperature effect to temperatures below zero degree centigrade, to which region the older investigators had not carried their work.

The work was carried on in the constant temperature rooms of the Palmer Physical Laboratory. The larger of the two rooms was provided with a thermostat which responded readily to changes of temperature as small as  $0^{\circ}.1$ . The smaller room had no temperature regulating device but its temperature could be lowered still further. Two separate determinations were made. The first, made in the larger room, covered the range of temperatures between  $-3^{\circ}.6$  C. and  $56^{\circ}.8$  C. The second, made in the smaller room covered the range of temperatures between  $-25^{\circ}.8$  C. and  $21^{\circ}.2$  C.

The W.E. fork No. 488 and the Koenig fork were placed side by side on a table in the middle of the larger room and a box with open ends was built over them. A microscope with a scale in the ocular was focused upon a bright spot at the end of one of the prongs of each fork, in order to observe and adjust the amplitude of vibration. Two thermometers made by R. Fuess with ranges of  $-30^{\circ}$  C. to  $100^{\circ}$  C. and scales divided into tenths of one degree were suspended so that the bulb of one was placed between the prongs and near the stem of one of the forks. A Callander recorder was set up to observe the temperature without going into the constant temperature room in use, but it was found to be not quite sensitive enough to give the temperature to the desired degree of precision; therefore it was used only to keep a rough record of the changes. The following general method of procedure was used during the first experiment: About one half hour after the temperature of the room had reached a desired value the room was entered and the gaps of the forks were examined to see if there had been any changes in the lengths of the gaps from the value .05 mm. to which they had been adjusted. Each fork was then set into vibration and the amplitude adjusted, if necessary; fork No. 488 was left vibrating and the open ends of the box were covered with pieces of woolen cloth. After about ten minutes the room was entered again, the thermometers read (this could be done without opening the box), and the chronograph started. Twenty

minutes later the room was entered again, the thermometers were read, the Koenig fork was started and its record also taken for twenty minutes, after which the thermometers were read again. In this manner a large number of records were obtained which covered the range of temperatures between  $-3^{\circ}.6$  C. and  $56^{\circ}.8$  C. and extended over several days.

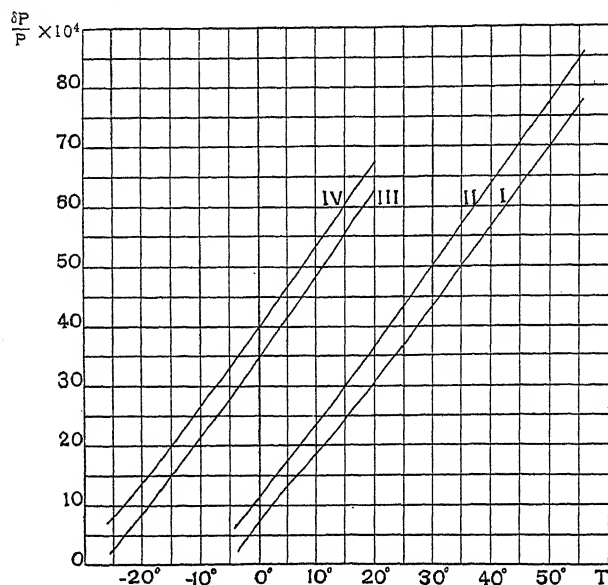


Fig. 11.

The results obtained with fork No. 488 were plotted on a large scale upon a millimeter cross-section paper with the temperatures as abscissas and the corresponding values of 50 complete periods as ordinates and a smooth curve was obtained with a continuously increasing slope. From this curve *II* of Fig. 11 was drawn, where the ordinates give the percentage increases in the period in ten-thousandths of the period at  $0^{\circ}$  C.

The period-temperature curve of the Koenig fork had an inflection point at about  $11^{\circ}$  and another at about  $35^{\circ}$ . There is no doubt that this behavior of the Koenig fork was due to slight variations in the spark gaps observed during inspections of the gaps made before taking records. The variation in the gaps must have been due to the expansion and contraction of the vulcanite piece which carried the contact points. In the regular W.E. forks this piece was of micanite which has a much smaller coefficient of expansion. Curve *I* which represents the results obtained by the Koenig fork is therefore of interest because it shows the importance of having spark gaps of constant length as well as because it supports the general conclusions derived from the other three curves of Fig. 11.

In the second experiment the Koenig fork was replaced by the W.E. fork No. 313 and the apparatus was set up in the smaller room. This room did not have a thermostat and consequently its temperature could not be kept constant; therefore the following method of procedure was adopted. The room was cooled several degrees below the temperature in the neighborhood of which it was desired to take a record; then the refrigerating machine was stopped and the temperature was allowed to rise for several hours and then records of the two forks were taken in the same manner as in the first experiment. This was repeated until ten pairs of records were taken while the temperature was lowered in the manner just described from  $21^{\circ}\text{C.}$  to  $-27^{\circ}\text{C.}$  The temperature of the room was then allowed to rise from  $-27^{\circ}$  to the normal room temperature and 15 more pairs of records were taken. The experiment took eight days and the rate of change of temperature was so slow during the intervals when records were taken that the temperature could be considered as constant. The results of this experiment are indicated by

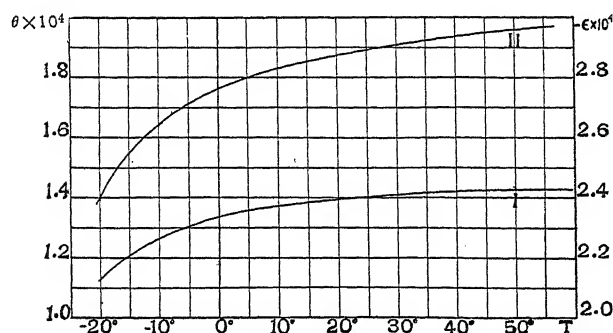


Fig. 12.

curves *III* and *IV* of Fig. 11, curve *III* corresponding to No. 488 and curve *IV* to No. 313.

The results indicated by the curves *II*, *III* and *IV* were plotted accurately on a large scale and the different rates of increase in the period at different temperatures were determined for each curve. The average values for the three curves are given in Table I., where the numbers in the first column represent the temperatures and those in the second column the corresponding values of the temperature coefficient  $\theta$  defined by

$$\theta = \frac{1}{P_0} \frac{\partial P}{\partial t},$$

or

$$P = P_0(1 + \theta t).$$

TABLE I.

$t.$	$\theta.$	$-\epsilon.$	$t.$	$\theta.$	$-\epsilon.$
$-20^{\circ}$	$1.14 \times 10^{-4}$	$2.39 \times 10^{-4}$	$20^{\circ}$	$1.38 \times 10^{-4}$	$2.87 \times 10^{-4}$
$-10^{\circ}$	1.27	2.65	$30^{\circ}$	1.40	2.91
$0^{\circ}$	1.33	2.77	$40^{\circ}$	1.42	2.95
$10^{\circ}$	1.36	2.83	$50^{\circ}$	1.43	2.97
			$-20$ to $50$	1.34	2.79

These results are represented graphically by curve *I* of Fig. 12. The values of  $\theta$  given in Table I. are from 10 to 40 per cent. higher than the average value of the results obtained by former observers as will be seen from Table II.

TABLE II.

Observer.	Range of Temp.	Temp. Coef.	Observer.	Range of Temp.	Temp. Coef.
Mercadier <sup>1</sup> .....	$3^{\circ}$ – $26^{\circ}$	$0.96 \times 10^{-4}$	Michelson <sup>5</sup> .....	$12^{\circ}$ – $24^{\circ}$	$1.00 \times 10^{-4}$
Kayser <sup>2</sup> .....	$0^{\circ}$ – $26^{\circ}$	0.98	Lang <sup>6</sup> .....	$14^{\circ}$ – $19^{\circ}$	1.11
Koenig <sup>3</sup> .....	$3^{\circ}$ – $26^{\circ}$	1.11	Pierpaoli <sup>7</sup> .....	$0^{\circ}$ – $30^{\circ}$	1.05
Koenig <sup>3</sup> .....	$26^{\circ}$ – $56^{\circ}$	1.07	Woodruff <sup>8</sup> .....	$20^{\circ}$ – $200^{\circ}$	1.08
McLeod & Clarke <sup>4</sup>	$15^{\circ}$ – $26^{\circ}$	1.10	Dadourian.....	$-26^{\circ}$ – $57^{\circ}$	1.34

It will be shown in Part XII. that the effect of temperature upon the period of a fork is mainly due to its effect upon the elasticity of the fork and that only about 5 per cent. of the former effect is due to the increase in the dimensions of the fork resulting from a rise of temperature.

#### IX. CONSTANCY OF PERIOD.

The constancy of the performance of the W.E. fork No. 488 was tested under the following conditions: The pole-pieces of the electromagnet were set at 1.37 mm. from the prongs. The contact points were adjusted so that the length of the gaps was .05 mm. A potential of 24 volts was applied directly to the binding posts on the fork. Under these conditions an amplitude of 1.58 mm. was obtained which corresponds to the stationary period of the fork under the given conditions. The fork was operated continuously from 8.43 a.m. until 12.45 M., and chronographic records were taken. The period was then determined for different parts

<sup>1</sup> Mercadier, Journ de Phys., 5, p. 201, 1876; C. R., 83, p. 822, 1876.

<sup>2</sup> Kayser, Wied. Ann., 8, p. 444, 1879.

<sup>3</sup> Koenig, Wied. Ann., 9, p. 394, 1880.

<sup>4</sup> McLeod & Clarke, Phil. Trans. Roy. Soc., 171, part I., p. 1, 1880.

<sup>5</sup> Michelson, Am. Journ. Sci., 25, p. 61, 1883.

<sup>6</sup> Lang, Wied. Ann., 29, p. 132, 1886.

<sup>7</sup> Pierpaoli, Rend Linc., 4 (1), p. 714, 1888; 5 (2), p. 265, 1889.

<sup>8</sup> Woodruff, PHYS. REV., 16, p. 325, 1903.

of the run from careful measurements of the record, taking intervals of 20 minutes. The results of these determinations are given in Table III. where the numbers in the first column give the temperatures at the middle of the different 20-minute intervals, those in the second column are the corresponding observed values of 50 complete periods, the numbers of the third column are these values corrected to 20°.50 C. and those of the last column are the percentage deviations of the values of the periods thus determined from their mean value.

TABLE III.

Temperature.	50 <i>P.</i>	50 <i>P</i> <sub>c</sub> .	$\delta P/P$ .
20°.30	1.000885 sec.	1.000913 sec.	$-8.4 \times 10^{-6}$
20°.34	1.000903	1.000925	3.6
20°.38	1.000910	1.000927	5.6
20°.46	1.000925	1.000931	9.4
20°.50	1.000336	1.000934	13.6
20°.56	1.000934	1.000926	4.6
20°.66	1.000923	1.000901	-20.4
20°.69	1.000940	1.000913	-8.4
20°.72	1.000954	1.000923	1.6
		1.0009214	$\pm 8.4$

Another set of determinations was made the next day with the spark gaps equal to 0.3 mm. the pole-pieces at 1.0 mm. from the prongs, and an amplitude of 1.75 mm. The fork was operated from 12.55 until 4.19 P.M. and a continuous record taken. The values of 50 periods were then determined as in the preceding experiment and the results given in Table IV. were obtained.

TABLE IV.

Temperature.	50 <i>P.</i>	50 <i>P</i> <sub>c</sub> .	$\delta P/P$ .
20°.77	1.004153 sec.	1.004134 sec.	$-10 \times 10^{-6}$
20°.74	1.004168	1.004155	11
20°.72	1.004186	1.004175	31
20°.70	1.004181	1.004170	26
20°.67	1.004152	1.004148	4
20°.65	1.004153	1.004151	7
20°.63	1.004136	1.004137	-7
20°.62	1.004126	1.004136	-8
20°.61	1.004115	1.004122	-22
20°.60	1.004114	1.004121	-23
		1.0041443	$\pm 14.9$

A glance at the last columns of Tables III. and IV. shows the remarkable constancy of performance of the fork. In the first run the maximum

deviation from the mean for a run of four hours is only one part in 50,000 while the average deviation is less than one part in 100,000. In the second run the deviation from the mean is larger. But in this case the deviation is systematic indicating that the conditions affecting the period were not kept as constant as during the first run. The great constancy of performance shown by the fork is all the more remarkable because it is a commercial article in the manufacture of which no great effort had been made to secure such important requirements as a high degree of uniformity of thickness of prongs and symmetry relative to the axis of the fork. This investigation shows conclusively that a proper control of the factors affecting the period is of prime importance.

#### X. EXPRESSION FOR PERIOD OF VIBRATION.

It has been shown by Mercadier<sup>1</sup> that the expression for the period of transverse vibration of a bar obtained from the theory of elasticity holds good for a tuning fork provided the projection of the median line of the prongs upon the geometric axis of the fork is taken for the length of the prongs and a small correction term is added to this. The expression for the period of vibration of a bar is

$$P = \frac{4\pi\sqrt{3}}{r^2v} \frac{l^2}{a} \quad (1)$$

where  $l$  is the length and  $a$  the thickness of the bar,  $v$  the velocity of sound in the bar, and  $r$  the root of the equation

$$(e^r + e^{-r}) \sin 2r + 2 = 0 \quad (2)$$

and equals 1.87011.<sup>2</sup> For rods of the same material  $v$  is constant, therefore we can write

$$P = K \frac{l^2}{a} \quad (3)$$

Mercadier's modified expression for tuning forks is

$$P = K \frac{(l + \lambda)^2}{a} \quad (3')$$

where  $\lambda$  is a correction term introduced in order to satisfy Mercadier's experimental results and equals .012 $l$ . Evidently the value of  $\lambda$  will depend upon the shape of the fork near the shoulders of the prongs. Since the W.E. forks had a slightly different shape from those used by Mercadier it was thought desirable to determine the value of  $\lambda$  for these forks. Using Mercadier's experimental value of  $K$ , namely, 1/81,827

<sup>1</sup> E. Mercadier, C. R., 79, pp. 1001, 1069, 1874; Journ. de Phys., V., p. 201, 1876.

<sup>2</sup> Poisson, Traité Mécanique, II., p. 390.

sec./cm., the values given in the first column of Table V. were obtained for ten different forks.

TABLE V.

$\lambda$ .	$\lambda/l$ .	$E$ .	$\delta E/E$ .
+ 0.16 cm.	+ 0.0040	$18.80 \times 10^{11}$	- 0.016
+ 0.07 "	+ 0.0018	18.92	- 0.009
+ 0.06 "	+ 0.0015	18.98	- 0.006
+ 0.06 "	+ 0.0015	19.03	- 0.004
+ 0.04 "	+ 0.0010	19.07	- 0.002
- 0.03 "	- 0.0007	19.16	- 0.003
- 0.05 "	- 0.0012	19.02	+ 0.004
- 0.09 "	- 0.0022	19.30	+ 0.010
- 0.10 "	- 0.0025	19.30	+ 0.011
- 0.17 "	- 0.0042	19.42	+ 0.017
- 0.005 "	- 0.0001	19.10	.0092

It will be observed (a) that the highest value of  $\lambda$  obtained is only one third of 0.48 cm. required for the W.E. forks by Mercadier's empirical relation  $\lambda = 0.012 l$ , (b) that the values are fairly evenly distributed on the two sides of zero, and (c) that their algebraic sum differs from zero by an amount which falls within the experimental errors of the measurement of the lengths of the prongs. Therefore the unmodified equation (3) gives closer results for the W.E. forks than Mercadier's equation (3').

#### XI. THE MODULUS OF ELASTICITY AND VELOCITY OF SOUND IN STEEL.

Replacing  $v$  in equation (1) by its expression in terms of the modulus of elasticity  $E$  and the density  $\rho$  we have

$$P = \frac{4\pi\sqrt{3}}{r^2} \frac{l^2}{a} \sqrt{\frac{\rho}{E}}. \quad (4)$$

Putting in the last equation 1.87011 for  $r$ , 7.363 for  $\rho$  and the values of  $P$ ,  $l$ , and  $a$  obtained from the ten W.E. forks the values of  $E$  given in the third column of Table V. were obtained. The value 7.363 for  $\rho$  was obtained from a careful determination of the density of four pieces which were cut off the ends of the prongs of different forks in adjusting their periods. Using this value of  $\rho$  and the average value of  $E$  given at the bottom of the third column, the value  $5.093 \times 10^5$  cm. per sec. was obtained for the velocity of sound in steel.

The deviations of the values of  $\lambda$  and of  $E$  from their mean values are small and undoubtedly due to the facts that (a) the prongs were not as uniform as might have been reasonably expected, and that (b) some of the forks were cut in more at the shoulders than others.

## XII. TEMPERATURE COEFFICIENT OF MODULUS OF ELASTICITY OF STEEL.

Let  $P_0$ ,  $a_0$ ,  $l_0$ ,  $\rho_0$  and  $E_0$  denote the values at  $0^\circ$  C. of the quantities involved in equation (4), and  $P$ ,  $a$ ,  $l$ ,  $\rho$  and  $E$  denote their values at any other temperature  $t$ , then we can write

$$\frac{P}{P_0} = \frac{a_0}{a} \cdot \frac{l}{l_0} \cdot \left(\frac{\rho}{\rho_0}\right)^{1/2} \left(\frac{E}{E_0}\right)^{1/2}. \quad (5)$$

Introducing in the last equation the temperature coefficients  $\alpha$ ,  $\theta$ , and  $\epsilon$  defined by

$$l = l_0(1 + \alpha t),$$

$$P = P_0(1 + \theta t),$$

$$E = E_0(1 + \epsilon t),$$

we obtain, to a first order of approximation, the relation

$$\theta = -\frac{\alpha + \epsilon}{2},$$

or

$$\epsilon = -(2\theta + \alpha). \quad (6)$$

Since  $\alpha$  and  $\theta$  are both positive magnitudes  $\epsilon$ , the temperature coefficient of the modulus of elasticity must be negative. In other words the modulus of elasticity decreases with increasing temperature. Taking the coefficient of linear expansion of steel to be  $1.1 \times 10^{-5}$  and substituting in equation (6) the values of  $\theta$  given in the second column of Table I. we obtain the figures of the third column for the values of  $\epsilon$  corresponding to the temperatures given in the first column. The variation of  $\epsilon$  with the temperature is shown graphically by curve *II* of Fig. 12. The unmistakable increase of the absolute value of  $\epsilon$  with the increase of temperature indicated by Table I. and curve *II* is in agreement with results obtained by Pizati<sup>8</sup> and Dodge.<sup>10</sup>

The relatively large differences among the values of  $\epsilon$  obtained by different observers must be due to the variety of the specimens of steel used and also due to experimental errors. It may be stated that the indirect method used in this paper is more accurate than the more direct methods by which the other observers have determined the temperature coefficient of elasticity.

In conclusion the author wishes to acknowledge his indebtedness and express his thanks to Professor W. F. Magie for placing the facilities of the Palmer Physical Laboratory at his disposal; to Professor E. H. Loomis for helping him place the constant temperature plant in working order; to Professor H. N. Russell for the use of the observatory chrono-



TABLE VI.

Observer.	$-\epsilon$ .	Observer.	$-\epsilon$ .
Wertheim <sup>1</sup> .....	$2.8 \times 10^{-4}$	Gray, etc. <sup>7</sup> .....	$2.47 \times 10^{-4}$
Kohlrausch & Loomis <sup>2</sup> .....	4.6	Wassmuth <sup>8</sup> .....	2.64
Pizati <sup>3</sup> .....	1.95	Walker <sup>9</sup> .....	2.5
Mayer <sup>4</sup> .....	2.21	Dodge <sup>10</sup> .....	1.9
Miss Noyes <sup>5</sup> .....	3.4	Dadourian.....	2.79
Shakespeare <sup>6</sup> .....	3.87		

graph; and to Captain H. B. Williams, who had charge of the sound ranging work of which the present investigation formed a part, for his interest in the work.

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<sup>1</sup> Wertheim, Pogg. Ann., 2, p. 1, 1848.

<sup>2</sup> Kohlrausch & Loomis, Pogg. Ann., 141, p. 481, 1870.

<sup>3</sup> Pizati, Nuo. Cim., 1, p. 181; 2, p. 137, 1877; 3, p. 152, 1879; 5, pp. 34, 135, 145, 1879.

<sup>4</sup> Mayer, Brit. Assoc. Rep., p. 573, 1894.

<sup>5</sup> Miss Noyes, Phys. Rev., 3, p. 433, 1896.

<sup>6</sup> Shakespeare, Phil. Mag., 47, p. 539, 1899.

<sup>7</sup> Gray, Blyth and Dunlop, Proc. Roy. Soc., 67, p. 180, 1900.

<sup>8</sup> Wassmuth, Phys. Zeits., 6, p. 755, 1906.

<sup>9</sup> Walker, Proc. Roy. Soc. Edin., 28, p. 562, 1907.

<sup>10</sup> Dodge, Phys. Rev., 5, p. 373, 1915.

## THE MATHEMATICAL STRUCTURE OF BAND SERIES II.

BY RAYMOND T. BIRGE.

SYNOPSIS.—In the first article of this series, the author proposed a hyperbolic formula, for the accurate representation of long band series. The present paper initiates a more general discussion of this formula, and of the experimental results which the author has since obtained.

Detailed analytic work has shown the new formula accurate, within limits of experimental error, for those band series showing the most radical deviations from the simple Deslandres' law. Preliminary work indicates that the hyperbolic formula is equally satisfactory in the case of series showing positive deviations from Deslandres' law, instead of the customary negative deviations.

A complete investigation of any band series is greatly facilitated by a preliminary study of its deviations from Deslandres' law. New methods have accordingly been developed for making this comparison in a rapid, yet accurate manner. In particular, there is developed a new Least Squares' formula, giving directly the value of the only desired constant in a Deslandres' formula, thus avoiding the more extended calculations necessary when all three constants are simultaneously evaluated.

A comparative study of band and line series formulæ suggests for band series a modified hyperbolic formula. This modified formula has been tested on several series and has been found very satisfactory. New line series formulæ are also suggested, but are not seriously recommended at this time.

## INTRODUCTION.

IN the first paper on this subject,<sup>1</sup> there was proposed a new formula for band series, together with the detailed quantitative results for the main ( $A_1$ ) "singlet" series of the 3883 CN. band. It was shown also that preliminary work indicated that the formula would hold equally well for the other series of this band. In particular, the singlet series from the third head ( $C_1$  series) exhibits the most radical deviations from Deslandres' law of any known band series. It therefore forms a crucial test for any new formula. This series has since been extended at both ends, the new lines having been identified from the author's own spectrograms. The complete quantitative results for this series were presented to the American Physical Society at the October, 1917, meeting.<sup>2</sup>

Certain remarkably systematic irregularities in the  $C_1$  series, brought out by these computations, made it seem desirable to study also the  $B_1$  series. This work has now been completed, much of the actual computing

<sup>1</sup> Astrophysical Journal, 46, 85-103, 1917. In the future, it will be referred to as I.

<sup>2</sup> PHYS. REV., (2), 11, 136, 1918.

having been done by an assistant, for whose services the author is indebted to the Rumford Committee of the American Academy of Sciences. At the same time, a general survey was made of practically all band series which seemed to show large deviations from Deslandres' law. A very considerable amount of material, both quantitative and qualitative, has now been collected, and it has seemed desirable to present a general review of the progress of the work to date. Such detailed results as appear of sufficient importance will be published later.

The present communication includes:

1. A discussion of methods of handling band series formulæ, in particular, Deslandres' parabolic formula.
2. A new viewpoint as to the original hyperbolic formula, together with a possible alternative formula in which the necessity of making a summation is avoided.

In the next paper there will be presented a review of typical band series, especially of those showing radical deviations from Deslandres' law, in a positive, as well as in a negative sense.

#### THE MATHEMATICAL ANALYSIS OF BAND SERIES.

It is well known<sup>1</sup> that all band series seem to conform to Deslandres' law<sup>2</sup> for at least an initial portion of their extent. The investigations of the author have convinced him that *no* band series strictly obeys Deslandres' law over *any* portion of its extent. But for short series (*i. e.*, series covering a small frequency interval) the deviations from this law may easily be far less than the experimental errors.<sup>3</sup> The first step however in the examination of any given series consists in determining how closely it conforms to the simple Deslandres' formula. Various methods for doing this have been used by previous investigators, but no general discussion of the matter has yet been published.

Deslandres' formula, in its original form,<sup>4</sup> states that the frequencies of successive lines of a band series are given by the parabola

$$\nu = \nu_0 + am + bm^2 \quad (I)$$

or

$$\nu = A + B(m + c)^2, \quad (I')$$

<sup>1</sup> For the most complete summarized description of all bands discussed in this series of articles, see Konen, "Das Leuchten der Gase und Dämpfe," pp. 199-278.

<sup>2</sup> Deslandres, *Comptes Rendus*, 103, 375, 1886.

<sup>3</sup> It may be remarked in passing that the amount of deviation from Deslandres' law seems to have no connection with the spectral extent of the series, nor with the number of terms in the series. Some series of 100 or more terms show a scarcely perceptible deviation, others of the same length a most radical deviation. Many series of few terms (30 or less) but of large spectral range also show radical deviations.

<sup>4</sup> For discussion see I., pp. 86-87.

where  $m$  takes successive integral values 0, 1, 2, 3, etc.<sup>1</sup> The successive first frequency differences are then given by the straight line

$$\Delta\nu = a + 2bm = 2B(m + c), \quad (2)$$

where  $m$  has the successive *fractional* values 0.5, 1.5, 2.5, etc., while the second frequency differences are given by the constant

$$\Delta\rho = 2b = 2B. \quad (3)$$

Any series for which the second frequency difference is constant therefore, per se, obeys Deslandres' law, and the first obvious method of testing this law is to compute these second differences.

But because of experimental errors, true perturbations, etc., these differences will never be constant, and in many cases it is difficult to tell from mere inspection whether they exhibit any definite trend from constancy. A far more delicate method of detecting any such trend, if present, is to plot the first frequency differences against  $m$ , on a fairly large scale. (50 × 50 cm. is recommended.) This curve, by Deslandres' law, should be a straight line, of slope  $2b$  (*i. e.*, secondary frequency difference), and intercept ( $\Delta\nu = 0$ )  $m = -a/2b = -c$ . The author has called it the "slope" form, and has found it to be actually an hyperbola.

In practically all long series the slope curve exhibits a very definite trend from linearity. Yet in instance after instance investigators have analytically fitted the series to a Deslandres' formula, and have concluded that the agreement was satisfactory, when the slope curve (*if* it had been plotted) would instantly have shown the falsity of such a conclusion.

But let us assume that a casual inspection of the  $\Delta\nu : m$  curve does not indicate any definite non-linearity. This condition is more likely to arise when the experimental errors are rather large (over 0.02 Å). A case in point is the main series of the 3371 Nitrogen band, which Lewis<sup>2</sup> extended, by using self-induction in the circuit, until it entered the next band at 3159 Å. The  $\Delta\nu : m$  curve for this band is apparently a straight line and the equation given by Lewis is the best obtainable when the simplified Deslandres' formula

$$\nu = A + Bm^2 \quad (4)$$

<sup>1</sup> Comparing (1) and (1')  $b = B$ ;  $c = a/2b$ ;  $\nu_0 = A + Bc^2$ . There seems to be some confusion as to just what is to be considered the "head" of a band. The ordinary idea, stated implicitly if not explicitly, is that this head is given by the line corresponding to  $m = 0$ , in any satisfactory formula. The author has tried in I. to follow consistently this usage. But (1') is often written  $\nu = \nu_0 + B(m + c)^2$ . This makes  $\nu_0$  (the head) correspond to  $m = -c$ . I doubt if anyone actually thinks of the head in that way, and so the form is objectionable. Of course when  $c = 0$ , the two view-points become identical.

<sup>2</sup> Astrophysical Journal, 40, 148, 1914.

is used. As has been remarked<sup>1</sup> (4) is *not* Deslandres' law, and its use is permissible only when the inaccuracy of the data makes the true determination of the intercept ( $-a/2b = -c = -$  "the phase") impossible. In this case, the data *are* rather inaccurate, but a slight improvement can be made by using (1') rather than (4).

The problem now becomes,—what is the best analytical method of handling the experimental data, in order to determine whether or not the Deslandres' formula holds, *i. e.*, whether a straight line is really the best simple smooth curve that can be passed through the experimental points of the  $\Delta\nu : m$  plot, and if so, what is the slope of this curve? This problem is identical with that of determining the constancy of the acceleration of gravity, and its value, if constant, given the position of a series of crests on an Atwood's Machine curve, but with the time and point of initial descent not known. For this latter problem we have

$$s = s_0 + v_0 t + 1/2 g t^2 \quad (5)$$

or

$$s = (s_0 - v_0^2/2g) + g/2(t + v_0/g)^2, \quad (6)$$

*i. e.*,

$$s = \alpha + \beta(t + \lambda)^2, \quad (6')$$

where  $\lambda$  = the "phase," *i. e.*, the fractional part of a second before  $t = 0$  at which descent began.<sup>2</sup> In the case of Deslandres' law, (1) and (1') correspond to (5) and (6') respectively.

If a study of the actual plotted data, together with a knowledge of the experimental conditions, indicates that the deviations of the data from any simple smooth curve are due solely to chance experimental errors, then the method of Least Squares leads to the "most probable" values of the constants of the equation of the curve. In the case of spectral series, the conditions necessary for the legitimate application of Least Squares are actually present,—*i. e.*, in a well-measured series, using the new International standards, there is practically no opportunity for unsuspected systematic errors of any appreciable amount.

The full details of the method of Least Squares, as applied to the Deslandres' formula, are given by V. Carlheim-Gyllensköld.<sup>3</sup> The necessary calculations are, however, rather extended, especially in the case of long series. Moreover we do not, at the outset, wish to assume the truth of Deslandres' law. Instead, we wish to investigate whether or not the

<sup>1</sup> I., p. 89.

<sup>2</sup> *I. e.*, we assume  $t = 0$  for the nearest even second to the time of starting, just as in band series we choose  $m$  so as to make  $c$  (the "phase") some fraction less than 0.5. This may possibly be the analogy Thiele had in mind when he called  $c$  the "phase." The question of the origin of the term has recently been raised. (Proc. Phys. Soc. of London, 30, 130, 1918.)

<sup>3</sup> Svenska Vet. Ak. Handl., 42 N : r 8, 1907.

best smooth  $\Delta\nu : m$  curve has a constant slope or a varying slope. But in the case of a long band series, such as the 3371 N series, the method of Least Squares can be applied, with great advantage, to this problem as well. For we may break up the  $\Delta\nu : m$  curve into several sections, and determine, by Least Squares, the most probable value of the slope for each section. With 30 or more lines to a section, the probable error of this procedure is far less than that of any graphical method,—a fact which is frequently overlooked, but which the present series of investigations has repeatedly demonstrated.

The author has tried also various approximate methods for computing the second difference and of testing its constancy, but has come finally to the conclusion that the Least Squares solution is the only really feasible procedure in a case like that under consideration, where a plotted  $\Delta\nu : m$  curve does not definitely indicate any curvature. Fortunately however it has been possible to very considerably reduce the amount of necessary calculations, so that the Least Squares method becomes but slightly longer than various approximate methods.

In equation (1) we have three undetermined coefficients, but in this particular case, we are interested only in the value of  $b$ . Moreover, the variable  $m$  takes on the regular series of values 1, 2, 3, etc. Now in the case of the simpler equation

$$y = A + Bx \quad (7)$$

it is possible to compute directly, by Least Squares, the value of  $B$ , when  $x = 1, 2, 3, \dots, n$ . In fact<sup>1</sup>

$$B = \frac{(n-1)(y_n - y_1) + (n-3)(y_{n-1} - y_2) + \dots}{1/6n(n^2 - 1)}. \quad (8)$$

The "physical meaning" of (8) is that we are to combine the first and last observations, the second and next to last, etc., and each difference thus obtained is to be multiplied by a factor equal to the length of the interval (in terms of the corresponding change in  $x$ ). Finally, we divide the numerator by the total number of corresponding changes in  $x$ . Thus for 6 observations, (8) can be written

$$B = \frac{5(y_6 - y_1) + 3(y_5 - y_2) + (y_4 - y_3)}{5^2 + 3^2 + 1^2}. \quad (8')$$

In the case of equation (1), it is to be expected that a similar formula for the value of  $b$  can be derived. But so far as the author has been able to ascertain, no such formula has been published. Accordingly this work has been carried out, and the desired expression obtained. As is to be

<sup>1</sup> See Kohlrausch, "Physical Measurements," p. 17 (English translation of 7th German edition).

expected, the necessary reductions and the final result are much more complicated than in the case of (7) and (8). This result is

$$b = \frac{30}{n(n^2 - 1)(n^2 - 4)} \sum \{ (y_r + y_{n+1-r})(A + [r-1]B + 6[r-1][r-2]) \}, \quad (9)$$

where  $A = (n-2)(n-1)$ ,

$B = 6(2-n)$ ,

$n$  = number of observations ( $y$ ) [ $y = \nu$  of (1)],

$r = \begin{cases} 1, 2, 3, \dots n/2 \text{ for } n \text{ even.} \\ 1, 2, 3, \dots (n+1)/2 \text{ for } n \text{ odd.}^1 \end{cases}$

Thus for  $n = 6$ , we have

$$b = \frac{5(y_6 + y_1) - (y_5 + y_2) - 4(y_4 + y_3)}{56}. \quad (9')$$

For  $n = 5$

$$b = \frac{2(y_5 + y_1) - (y_2 + y_4) - 2(y_3)}{14}. \quad (9'')$$

The "physical meaning" of (9) is this: The coefficients of the first and last observations, the second and next to last, etc., are the same. Hence the readings may be combined in pairs, as in the case of (8). (But note that the pairs of observations are added while in (8) they are subtracted.) The successive coefficients of these pairs of observations follow a parabolic law, in which the second difference is constant ( $= 12$ ). Hence the calculation of the coefficients is very simple. In (9)  $A$  is the initial coefficient, and  $B$  the initial first difference. Thus formula (9) may be written

$$b = \frac{30}{n(n^2 - 1)(n^2 - 4)} \{ A(y_1 + y_n) \\ + (A + B)(y_2 + y_{n-1}) + (A + 2B + 12)(y_3 + y_{n-2}) \\ + (A + 3B + 36)(y_4 + y_{n-3}) + \dots \}. \quad (10)$$

In applying (9) to the N3371 series, the 84 observed frequencies (all weighted equally) were divided into two equal sets. For the first 42 (small  $m$ 's) we obtain  $b = 3.681$ ; for the second 42 (large  $m$ 's)  $b = 3.605$ .<sup>2</sup> The mean is 3.643, very close to Lewis' value of 3.634, using the simplified equation (4).

Because of the large number of observations, the probable error in these results is very small. There is thus clear evidence of a variation

<sup>1</sup> Note that for  $n$  odd, the last term in the summation contains but one  $y$ , i. e.,  $y_{n+1/2}$  with the coefficient  $(1-n)(1+n)/2$ .

<sup>2</sup> The best approximate methods which the author has been able to devise give for the first 42,  $b = 3.645$  or  $3.660$ ; for the last 42,  $-3.585$  or  $3.555$ ,—showing the uncertainty of the methods.

of slope of about 2 per cent. But since 3.681 is the probable slope at the center of the first half of the series, while 3.605 applies to the center of the second half, we have a probable total variation, over the entire extent of the observations, of about 4 per cent. It is thus evident that the second difference *decreases*, and that the series shows deviations from Deslandres' law in the *negative* sense. Moreover, we have thus detected this 4 per cent. variation in a case where a really good analytic fitting of Deslandres' formula reveals practically no trace of systematic deviations. Even when such deviations *do* appear in the analytic work, they are difficult to interpret. This 3371 series has been used for illustration because it is the only really long series, among those studied, which from the plotted  $\Delta\nu : m$  curve *seemed* to obey the parabolic law.

In the case of more complex formulæ, such as the hyperbolic law proposed by the author, it is practically necessary to use the ordinary methods of calculation. After plotting the data and drawing the best smooth curve through the results, a number of points on this curve, equal to the number of arbitrary constants in the assumed formula, are chosen and the theoretical curve is made to pass through these points, by solving the simultaneous equations of condition. The Obs-Calc value for each observation is then calculated and plotted (as ordinate). A smooth curve is drawn through the results (the so-called "Residual Plot"<sup>1</sup>). The object is then to "warp" the  $x$ -axis until it fits this smooth curve as well as possible. With practice one can predict roughly whether it is possible to so warp the axis as to give a really good agreement. The criteria depend entirely upon the form of equation used, and general rules cannot be given. Each warping means, of course, the determination of a new set of constants. In the case of the hyperbola, four trials are usually sufficient, if the data are all equally trustworthy. For the  $B_1$  and  $C_1$  series of the 3883 CN band, many more trials were necessary, since the systematic deviations, to be discussed later, made it difficult to determine, at first, just what would constitute the best agreement. Much time was thus lost trying to do (in the light of future knowledge) quite impossible things.

Also, the hyperbolic formula refers to the  $\Delta\nu : m$  curve. The Obs-Calc values are however calculated and plotted for the  $\nu : m$  curve. The warping of the  $x$ -axis of the Residual Plot then consists in giving it a definite new *slope* (*i. e.*, a definite new  $\Delta\nu$ ) at selected points, and solving, instead of giving it a definite new *ordinate*. The judging of the proper slope is far more difficult than the judging of the proper ordinate, and this is the one real objection to the use of the "slope" form of the equa-

<sup>1</sup> Goodwin, Precision of Measurements, p. 60.



tion. But the greater simplicity of this form more than compensates. The fact that, for the proposed hyperbolic formula, the  $\nu : m$  equation is a summation, instead of an integral, is an objection to this particular formula but not to the use in general of the  $\Delta\nu : m$  curve, rather than the  $\nu : m$  curve.

#### THE HYPERBOLIC FORMULA, AND A POSSIBLE ALTERNATIVE FORM.

As shown in Fig. 2, page 92 of I., the  $\Delta\nu : m$  curves<sup>1</sup> for the  $A_1$ ,  $B_1$ , and  $C_1$  series of the 3883 CN band look very much like hyperbolæ, instead of the straight line demanded by Deslandres' formula. These three series have now been found to fit accurately hyperbolic curves, save for certain systematic perturbations (previously known) and for small systematic irregularities, newly discovered. These latter amount to a change of frequency of only one part in 300,000 as a *maximum*, and so are inconsequential as far as the general form of the curve is concerned. Aside from the real periodic irregularities, the observed and computed curves agree to at least 0.005 Å throughout a series 200 Å long, *i. e.*, on a  $\nu : m$  curve extending ten feet in each direction, the agreement is to less than 0.1 mm. throughout, indicating the great accuracy of the hyperbolic formula.

The deviations from Deslandres' law in all of these series are in a *negative* sense, in that the successive frequency differences increase *less* rapidly than demanded by the law, *i. e.*, the second differences *decrease*. Series of this type will be referred to as *negative series*, and bands containing them as "bands of the negative type." The phrase "negative bands" is already in use, with a distinctly different meaning. In I. the statement was made that in only one band are series known in which the deviations are in a contrary sense, *i. e.*, the 2370 band of air. This statement is quite incorrect. At the time of discovery, these 2370 series *were* the only known examples of what we shall call *positive series*. Since then a number of similar series have been found. It is these series which the author has been particularly studying. In the 2370 band there are four

<sup>1</sup> In all detailed quantitative work, the author has used the frequency in vacuo, but for the qualitative work to be described in the following paper the reduction to vacuo is not necessary and has not been made. Wave-lengths should never be used, except incidentally. In this connection it might be added, since it seems not known everywhere, that there is published a table of reciprocals (Cotsworth's Reciprocals,—McCorquodale and Co., London.) giving to seven figures the reciprocals of all numbers to 100,000. By linear interpolation to hundredths one can obtain the reciprocal of any 7 figure quantity to seven figures. The maximum error is one unit in the last figure, the probable error considerably less than half a unit. The rule for interpolating the seventh figure, as given in the tables, is obviously incorrect. The author has found it more convenient to use a slide rule for all interpolations. When a succession of adjacent frequencies is desired, as in a band series, each can be obtained in a very few seconds,—quite as rapidly, in fact, as by the use of the best calculating machines.

positive series, and four negative series. We shall call such a band a "neutral band." At present the 2370 band is I believe the only known example.

In the only detailed study of this band,<sup>1</sup> a very good drawing but no data on the observed wave-lengths are recorded. (Schniederjost<sup>2</sup> resolves only a few lines of this band.) The only data given are the Obs-Cal values for every fifth line, for four of the series, together with the equations used. With this data it is possible to work back to the observed frequencies, for every fifth line, and thus to determine roughly the form of the  $\Delta\nu : m$  curves. These curves show but slight deviations from a straight line, the maximum variation in slope being about 9 per cent. (for the  $II_2$  and  $IV_2$  series). That is, the final slope (highest  $m$ ) is about 109 per cent. or 92 per cent. respectively, of the initial slope (at  $m = 0$ ). The  $\Delta\nu : m$  curve for the  $A_1$  series of the 3883 band has been given in Fig. 2 of I. (page 92) and for convenience in comparing other series the slope at various points, in terms of the initial slope (*i. e.*, the relative second frequency differences) is given herewith.

$m = 0$	100	per cent.	$m = 101$	76.5	per cent.
21	98	per cent.	121	61.5	per cent.
41	95	per cent.	141	29.0	per cent.
61	91	per cent.	151	14.0	per cent.
81	85.5	per cent.	156.5	0	per cent. (max. $\Delta\nu$ )

In respect to the magnitude of the departure from Deslandres' law, the 2370 series are thus not in a class with the 3883 series, or with many others studied, and the fitting of them to a six constant equation, such as that proposed by the author, would be a simple matter. The striking point about these 2370 series is however the fact that four of them show positive deviations, and four negative, and the deviations in corresponding series are *equal*. This shows that if the *negative* series are hyperbolic (and the  $\Delta\nu : m$  curves look quite similar to the initial portion of the  $A_1$  3883 curve) the *positive* series are *also* hyperbolic. It also suggests a new point of view on the entire subject.

We have assumed in I. that the hyperbolic law is due to the fundamental field of force of the molecule (all spectroscopists now agreeing that band series are due primarily to molecular structure, rather than to atomic). Let us assume however that the *tangent to the  $\Delta\nu : m$  curve at  $m = 0$*  represents this fundamental field of force, *i. e.*, a Deslandres' law, and that the deviations from this straight line indicate disturbing effects. Let us denote any  $\Delta\nu$  on the straight line by  $\Delta\nu_s$  and any  $\Delta\nu$

<sup>1</sup> Deslandres and Kannapell, *Comptes Rendus*, 139, 584, 1904.

<sup>2</sup> *Zeitschrift für wiss. Photographie*, 2, 265, 1904.

on the hyperbola by  $\Delta\nu_n$ . Let  $\Delta\nu_s - \Delta\nu_n = D$ . Then if the  $\Delta\nu_n : m$  curve is truly an hyperbola, the  $D : m$  curve will also be an hyperbola (and conversely) since we have made only a linear transformation of variables in which the derivatives  $ab - 4c^2$  remains invariant. (The transformation is mathematically the same as that from fixed to moving axes.) Moreover, the curve in  $-D$  and  $m$  is also an hyperbola, by symmetry, and thus the curve in  $\Delta\nu_s - (-D) = \Delta\nu_p$  is an hyperbola. (The converse transformation of variables.) Thus the four *positive* series ( $\Delta\nu_p : m$  curves) are hyperbolæ, if the four *negative* series ( $\Delta\nu_n : m$  curves) are. Also, we can state that the systematic deviations ( $D$ ) from the fundamental field of force of the molecule (assumed to be such as to give a Deslandres' law) are of such nature as to yield hyperbolic  $D : m$  curves, instead of saying that the field of force is such as to yield hyperbolic  $\Delta\nu : m$  curves. The two statements are actually only different ways of viewing the phenomena, for the physical facts are the same, in either case.

The chief use that has been made of the above ideas is in the handling of positive series. It is difficult to recognize the hyperbolic form, in such series, and I have therefore drawn the tangent at  $m = 0$ , reversed the deviations ( $D$ ) from this line, and drawn the resulting curve, thus obtaining the corresponding negative  $\Delta\nu : m$  curve. This curve, in all cases, has appeared to be hyperbolic, as nearly as could be judged by mere inspection. This procedure also gives a simple method of judging, in the case of positive series, how radical are the departures from Deslandres' law. In only one series (one of those in the 3064 water band) does the corresponding negative  $\Delta\nu : m$  curve extend past the point of maximum  $\Delta\nu$ . In this one case it has about the extent of the  $A_1$  3883 curve. Thus no positive series shows anything like the deviation of the  $C_1$  3883 series.

The so-called positive series are thus found (qualitatively, at least) to be no exception to the hyperbolic law, but to represent simply *positive* hyperbolic deviations from Deslandres' law, instead of *negative* deviations. Needless to say, the structure of *positive* band series is somewhat different from that of *negative* series. The former has no point of maximum  $\Delta\nu$  and no "tail." The remarks on page 101 of I. should therefore be considered as applying only to *negative* series.

The hyperbolic law proposed by the author has the form

$$\Delta\nu^2 + B\Delta\nu.m + Cm^2 + D\Delta\nu + Em + F = 0, \quad (11)$$

where  $F = 0$  for the  $A_1$  3883 series.

Solving for  $\Delta\nu$  we have

$$\Delta\nu = -Bm/2 - D/2 \pm \sqrt{\left(\frac{B^2 - 4c}{4}\right)m^2 + \left(\frac{BD}{2} - E\right)m + \frac{D^2}{4} - F}, \quad (12)$$

where  $m = 0.5, 1.5, 2.5$ , etc., and the negative sign before the radical is to be used. Then

$$\nu_n = \nu_0 + [\Sigma\Delta\nu]_{m=0.5}^{m=n-0.5}. \quad (13)$$

Since  $\Delta\nu$  is given by an hyperbola we might write (13) as

$$\nu = \nu_0 + \Sigma \text{ hyperbola}. \quad (14)$$

As explained in I., pages 100-101, the best convergent series that approximates this function is probably

$$\nu = a + bm^2 - cm^4 + dm^6, \text{ etc.}, \quad (15)$$

which Kilchling<sup>1</sup> uses as an empirical formula, but which Ritz<sup>2</sup> derived on theoretical grounds.

The formal analogy between line and band series has frequently been pointed out. The approximate formula for line series (Rydberg) is

$$\nu = \nu_\infty - \frac{N_0}{(m + \mu)^2}, \quad (16)$$

or

$$\frac{N_0}{\nu_\infty - \nu} = (m + \mu)^2. \quad (17)$$

The approximate formula for band series is

$$\nu = A + B(m + \mu)^2 \quad (1')^3$$

or

$$\frac{\nu - A}{B} = (m + \mu)^2. \quad (18)$$

(18) and (17) have the same functional form, and hence the formal analogy, previously stated by various authors. The following suggestions however are believed to be mainly original.

There seem to be two possible methods of modifying the  $f(m)$  occurring in (17) and (18), in order to obtain more accurate formulæ. (1) Compute successive differences, in (17) and (18). This gives

$$\Delta\left(\frac{N_0}{\nu_\infty - \nu}\right) = 2(m + \mu), \quad (19)$$

$$\Delta\left(\frac{\nu - A}{B}\right) = \frac{\Delta\nu}{B} = 2(m + \mu). \quad (20)$$

<sup>1</sup> Zeitschrift für wiss. Photographie, 15, 293, 1916.

<sup>2</sup> Ritz (Weiss) Comptes rendus, 152, 585, 1911, or Astrophysical Journal, 35, 75, 1912.

<sup>3</sup> Here, as explained,  $A$  is *not* exactly equal to  $\nu_0$  (the true head) but differs from it by  $Bc^2$ . In giving the formal analogy between line and band series, equation (1') is usually written with  $\nu_0$  in place of  $A$ . But to avoid inconsistency, this has not been done here.

But the author has found that  $\Delta\nu$  = hyperbola, instead of  $\Delta\nu$  = straight line. By analogy we might expect for line series

$$\Delta\left(\frac{N_0}{\nu_\infty - \nu}\right) = \text{hyperbola.} \quad (21)$$

The other method is (2) Compute the square roots of (17) and (18). This gives

$$\sqrt{\frac{N_0}{\nu_\infty - \nu}} = m + \mu, \quad (22)$$

$$\sqrt{\frac{\nu - A}{B}} = m + \mu. \quad (23)$$

In the case of line series, this second procedure has always been followed. The  $\sqrt{N_0/\nu_\infty - \nu}$  instead of being a straight line, is actually a much more complex  $f(m)$ . Nicholson<sup>1</sup> concludes that it must be some  $f(m + \mu)$ . Ritz uses  $f(m) = m + a + b/m^2$ . Hicks uses  $m + a + b/m$ . Nicholson,<sup>1</sup> in order to get accurate agreement for even the simple Helium series, has had to use

$$m + \mu + \frac{a}{m + \mu} + \frac{b}{(m + \mu)^2} + \frac{c}{(m + \mu)^3}.$$

But *possibly* this  $f(m)$  is an hyperbola. By analogy we would have, for band series,

$$\sqrt{\frac{\nu - A}{B}} = \text{hyperbola, or } \nu = A + (\text{hyperbola})^2 \quad (24)$$

where  $B$  has been incorporated into the hyperbola. By comparing (24) with (14) it is seen that we have thus replaced the summation by a square, and have thus greatly simplified the necessary calculations.

Formula (24) has been carefully tested on the  $A_1$  and  $C_1$  series of the 3883 band. It *seems* to fit these series almost as well as does (14). More trials might possibly result in an equally good agreement. If either the  $A_1$  or  $C_1$  series were slightly longer, it would be possible to decide definitely between the two formulæ. The  $C_1$  series extends much further (in terms of curvature) than the  $A_1$ , but because of the greater size of the perturbations and other irregularities of the  $C_1$  series, there is a greater uncertainty as to the best position of the theoretical curve. The two formulæ can be made to give practically identical results for small values of  $m$ , but not for larger values.

The methods of handling the two formulæ are quite different, but only the following brief statement will be made at this time. With formula (24) it is possible to handle the data directly in terms of  $\nu$  and  $m$ .

<sup>1</sup> Proc. Roy. Soc., (A), 91, 255, 1915.

But since the computations necessary in solving six simultaneous equations are several times as extended as those necessary for five, the author has preferred to work with the hyperbola, *i. e.*, with the curve in  $\sqrt{\nu - A}$  and  $m$ . Now the experimental values of  $\sqrt{\nu - A}$  are indeterminate, since  $A$  is as yet unknown. In the case of line series [compare (22) with (23)] it is possible to determine  $\nu_\infty$  with great accuracy, regardless of the type of function used. The same thing is roughly true for  $A$  in band series. Any simple formula, such as Deslandres', applied to the known smaller values of  $m$ , will give an approximate value of  $A$ . It is moreover known that the curve in  $\sqrt{\nu - A}$  and  $m$  must approach linearity, with decreasing  $m$ , since all band series approach Deslandres' law in this region, and this curve is very sensitive to small changes in  $A$ , when  $m$  is small. It is thus possible to determine accurately the value of  $A$ , necessary to get even approximate linearity. All computations can be made with a 20-inch slide rule. It is not necessary to have a good agreement between the observed and computed  $\sqrt{\nu - A} : m$  curves, for  $m$  small, since in this case a relatively large change in  $\sqrt{\nu - A}$  produces a relatively small change in  $\nu - A$ . The converse is true for large values of  $m$ , so that in this region the observed and computed  $\sqrt{\nu - A} : m$  curves must agree very exactly.

By using the above method, the time required to determine a given set of constants is the same for the two formulæ. But for the new formula the time required for testing the constants is far less, and this constitutes, with the old formula, the major portion of the work. Since either of the formulæ give practically satisfactory results for the 3883 series, it is immaterial which form is used, in testing other simpler series. But since the original form allows the data for the hyperbolic curve ( $\Delta\nu : m$ ) to be computed directly from the observations, this form has been used in all the general work to date. Each form has six coefficients, or five, if the hyperbola cuts the origin.

In lines series, by analogy, we would have instead of (22)

$$\nu = \nu_\infty - \frac{N_0}{(\text{hyperbola})^2}. \quad (25)$$

The author has plotted the experimental curves, for certain well-known line series, in terms of  $\Delta(N_0/\nu_\infty - \nu)$  and  $m$ , and also in terms of  $\sqrt{N_0/\nu_\infty - \nu}$  and  $m$ . The hyperbolæ (if either of the curves are such) are reversed, compared to band series, approaching linearity for large values of  $m$ , while the vertex occurs near the origin.<sup>1</sup> Unfortunately, line series do not

<sup>1</sup> The limiting slope ( $m = \infty$ ) equals *two*, for the first form, or *unity*, for the second, regardless of the equation used, and thus should be the same for all line series, if  $N_0$  is truly a universal constant. (Rydberg Constant). In band series one finds a great diversity of initial slopes and there is no such corresponding universal constant.

yield themselves to graphical methods, for if an error of 0.01 Å in term  $m = 2$  is represented by, let us say, 0.01 mm., an equal error in  $m = 30$  may be represented by about 5 mm. or more.

The author is not prepared, at the present time, even to suggest the use of either (21) or (25) for line series. The present trend of line series work is to derive formulæ directly from theoretical considerations. Because of the recent success of such efforts, by the use of various modifications of the Bohr atom,<sup>1</sup> it does not seem desirable to introduce any new purely empirical formula. In the case of band series, we should expect, from theoretical considerations, a formula of the type of (14) rather than of (24), since any application of the quantum idea must involve a summation, implicitly if not explicitly.

#### ADDENDUM.

The author has just received a reprint of a long paper by T. Heurlinger entitled "Untersuchungen Über Die Struktur der Bandenspektren" (Lund, 1918). A small portion of Heurlinger's work duplicates work (unpublished) of the author. In general however the ground covered is different. Heurlinger is not primarily interested in a new type of formula. He uses simply the power series (15), or a modification, and compares the numerical constants, for the various series. His main object is to relate together all the series in any one band, or group of bands, and to thus formulate general rules regarding band structure. The paper contains several ideas on band structure, radically new, but very suggestive, and apparently supported by a large amount of experimental evidence. The author intends to discuss these ideas more fully in a future paper.

UNIVERSITY OF CALIFORNIA,  
December, 1918.

<sup>1</sup> See particularly H. S. Allen, Proc. London Phys. Soc., 30, 127, 1918. Vegard, Phil. Mag., 35, 293, 1918. Ishiwara, Proc. Math. Phys. Soc. of Tokio, (2), 8, 106, 1915; (2), 8, 173, 1915; (2), 8, 540, 1916; (2), 9, 20, 1917; (2), 9, 160, 1917.

## THE PASSAGE OF PHOTOELECTRONS THROUGH METALS.

BY K. T. COMPTON AND L. W. ROSS.

SYNOPSIS.—Theoretical expressions are derived giving the rate of escape of photo-electrons from the surface of a thin metallic film as a function of its thickness. These expressions result from various assumptions regarding the probability of escape of an electron photoelectrically excited at a given depth within the metal.

The photoelectric currents from films of platinum and of gold of various thicknesses are measured for different wave-lengths of light, and the results are found to be consistent with the view that the average distance in any direction which an excited electron may move without losing its ability to escape is about  $2.67 (10)^{-7}$  cm. in platinum and  $5.0 (10)^{-7}$  cm. in gold, and that the probability of going a given distance without losing its ability to escape falls off exponentially with the distance. This average distance is shown to be independent of the initial kinetic energy of the electron. There is evidence that platinum is first deposited in a relatively unstable and electropositive state, and that it spontaneously changes into the ordinary stable form.

A critical discussion and comparison of methods of measuring film thicknesses shows that they may be relied upon if proper precautions are taken. Various interesting properties of these thin films are illustrated and discussed.

INTRODUCTION.—Ladenburg<sup>1</sup> was the first to prove that electrons, excited photoelectrically within a metal by ultra-violet light, may travel some distance through the metal before escaping from its surface. He deposited nickel electrolytically on glass, and showed that the photoelectric currents from the films increased with increasing film thickness up to about  $2(10)^{-4}$  cm. This result was qualitatively confirmed by Rubens and Ladenburg,<sup>2</sup> who showed that the ratio of the photoelectric currents on the incident and emergent sides of a piece of gold foil was about 100 : 1, while the ratio of the incident and emergent light intensities was about 1,000 : 1.

In 1913, Partzsch and Hallwachs<sup>3</sup> published a criticism of the quantitative results of Ladenburg's measurements, and also made a theoretical and experimental study of the optical and photoelectric properties of thin metal films deposited on quartz. The theoretical treatment in the present paper is based on, and is an extension of, the work of Partzsch and Hallwachs.

The experimental measurements here recorded are in many respects

<sup>1</sup> Ann. d. Phys., 12, p. 558, 1903.

<sup>2</sup> Ber. d. D. Phys. Ges., p. 749, 1907.

<sup>3</sup> Ann. d. Phys., 41, p. 247, 1913.



similar to measurements published by Robinson,<sup>1</sup> and in the main confirm his results. Certain refinements had to be added to his experimental procedure, however, in order to attain sufficient accuracy to permit a quantitative application of the theory.

METHOD.—The general plan of the experiment was to allow ultra-violet light of definite wave-length to enter a metal film, deposited on a quartz plate, from the quartz side and to measure the total photoelectric emission from the film. This was done with films of various thicknesses, maintaining other conditions constant, so that the photoelectric emission was obtained as a function of the film thickness. The nature of the variation of emission with thickness depends on the distance rate of absorption of light in the film and on the probability that an electron, photoelectrically excited by the light at any point in the film, may reach and escape from the surface of the film. The rate of absorption of light in the film is known, or may easily be measured. Thus the experimental results depend ultimately on the probability of the escape of an electron from any point in the film. Various assumptions regarding the form of the expression of this probability, associated with various assumptions regarding the passage of electrons through the metal, may therefore be tested by the experimental results. In the following section several such assumptions are put into forms suitable for quantitative tests.

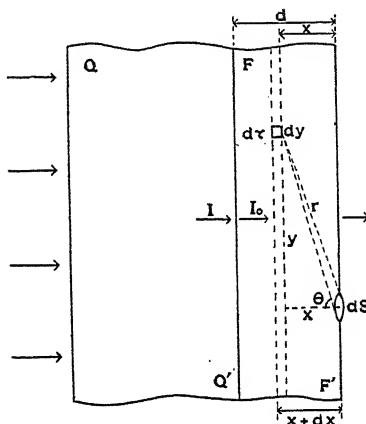


Fig. 1.

THEORY.—Let  $QQ'$  and  $FF'$  in Fig. 1 represent sections of the quartz plate and metal film, respectively, the thickness of the latter being  $d$ . Light of intensity  $I$  strikes the film from the quartz side, but part of it is reflected, so that  $I_0$  represents the intensity of the light which enters the film. Since the reflecting power of the film varies with its thickness,  $I_0$  is a function of  $d$ .

Let  $dx$  be the thickness of a layer of the film distant  $x$  from its free surface. The intensity of the light in this layer is  $I_0 e^{-\alpha(d-x)}$ , where  $\alpha$  is the coefficient of absorption of the light. If we let  $\nu$  represent the number of electrons which are photoelectrically excited per unit time, per unit volume, per unit light intensity, then the rate at which electrons

<sup>1</sup> Phil. Mag., 25, p. 115, 1913; *ibid.*, 32, p. 421, 1916.

are excited in the layer  $dx$  is

$$\nu A I_0 e^{-a(d-x)} dx,$$

where  $A$  is the area of the film. From symmetry it is evident that half of these start moving toward the free surface.<sup>1</sup>

Let  $F(x)$  be the probability that an electron will move a distance whose component normal to the surface is  $x$  without losing its ability to escape. Then the rate of escape from the film of electrons which start from the layer  $dx$  is

$$dN = \frac{\nu A I_0}{2} e^{-a(d-x)} F(x) dx.$$

The total rate of emission of electrons is therefore

$$N = \frac{\nu A I_0}{2} e^{-ad} \int_0^d e^{ax} F(x) dx. \quad (1)$$

SPECIAL CASES.—(a) Assume that the number of electrons which retain ability to escape falls off exponentially with distance moved normally to the surface, or

$$F(x) = e^{-\beta x}, \quad (2)$$

where  $1/\beta$  is the average distance which an electron can move normally to the surface without losing its ability to escape. Then the integration in equation (1) is easily performed, giving

$$N = \frac{\nu A I_0}{2} \frac{e^{-ad} - e^{-\beta d}}{\beta - a}, \quad (3)$$

which is identical with the equation derived by Partzsch and Hallwachs.<sup>2</sup>

(b) Assume that the number of electrons which retain ability to escape falls off exponentially with the distance moved (in any direction).—Consider those electrons which start toward the surface from an element of volume  $d\tau$  in the layer  $dx$  and escape from an element of the free surface  $dS$ . Of these, the fraction  $dS \cos \theta / 2\pi r^2$  start toward  $dS$ ; but of these, only the fraction  $e^{-\gamma r}$  succeed in escaping, according to our present assumptions. Thus

$$\frac{dS \cos \theta}{2\pi r^2} e^{-\gamma r}$$

<sup>1</sup> This assumes that the electrons begin to move with equal probabilities of motion in all directions and therefore in directions which are independent of the orientation of the electric vector in the light wave. There is possibly some experimental evidence that this assumption is not strictly true. If the electron is ejected from the atom by a direct pull from the electric field of the light wave the present assumption is probably seriously in error. If, however, the photoelectric emission is a secondary effect caused by a storing up of energy in the atom and its consequent instability, the assumption is valid.

<sup>2</sup> *Loc. cit.*

is the probability that an electron, starting toward the surface from  $d\tau$ , will escape from  $dS$ . It is to be noted that this probability is the same for electrons starting from any point in  $dx$  at a distance between  $y$  and  $y + dy$  from  $O$ . Thus the probability that an electron, starting toward the surface from any point in the layer  $dx$ , will escape from  $dS$  is

$$\frac{1}{A} \int_{y=0}^{y=\infty} \frac{dS \cos \theta}{2\pi r^2} e^{-\gamma r} \cdot 2\pi y dy,$$

since the layer may be considered to extend to infinity in comparison with the depth  $x$ . Finally, the probability that an electron, starting toward the surface from  $dx$ , will escape, is found by substituting  $A$  for  $dS$ , and is

$$F(x) = \int_0^\infty \frac{\cos \theta}{r^2} y e^{-\gamma r} dy.$$

This expression may be written

$$F(x) = x \int_x^\infty \frac{1}{r^2} e^{-\gamma r} dr,$$

of which the solution is

$$F(x) = e^{-\gamma x} + \gamma x Ei(-\gamma x), \quad (4)$$

where  $Ei(-\gamma x)$  represents the exponential integral

$$Ei(-\gamma x) = - \int_x^\infty \frac{1}{r} e^{-\gamma r} dr,$$

whose value for various values of  $(\gamma x)$  is given in Laska's "Sammlung von Formeln."

On this assumption, therefore, the total rate of electron emission from the film should be, by equation (1),

$$N = \frac{\nu A I_0}{2} e^{-\alpha d} \int_0^d e^{\alpha x} [e^{-\gamma x} + \gamma x Ei(-\gamma x)] dx. \quad (5)$$

The integral of this expression may be easily evaluated by graphical methods.<sup>1</sup>

<sup>1</sup> This equation may be integrated by parts after substituting for  $Ei(-rx)$  its alternative form given above. In performing this integration it is necessary to express the limits of integration of one of the terms by

$$\int_0^d f(x) dx = \int_0^\infty f(x) dx - \int_d^\infty f(x) dx$$

and to remember that

$$\int_0^\infty \frac{e^{-rx} - e^{-(r-\alpha)x}}{x} dx = \log \frac{r-\alpha}{r}.$$

We thus find for the solution

$$N = \frac{\nu I_0 A}{2} \left\{ \frac{1}{\alpha} (e^{-rd} - e^{-\alpha d}) + r \left( \frac{d}{\alpha} - \frac{1}{\alpha^2} \right) Ei(-rd) + \frac{r}{\alpha^2} e^{-\alpha d} Ei(-(r-\alpha)d) + \frac{r}{\alpha^2} e^{-\alpha d} \log \frac{r}{r-\alpha} \right\}.$$

In this expression,  $1/\gamma$  represents the average distance in any direction which an electron may move without losing its ability to escape.<sup>1</sup>

(c) Assume that an electron loses energy in proportion to the distance moved through the metal.—Let  $\sigma$  be the energy lost per centimeter path. The electron starts with an amount of energy  $h\nu$ , where  $h$  is Planck's constant and  $\nu$  is the frequency of the exciting light; of this energy, an amount  $h\nu_0$  is necessary to enable it to escape from the surface of the metal. Therefore

$$\sigma r_0 = h(\nu - \nu_0) \quad (6)$$

determines the greatest distance  $r_0$  which an electron can move in the metal without losing its ability to escape. As in case (b), the fraction  $dS \cos \theta / 2\pi r^2$  of the electrons starting toward the free surface from  $d\tau$  move toward  $dS$ . All these will escape if  $r < r_0$ , and none if  $r > r_0$ . Thus the probability that an electron starting toward the surface from the layer  $dx$  will escape from  $dS$  is

$$\frac{1}{A} \int_0^{\sqrt{r_0^2 - x^2}} \frac{dS \cos \theta}{2\pi r^2} \cdot 2\pi y dy = \frac{dS}{A} \left( 1 - \frac{x}{r_0} \right)$$

if  $x < r_0$  and is zero if  $x > r_0$ . Therefore

$$F(x) = \begin{cases} \left( 1 - \frac{x}{r_0} \right) & \text{if } x < r_0; \\ 0 & \text{if } x > r_0. \end{cases} \quad (7)$$

By equation (1), if  $d > r_0$ ,

$$\begin{aligned} N &= \frac{\nu A I_0}{2} \epsilon^{-ad} \int_0^{r_0} \left( 1 - \frac{x}{r_0} \right) \epsilon^{ax} dx \\ &= \frac{\nu A I_0}{2} \epsilon^{-ad} \left[ \frac{1}{\alpha^2 r_0} (\epsilon^{ar_0} - 1) - \frac{1}{\alpha} \right], \end{aligned} \quad (8)$$

Although this equation permits numerical computations to be made, such computations are neither as rapid nor as accurate as those made by a graphical solution of equation (5). The inaccuracy is due to the fact that the value for  $N$  is expressed as the difference between relatively large terms, and the tables of  $Ei(-rx)$  are not sufficiently accurate to give the difference with high percentage accuracy.

<sup>1</sup> We have implicitly assumed that the "ability to escape" depends on the velocity but not on the direction of motion of an electron at the surface. This is objectionable, since the chance of escaping against the attraction of the induced positive charge left on the film is certainly dependent on direction as well as speed of motion of an electron. The mechanism of "escape" and the nature of the work done in escaping is so little understood, however, that we have neglected this factor. If most of the work done in escaping is done within distances from the surface which are of atomic order of magnitude, the present assumption will probably not be seriously in error. It can furthermore be shown that the relations between  $N$ ,  $\alpha$  and  $r$  are such as to reduce any error from this cause to a minimum.

and if  $d < r_0$

$$N = \frac{\nu A I_0}{2} \epsilon^{-\alpha d} \int_0^d \left(1 - \frac{x}{r_0}\right) \epsilon^{\alpha x} dx$$

$$= \frac{\nu A I_0}{2} \left[ (1 - \epsilon^{-\alpha d}) \left( \frac{1}{\alpha} + \frac{1}{\alpha^2 r_0} \right) - \frac{d}{\alpha r_0} \right]. \quad (9)$$

*Thickness of Film giving Maximum Photoelectric Emission.*—It is much easier, in practice, to determine or test values of  $\beta$ ,  $\gamma$  and  $\sigma$  by finding the thickness of film giving maximum emission than by attempting to fit the entire graphs of the theoretical equations to the experimental curves. We shall therefore discuss the conditions under which maximum emission is obtained in the three cases considered above.

(a) By equating to zero the derivative of equation (3) with respect to film thickness  $d$  we find the thickness  $d_m$  for maximum emission to be given by

$$d_m = \frac{1}{\beta - \alpha} \log \frac{\beta - \left( \frac{1}{I_0} \frac{\partial I_0}{\partial d} \right)_m}{\alpha - \left( \frac{1}{I_0} \frac{\partial I_0}{\partial d} \right)_m}, \quad (10)$$

in which the term  $1/I_0 \partial I_0 / \partial d$  arises from the variation of the reflecting power of the quartz-film surface with thickness and may be determined experimentally, as described later. Taking experimental values of this term and of the coefficient of light absorption  $\alpha$  we find the values of  $d_m$  for various values of  $\alpha$  and  $\beta$  shown graphically in Fig. 2. If  $d_m$  and  $\alpha$

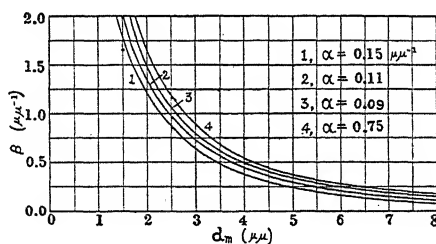


Fig. 2.

are determined experimentally, the appropriate value of  $\beta$  may be found from these curves.

(b) To find the maximum value of  $N$  from equation (5) the easiest method is to actually plot the equation, as is done in Fig. 3 for various values of  $\gamma$ .<sup>1</sup> From these and similar curves the values of  $d_m$  appropriate

<sup>1</sup> In Fig. 3 the curves for various values of  $r$  are drawn so as to merge into the curve giving the percentage of transmitted light, thus calling attention to the fact that for thick films the photoelectric emission varies with thickness in practically the same manner as does the transmitted light. To do this, however, the four curves had to be plotted to four different scales. If drawn to the same scale the maximum values of  $N$  for the four curves 1, 2, 3 and 4 would be 21.3, 39.9, 70.9 and 121.7 respectively.

to various values of  $\gamma$  and  $\alpha$  have been determined and plotted in Fig. 4, whence  $\gamma$  may be found if  $\alpha$  and  $d_m$  are experimentally determined.

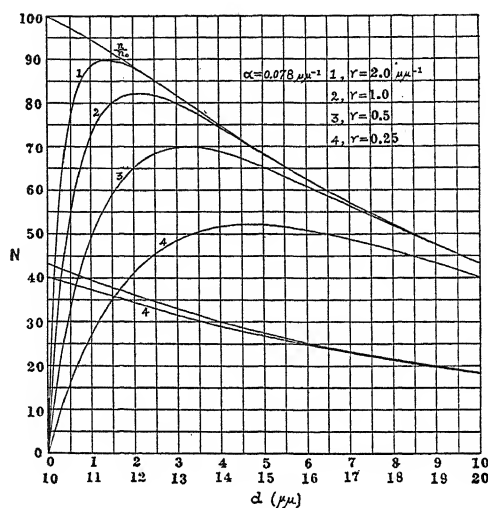


Fig. 3.

(c) In case energy is lost in proportion to the distance moved through the metal, the thickness for maximum emission is given by differentiation of equation (9), whence

$$d_m = \frac{1}{\alpha} \log (1 + \alpha r_0), \quad (11)$$

where  $r_0$  is defined by equation (6) as the greatest distance which an electron can go through the metal without losing its ability to escape.

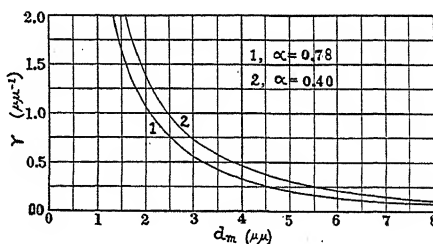


Fig. 4.

Since  $\alpha d_m$  is an extremely small quantity, equation (11) may be written without appreciable error,  $\alpha d_m = \alpha r_0$ , whence, by equation (6)

$$d_m = \frac{h(\nu - \nu_0)}{\sigma}. \quad (12)$$

If this were the actual type of energy loss, the thickness of film giving maximum current should be proportional to  $(\nu - \nu_0)$ , which is the difference between the frequency of the exciting light and the least frequency of light which will produce photoelectric emission.

**APPARATUS.**—In order to test the foregoing assumptions and to measure some of the quantities involved, metallic films were deposited on quartz plates by the method of "sputtering," in a vessel whose essential features are shown by Fig. 5. Transparent quartz plates one centimeter square and one millimeter thick were carefully cleaned in boiling caustic potash, nitric acid and distilled water and mounted by a touch of sealing wax over holes in a brass disk, as at *P*. Twelve such plates were mounted on the disk, which was then placed in the vacuum chamber shown in Fig. 5, so that the plates could be placed in turn under the cathode *C* by turning the ground glass joint *J*. A stationary brass shield *SS'* was held just above the disk *NN'* so as to shield from the discharge all the plates except the one *P* below the cathode. The vessel was rendered air tight by melting wax in the groove *GG'*. Great care was taken to keep the inside of the vessel free from wax or grease, and where wax was necessary it was entirely shielded from the discharge. The exhaust tube was connected through a liquid air trap to a McLeod gauge and Gaede pump. The source of the discharge current was a Snook high potential transformer designed to give a very steady discharge.

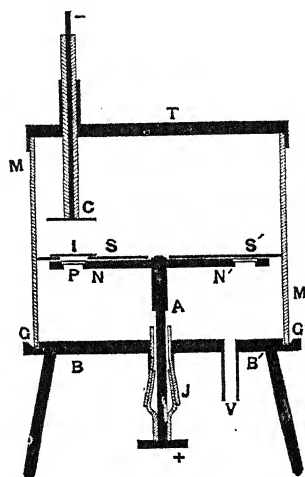


Fig. 5.

**Measurement of Thickness of Films.**—Three methods were employed to measure the thickness of the films. Of these, the most direct method was to weigh the thickest film, thence calculate its thickness and take the thickness of the other films proportional to their times of exposure to "sputtering." B. Pogány<sup>1</sup> has shown that this method is accurate if the sputtering conditions are maintained constant. In order to have constant conditions we adopted the following procedure. An iron disk *I* was placed over the hole in the shield *SS'* so that none of the quartz plates was exposed to the discharge. The discharge in the vessel was then continued for about two hours, until no more gas could be detected coming from the walls of the vessel under the influence of the discharge.

<sup>1</sup> Phys. Zeit., 15, p. 685, 1914.

Then the iron disk  $I$  was removed by a magnet and the plates were exposed successively to the discharge. They were exposed in the following order: 1, 2, 3,  $\dots$ , 11, 12, 12, 11,  $\dots$ , 2, 1, 1, 2,  $\dots$ , 12, 12,  $\dots$ , 1; so that the effect of any change in sputtering conditions would be evenly distributed among the plates. During the entire process the gas pressure was maintained constant by a regulating valve. Ordinarily, the times of deposit ranged from 1 minute to 100 or more minutes.

The second method employed involved a measurement of the absorption of light in the films. The disk  $NN'$  was mounted on an optical bench so that light from a mercury arc could be focused on any one of the films. The reflected or transmitted light was focused on the slit of a Hilger monochromatic illuminator provided with a linear thermopile, which served to measure the intensity of the light. By thus determining the intensity of the incident light, the reflection and absorption by the quartz plate and the intensities of the lights transmitted and reflected by the film covered plates, we found the fraction of the light entering the film which was absorbed by it. The coefficient of absorption of light of the wave-lengths used,  $576.9 \mu\mu$  and  $579.1 \mu\mu$ , is accurately known, whence the thickness of each film could be calculated.

The third method involved the use of data concerning the variation of reflection of ultra-violet light with thickness of platinum films on quartz, published by Partzsch and Hallwachs.<sup>1</sup> When light strikes a platinum film from the quartz side, they found that the intensity  $I_0$  of light entering the film is less than the intensity  $I$  of light incident on the quartz by an amount shown by curve 1, Fig. 6, for films of various thicknesses  $d$ . Ninety-two per cent. of the incident light is transmitted by a blank plate. From this we find that the ratio of the intensity of light transmitted by a blank plate to that transmitted by a plate with film of thickness  $d$  is given by

$$\frac{n}{n_0} = \frac{I_0}{.92} e^{-\alpha d}, \quad (13)$$

in which  $\alpha$  is known with considerable accuracy for the wave-length  $253.6 \mu\mu$  as a result of a slight extra-polation of measurements made by Meier.<sup>2</sup> Taking  $\alpha = .078 \mu\mu^{-1}$ , we obtain curve 2, Fig. 6. The intensity of the light transmitted by any film was taken to be proportional to the photoelectric current produced when this light fell upon a plate  $P$  (Fig. 7) which could be placed in the path of the light. The ratio of this current to that produced by light which has passed through a blank plate was therefore equal to  $n/n_0$ . Thus the thickness of the film  $d$  could be

<sup>1</sup> *Loc. cit.*

<sup>2</sup> *Ann. d. Phys.*, 31, p. 1017, 1909.



found directly from curve 2, Fig. 6, as soon as the ratio  $n/n_0$  was determined.

The reliability of these methods of estimating film thickness is proven by a comparison of their results when applied to the same set of films.

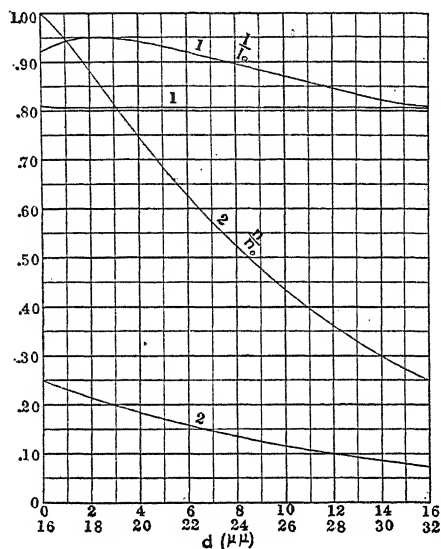


Fig. 6.

In Table I.,  $t$  represents the time of sputtering and  $d_1$ ,  $d_2$ ,  $d_3$  are the thicknesses calculated by the three methods, respectively. The area

TABLE I.

No.	$t$ (sec.).	$d_1$ ( $\mu\mu$ ).	$d_2$ ( $\mu\mu$ ).	$d_3$ ( $\mu\mu$ ).
1	0	0	0	0
2	20	1.5	1.8	1.7
3	30	2.3	2.5	2.5
4	40	3.1	3.0	3.0
5	50	3.8	3.8	3.9
6	60	4.5	4.4	4.6
7	70	5.3	5.5	5.5
8	80	6.0	6.5	6.2
9	90	6.8	6.8	6.6
10	120	9.0	8.6	8.6
11	150	11.3	11.8	11.6
12	6000	452.0	?	?

of plate no. 12 was 0.95 sq. cm. and the mass of film deposited on it was 0.000925 gm. This film was so opaque that no estimates of its thickness could be made by the second and third methods. The experimental

data which were used, with Curve 2, Fig. 6, to find the values of  $d_s$  are found in Table II.

Of these methods, the third was most convenient, and was used for most of the work with platinum films. The data for its use in dealing with gold films are lacking, so that we employed the second method in this case.

*Photoelectric Measurements.*—The photoelectric properties of these films were tested in the brass vessel shown in Fig. 7. Electrical contact between the films and the brass disk  $NN'$  was made by gold leaf, cemented with a little India ink. The disk was mounted on a shaft  $A$  in such a

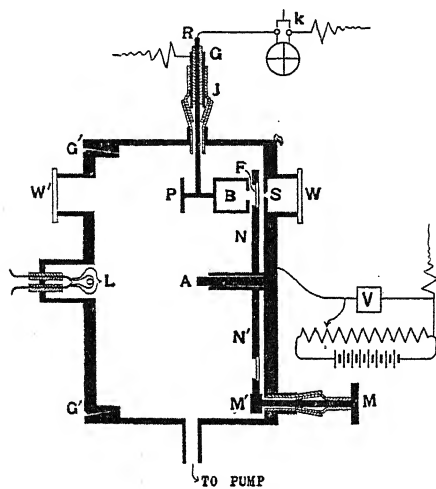


Fig. 7.

manner that any film  $F$  could be placed in the path of ultraviolet light from a quartz mercury arc and a Hilger monochromatic illuminator. This light entered through a quartz window  $W$ , was brought to a focus at the slit  $S$ , passed through the quartz plate and film and entered the hollow brass box  $B$ . This box was designed to reduce the amount of light returning to the film after reflection from the box, and a calculation proved that less than one per cent. of the photoelectric current from the

film could be due to light reflected from the box.

The photoelectric current  $N$  from the film  $F$  passed to the box  $B$  which served as the receiving electrode, and was connected with an electrometer which was used at a sensitiveness between 2,000 and 50,000 mm. per volt, depending on the type of measurement being made. The vessel and films were charged to a negative potential of about 10 volts, indicated by the voltmeter  $V$ , and the insulation of the electrometer system was protected by the guard ring  $G$ .

In order to measure the intensity of the light transmitted by the film, the platinum plate  $P$  was turned to the position of the box  $B$  and the potential of the vessel changed to +10 volts, so that the electrometer measured the photoelectric current  $n$  from the plate.

To facilitate quick substitution of one film for another, a second toothed wheel  $M'$ , which could be rotated by the outside handle  $M$ , was made to engage the toothed disk  $NN'$ . A small lamp  $L$  illuminated the

inside of the vessel, and the adjustment of any film to the position  $F$  was visible when looking through the window  $W'$ .

EXPERIMENTAL RESULTS. PLATINUM.—Ten sets of platinum films were tested, each set being examined to determine (a) the thickness of film giving maximum photoelectric emission, (b) the effect of using different wave-lengths of exciting light and (c) the nature of changes which occur in the films after they are deposited on the quartz plates. These points will be taken up separately.

(a) *The Thickness of Film Giving Maximum Photoelectric Emission.*—Table II. contains typical experimental measurements. The numbers

TABLE II.

No.	$N_1$	$N_2$	$N$	$n_1$	$n_2$	$n$	$d(\mu\mu)$
1	0	0	0	191	186	188	0.0
2	47	45	46	170	168	169	1.7
3	74	68	71	162	158	160	2.5
4	83	82	82	153	152	152	3.0
5	81	81	81	145	139	142	3.9
6	77	72	75	134	131	133	4.6
7	67	68	67	122	124	123	5.5
8	65	62	63	117	113	115	6.2
9	67	67	67	112	112	112	6.6
10	58	60	59	92	93	93	8.6
11	31	29	30	71	71	71	11.6
12	0	1	1	0	0	0	?

in the columns marked  $N$  refer to the electrometer deflections per half minute due to the photoelectric currents from the films. Those marked  $n$  refer to currents from the plate  $P$  and measure the intensity of the light transmitted by the films; these data are used with Fig. 6 to determine the film thickness  $d$ . In order to avoid error due to "time" changes, such as photoelectric fatigue, all measurements were taken in order 1, 2, 3, ..., 11, 12 and back again 12, 11, ..., 2, 1, as shown in the columns marked with subscripts 1 and 2. The means of these measurements are given in columns  $N$  and  $n$ .

These results are shown graphically in Fig. 8 (c). It is seen that there are two maxima, instead of the one predicted by the theory. It might be thought that this is due to some erratic photoelectric condition of films 9 and 10, if it were not for the fact that they were found in every one of the ten sets of platinum films tested. Other examples are shown in curves (a), (b) and (d), in which the ordinates have been changed by constant factors in order to avoid confusion in plotting. It appears that there is one maximum at  $d = 3.75 \mu\mu \pm .25 \mu\mu$  and another in the neighborhood of  $d = 7.5 \mu\mu \pm 1.0 \mu\mu$ .

Numerous tests were made in an attempt to discover the cause of the second maximum. For one set of films mercury vapor was allowed free access to the sputtering and testing vessels, while in other tests all parts were carefully cleaned and mercury vapor excluded by liquid air traps, but no difference was observed. Apparently it made no difference

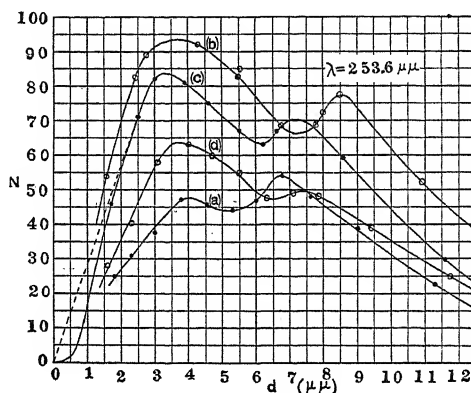


Fig. 8.

whether the films were allowed to stand in air or in *vacuo*. We have concluded that both maxima are really due to properties of the platinum films. The interpretation of these maxima is suggested by experiments discussed in the following sections.

One other feature of these curves is of interest. All the curves near the origin show an inflection, as if the photoelectric currents increased with thickness of film, first slowly and then more rapidly. If the quantity  $\nu$  in our theoretical equations were constant, this portion of the curve would be straight, indicating proportionality between emission and thickness. Robinson,<sup>1</sup> who has noticed this phenomenon, has supposed that it indicates less intrinsic photoelectric sensitiveness for thin than for thicker films. We have been able to prove, however, that  $\nu$  really is constant, and that the inflection in the curves is due to a secondary cause, viz., lack of continuity of very thin films. When a very thin film is first exposed to the ultra-violet light, the rate of electrometer deflection falls off rapidly as the exposure is continued. If the light is shut off the film regains its original apparent sensitiveness only after a lapse of some minutes. If the *initial* rate of deflection instead of the total deflection in a relatively long time interval is used to determine  $N$ , it is found that the values of  $N$  tend to lie on the theoretical straight line, such as the dotted line in curve (c). This is much more marked

<sup>1</sup> *Loc. cit.*

in the case of gold films than in the case of platinum films—which is in line with the observation that a greater thickness of gold than of platinum is required to give an electrically conducting layer.<sup>1</sup> The upward concavity of the curves near the origin is apparently to be explained, therefore, by the effect of isolated regions of the film which quickly charge up to such positive potentials as to reduce the rate of emission from these and contiguous parts of the film. This phenomenon is easily detected by the failure to get electrometer deflections which are proportional to time, but it was never observed in the case of a platinum film thicker than  $2\ \mu\mu$ .

(b) *The Effect of Using Different Wave-lengths of Exciting Light.*—The influence of the wave-length of exciting light upon the character of the emission is shown by the curves of Fig. 9. All show the double maximum,

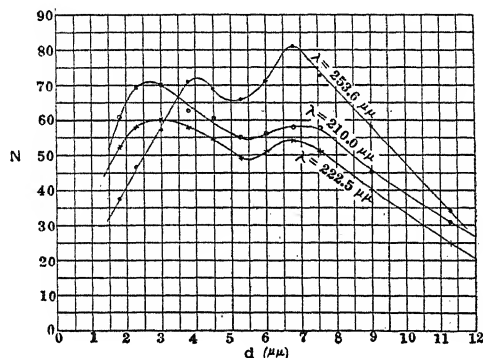


Fig. 9.

but there are two points of difference. The second maximum is relatively smaller and the first maximum is displaced toward the left when dealing with the shorter wave-lengths. Every set of films tested showed these characteristics, although they were more pronounced in some sets than in others.

(c) *Time Changes.*—An examination of Fig. 8 shows that even for a given wave-length, the two maxima are not always of the same relative magnitude. This is due to a slow time change in the properties of the films, an actual example of which is shown by Fig. 10. The second maximum becomes relatively less important as the film stands, and it makes no apparent difference whether the films stand in *vacuo*, or exposed to air. Heating the films in an oven at about  $150^\circ\text{C}$ ., however, greatly accelerates the change, and after this treatment the second hump is scarcely noticeable.

<sup>1</sup> King, PHYS. REV., 10, p. 291, 1917.

DISCUSSION OF RESULTS. *Interpretation of the Two Maxima.*—Apparently the two maxima of the curves must be ascribed to platinum in two conditions. Of these conditions, the one giving rise to the right hand maximum is evidently more unstable and more electropositive than the other, since it is relatively more important soon after the formation of the films and relatively more sensitive to light of long wave-lengths. We therefore take the first maximum to be characteristic of platinum in its ordinary stable state, while the second maximum is due

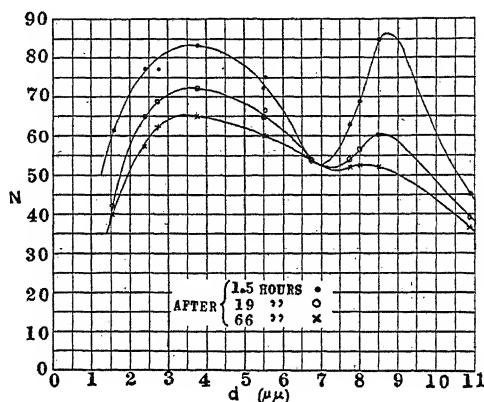


Fig. 10.

to platinum in the amorphous or spongy state in which it is first deposited. Since the change from the amorphous to the stable state is spontaneous, the latter state must be one of less internal potential energy, which is in accord with the observation that the platinum in this state is more electronegative. It is also seen, by reference to Figs. 2 and 4, that the average distance,  $1/\beta$  or  $1/\gamma$ , moved by an electron without losing its ability to escape is greater in the less compact state. These deductions are all consistent with each other and are also in line with well known facts regarding the abnormal resistivity of freshly sputtered films and the change to normal resistivity produced by "aging." We shall therefore confine the remaining discussion to the information regarding ordinary platinum which is given by a study of the first maximum.

*Comparison of the Three Assumptions Regarding the Process by which an Electron Loses its Ability to Escape.*—The third assumption, viz., that an escaping electron loses energy in direct proportion to the distance traveled through the metal, may at once be discarded. According to it, the thickness giving maximum emission should be directly proportional to  $(\nu - \nu_0)$  by equation 12, or to  $(1/\lambda - 1/\lambda_0)$ , where  $\lambda$  is the wave-length of the exciting light and  $\lambda_0$  is the longest wave-length of light which will

cause photoelectric emission.  $\lambda_0$  is known to be about  $280 \mu\mu$  for platinum.<sup>1</sup> The values of  $\lambda$  employed ranged from  $210 \mu\mu$  to  $265 \mu\mu$ . Thus the assumption requires that  $d_m$  should be four or five times larger for the shorter than for the longer wave-lengths. Not only is this not the case, but what little variation of  $d_m$  with  $\lambda$  is found is in the opposite direction to that required by the assumption.

As far as we can see, either the first or the second assumptions are in conformity with the facts. We are unable to decide between them experimentally, since either equation (2) or equation (5) satisfies the experimental results within the limits of uncertainty introduced by the presence of the second maximum. Of the two assumptions, however, the more reasonable one is the second—viz., that the probability that an electron will retain its ability to escape decreases exponentially with the distance traveled from its starting point.

On this assumption, we may determine  $1/\gamma$ , the average distance which an electron may move without losing its ability to escape. For  $\lambda = 253.6 \mu\mu$ , we may take  $d_m = 3.75 \mu\mu$  as the mean of all our determinations. Taking  $\alpha = .078 \mu\mu^{-1}$ , we find  $\gamma$ , from Fig. 4, to be  $0.375 \mu\mu^{-1}$ .

For shorter wave-lengths,  $d_m$  has slightly smaller values. This may be due to a larger value of  $\alpha$  for shorter wave-lengths or to a larger value of  $\gamma$ . This latter possibility is very unlikely, since it would mean that the rapidly moving electrons have less average penetrating power than the slow ones. We attempted measurements of  $\alpha$  for the shorter wave-lengths and did succeed in proving that  $\alpha$  increases as  $\lambda$  is decreased below about  $245 \mu\mu$ , but our measurements were not accurate enough to prove that the shift of  $d_m$  with wave-length is due entirely to this cause. We are, however, safe in saying that there is no evidence that  $1/\gamma$  increases as  $\lambda$  decreases, *i. e.*, *there is no evidence that electrons with large initial velocities retain their ability to escape over longer paths than the slower electrons.*

This result is rather surprising and is of considerable importance, for it proves that an electron loses its ability to escape at a single catastrophe, and not by gradual process or a succession of small energy losses. If we define one of these catastrophes as a "collision," then we may identify  $1/\gamma$  with the "mean free path" of the electron in the metal. This "mean free path" is apparently independent of the velocity of the electron, within the limits of velocity of photoelectrons. It is not easy to see how the theory of free electrons moving like molecules of a perfect gas in a metal can be reconciled with these results which indicate definite collisions of an inelastic type. These experiments are therefore consistent

<sup>1</sup> Compton and Richardson, Phil. Mag., 26, p. 549, 1913.

with the facts of specific heats of metals in indicating that electrons cannot exist in metals freely and independently sharing the thermal energy with the atoms.

The magnitude of the mean free path  $1/\gamma$  is  $2.67 (10)^{-7}$  cm., which is greater than, but of the same order of magnitude as the distance between atomic centers in platinum. It is therefore probable that the electrons lose at least the greater part of their kinetic energy at encounters with atoms, and they may entirely lose their freedom at these encounters.

Patterson<sup>1</sup> has calculated the mean free path of conducting electrons by data regarding the Hall effect in platinum and also by applying J. J. Thomson's theory of the resistance of thin films. By each method he finds the mean free path to be about  $6(10)^{-7}$  cm., not far different from the value we have calculated. Strict comparison is inadmissible, however, because the "mean free path" does not mean quite the same thing in the two cases, and because the assumptions underlying the formulæ used by Patterson cannot be accepted in their present form.

**GOLD FILMS.** In dealing with gold films the experimental difficulties were so great as to preclude the possibility of any great accuracy, and no extensive tests were made. The difficulties arise from the high resistance of films of the thickness necessary for the present tests, and are apparently due to lack of continuity of the films. Isolated portions of the films quickly charge up to retarding potentials, causing the rate of electrometer deflection to decrease rapidly. This phenomenon was noticed with only the very thinnest films of platinum.

In order to be more independent of errors from this cause, the films were kept shielded from light until the electrometer key was opened and we were ready to make an observation. A shutter was opened, so as to expose the film for two seconds. The deflection due to the electrons liberated in this interval was taken to represent the relative photoelectric sensitiveness of the film. The results of these tests are shown in Fig. 11. There is no evidence of a double maximum, though the determinations may not be accurate enough to make this conclusion certain. The mean free path  $1/\gamma$  may be calculated by Fig. 4 to be about  $5.0 (10)^{-7}$  cm. The value of  $\alpha$  used in this calculation is given by Meier<sup>2</sup> as  $\alpha = .056 \mu\mu^{-1}$  for wave-length  $253.6 \mu\mu$ . Patterson has suggested a value  $16(10)^{-7}$  cm. for the mean free path of the conducting electrons, while Partzsch and Hallwachs suggest

$$\frac{1}{\beta} = 11.6(10)^{-7} \text{ cm.}$$

(See the second assumption of the present paper.)

<sup>1</sup> Phil. Mag., 3, p. 655, 1902.

<sup>2</sup> Loc. cit.



SUMMARY.—The experiments described in this paper indicate that:

1. Photoelectrons, excited within a metal, lose their initial kinetic energy as the result of single and definite catastrophes, or "collisions," rather than by a gradual process or a succession of small energy losses.
2. The average distance which an electron can move before it loses its

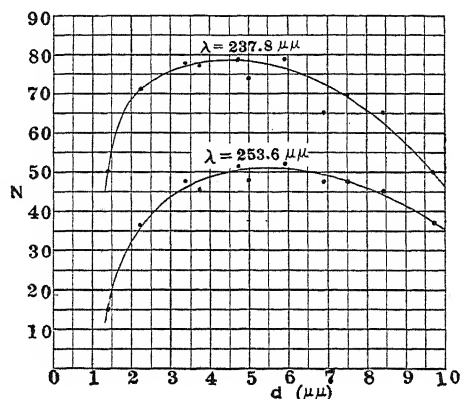


Fig. 11.

energy by one of these "collisions" is independent of the initial velocity and is of the same order as, but slightly larger than, the distance between atoms. The measurements indicate a mean free path of about  $2.67 (10)^{-7}$  cm. in platinum and  $5.0 (10)^{-7}$  cm. in gold.

3. Platinum films thinner than  $2 \mu\mu$  and gold films thinner than about  $8 \mu\mu$  are apt to be imperfectly conducting due to isolated regions which are not in conducting contact with the main body of the film. Platinum films therefore appear to be more homogeneous than gold films.

4. The small penetrating power of photo-electrons renders it extremely improbable that the thermionic emission from metals may be accounted for by considering it to be a photoelectric effect from the entire body of the metal integrated over the complete radiation spectrum.<sup>1</sup>

<sup>1</sup> This possibility is discussed in detail in Richardson's "The Emission of Electricity from Hot Bodies," pp. 95-102.

<sup>2</sup> Since the above paper was written, a paper dealing with the same general subject has been published in the February number of this Journal by Dr. Otto Stuhlman, Jr. His experimental results are not in good agreement with those in this paper, and his interpretation of them is quite different. We are at present unable to suggest a satisfactory explanation of the apparent lack of agreement.

PALMER PHYSICAL LABORATORY,  
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## NEW BOOKS.

*The Fundamental Equations of Dynamics and its Main Coördinate Systems Vectorially Treated and Illustrated from Rigid Dynamics.* By FREDERICK SLATE. Berkeley: University of California Press, 1918. Pp. vii + 233.

This is a critical exposition of dynamical principles in the form of a commentary rather than a text-book or treatise, covering the concepts associated with systems of particles and rigid bodies with some passing reference to deformable media. One declared purpose is to utilize the values that appear in some of the more recent aspects of the subject, especially the point of view of energetics, the theory of relativity, and the formulation in terms of vector algebra. Mention is made of the Einstein theory, but the only relativity actually considered is of the Galileo-Newton type, extended to cover the study of motion relative to rotating frames of reference and the forces of Coriolis. Extended treatment is given the Euler coördinates for a rigid body, which also supplies the main illustration given of the Lagrange equations. At the end of the volume are a series of notes, partly historical, partly on comparison references. The style is somewhat verbose and seems stilted at times, perhaps because of a conscious effort at sufficient novelty of wording to avoid the risk of conventionalism which is so real in such a familiar subject. For this reason and in view of its apparent purpose the value of the book will appear best to those who already have acquaintance with the matter and are interested to think through its foundations in a fresh way.

A. C. L.

## THE PHYSICAL REVIEW.

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### THE FUNCTION OF PHASE DIFFERENCE IN THE BINAURAL LOCATION OF PURE TONES.

BY R. V. L. HARTLEY.

**SYNOPSIS.**—This paper offers an explanation of the sound images observed when pure tones of the same frequency and intensity but of different phase are applied to the two ears. Use is made of theoretical curves calculated by Stewart and Fry giving the relation between the position of an actual source and the resulting phase difference  $P$  of the sound at the two ears. On the assumption that the listener subconsciously perceives the phase difference and places the image in the position that experience has taught him to associate with that phase difference, most of the observed phenomena follow directly from the curves.

For frequencies below about 600 cycles they account for (1) the observed rotation of the image with continuously varying  $P$ ; (2) its motion toward the ear followed by a range of uncertainty as  $P$  approaches  $180^\circ$ ; (3) the increase with frequency of the value of  $P$  for maximum lateral displacement of the image. For higher frequencies they explain the progressive decrease in definiteness of the image with increase in frequency as due to the presence of a number of simultaneous images. These multiple images as given by the curves are in very good agreement with those observed in the rather unique experiments of Bowlker.

The completeness with which the assumed theory of direct perception of phase difference explains the observed phenomena is strong evidence in its favor against the rival theories based on cross conduction through the head.

#### INTRODUCTION.

THE problem of the location of a source emitting a pure tone presents features which differentiate it quite sharply from the general problem of sound location. In the first place, pure tones are much less easily localized than are complex sounds. And in the second place, persons who are deaf in one ear have almost no sense of location for pure tones (provided the head is fixed), even though they may locate other sounds almost as well as a person with normal hearing.

Hence, while both monaural and binaural effects may be utilized in locating complex sounds, the location of a pure tone is essentially binaural, and so must depend upon differences in the characteristics of

the sound reaching the two ears. A sustained pure tone is completely determined by its frequency, amplitude and phase, and of these characteristics only the last two are subject to variation with the position of the source. It follows therefore that the sense of location can be dependent only on differences of intensity or phase or both. The existence of a connection between relative intensity and location was probably recognized before that of phase difference, but both have been known for a comparatively long time. In what follows it will be assumed that the effect of relative intensity is reduced to a minimum by keeping the intensities equal at the two ears.

Stewart,<sup>1</sup> in his article on Binaural Beats, includes a very good review of the literature on phase difference as affecting location. The experimental results may be very briefly summarized here. Two general methods have been used. In one, a particular phase difference is maintained by means of an adjustable difference between the paths from a single source to the two ears. In the other the phase difference is made to vary continuously by exciting the two ears from sources of slightly different frequency. The rate of variation is determined by the difference in frequency.

It is commonly observed that with zero phase difference, the apparent source of the sound, or for brevity the image, is in the median plane. Its location varies with different experimenters but more often appears to be in front. For frequencies below about 600 cycles it is agreed that the image lies on the side for which the phase at that ear leads. With continuously varying phase difference the image is described as starting from a position straight in front for zero phase difference, moving more or less in a circle to one side, till opposite the ear, then moving in toward the ear, entering the head, jumping quickly to the other ear for  $180^\circ$ , and continuing on around to the front along a similar path on that side. For higher frequencies most observers report a falling off in the definiteness of the location and finally a complete loss of the sense of direction.

In attempting to explain the mechanism by which the phase difference determines the position of the image, two general theories have been advanced. One of these, which we may term the direct perception theory, assumes that the listener is able to take cognizance of a difference in phase, as such, and by virtue of his past experience, or that of his ancestors, is able to associate any particular phase difference with the direction from which he has been accustomed to receive sounds having that phase difference.

<sup>1</sup> G. W. Stewart, *PHYS. REV.*, June, 1917, p. 502.

Lord Rayleigh<sup>1</sup> has attempted to explain in a general way the observed facts on the basis of this theory. He pointed out that for a fork of 128 cycles, moving around the head, the phase difference would never exceed about one eighth of a period, and commented on the fact that in the laboratory experiment the location effect persisted for phase differences right up to half a period. He noted also that at a frequency around 512, the phase difference with the source opposite one ear should be  $180^\circ$  and concluded there could be no discrimination at that frequency for sounds directly right or left. For somewhat higher frequencies he pointed out that the same phase difference could be produced by a sound on either the right or the left, and considered it fortunate that our sense of direction fails us at the point where it would begin to give misleading results. It should be noted that in arriving at the phase difference for an actual source at one side he made use of the very rough assumption that the difference in the paths to the two ears is one foot. He also drew other conclusions from the unjustifiable assumption that the sense of lateralness is a maximum when the phase difference is  $90^\circ$ . On the whole, his results were not particularly conclusive.

Against this theory certain psychologists advanced the argument that it involved a violation of the general principle that the sensation resulting from the stimulation of a sensory nerve is independent of the nature of the stimulus. In other words, they held that the characteristic phase of the sound stimulus at each ear could not be preserved in the nervous impulses reaching the brain as would be necessary for a perception of difference in phase. Apparently, however, this general principle, has been deduced from rather limited experimental evidence, and it is not considered by psychologists generally to be very definitely established. This objection, therefore, is not sufficient to warrant a rejection of the theory, provided it can be shown to be satisfactory in other respects.

Nevertheless, it was probably this argument which led to attempts on the part of physicists to show how differences in phase could give rise to differences in the intensity of the stimuli, which latter would cause a location on the side of the stronger stimulus. This resulted in various theories being advanced which we may designate as cross-conduction theories, since they are based on the assumption that an appreciable fraction of the sound entering each ear is conducted through the head to the other ear and there combines with that entering directly.

Myers and Wilson<sup>2</sup> present such a theory. In order to make the directions fit the experiments they find it necessary to introduce an

<sup>1</sup> Rayleigh, *Phil. Mag.*, 13, 1907, pp. 214 and 316.

<sup>2</sup> C. S. Myers and H. A. Wilson, *Proc. Roy. Soc.*, 80, 1908, p. 260.

arbitrary phase reversal in passing through the head. Even then their theory explains only the most general features of the experiments, since it makes the displacement of the image proportional to the sine of the phase difference. More recently Stewart<sup>1</sup> has embodied the same idea in a more elaborate theory and uses it to account for the location when the phase difference is between  $90^\circ$  and  $270^\circ$ . For the rest of the cycle he assumes direct perception of phase.

Against this type of theory numerous objections have been raised. Probably the most important of these is the experimental evidence showing that the amount of sound actually reaching the ear by cross-conduction is far too small to produce the effects ascribed to it. A good summary of this evidence is given by Peterson.<sup>2</sup> Further, if the location effect due to phase difference were determined purely by the resulting intensity relation it should be very strongly affected by changes in the relative intensity of the sounds coming to the two ears. Under certain conditions, however, it is found to be practically unaffected by very considerable changes of this sort. In order to give even a qualitative explanation of the phenomena it is necessary to introduce a purely arbitrary assumption of phase reversal within the head. To make the agreement at all quantitative even over a limited range of frequency, it has been necessary to set up a very complicated theory involving a rather large number of assumptions.

To put the situation briefly then, the cross-conduction theory, while it does give something of an explanation, is open to very serious objections. The direct perception theory, on the other hand, while it has nothing very serious against it, has so far had no very positive evidence advanced in its favor, such as would be furnished by a quantitative explanation of the experimental facts. The most obvious line of attack then is to find out whether or not any such explanation is possible on the basis of direct perception.

#### PURPOSE.

It is the present purpose to attack this question by attempting to trace a correlation between the phase differences produced at the ears by actual sources in various positions and the positions of the images which result when sounds of equal intensity and predetermined phase differences are applied experimentally to the two ears.

#### DATA AND DISCUSSION.

For this the first essential is a knowledge of the phase at the ear as a function of the position of the source for various frequencies. Stokes<sup>3</sup>

<sup>1</sup> G. W. Stewart, *PHYS. REV.*, June, 1917, p. 514.

<sup>2</sup> Joseph Peterson, *Psychol. Rev.*, Vol. 23, No. 5, p. 333, 1916.

Lord Rayleigh, *Theory of Sound*, Ch. 17.

has developed a method whereby it is possible to calculate, for any point on a great circle of a rigid sphere, the phase of a sound whose source is in the plane of the great circle. The wave-length of the sound and the distance of the source are expressed in terms of the radius of the sphere. Stewart<sup>1</sup> has applied this method to the calculation of certain special cases. At the time this work was undertaken, T. C. Fry, a colleague in the Western Electric Co., had, independently, applied the method to certain other cases. A study of these two sets of results led to the conclusions described below. Later Fry extended his calculations securing the results from which have been plotted the curves given in Figs. 1 to 5.

Stewart assumes the head to be such a rigid sphere of 60 cm. circumference, the ears being at opposite ends of a diameter. For a single case, corresponding to a source, of frequency about 280 cycles, distant about 480 cm., he plots the difference in phase at the two ears as a function of the direction of the source. The only deduction which he draws from this curve is that, "The phase difference changes the most rapidly when the source of sound is in front or behind the hearer."

In plotting the curves of Figs. 1 to 5, the circumference of the head was assumed to be 55 cm. (radius,  $r = 8.75$  cm.). The angular separation of the ears in the horizontal plane was taken as  $165^\circ$  and the velocity of sound as 340 m. per second. The figures represent progressively increasing frequencies. The individual curves of each figure are for sources at various distances  $R$  from the center of the head, ranging from  $2r$  to infinity. (Unfortunately, the method of calculation was not adaptable to sources at the surface of the head where  $R = r$ .) Abscissæ represent the angular displacement  $\phi$  of the source from a point straight in front, being taken positive to the right and negative to the left,  $180^\circ$  being at the back. Ordinates represent the phase lead  $P$  at the right ear over the left. Negative values then mean that the phase is leading at the left ear. A phase lead  $180^\circ + x$  ( $x < 180^\circ$ ) at one ear is equivalent to a lead of  $180 - x$  at the other ear. In other words, those parts of the curve where the phase lead exceeds  $180^\circ$  in either direction must have  $360^\circ$  either added or subtracted so as to keep the ordinates between  $-180^\circ$  and  $+180^\circ$ . This has been done in Figs. 3, 4 and 5.

From this theoretical information it is possible to predict, on the basis of the direct perception theory, where a listener should place the image when the phase difference at the ears is given any arbitrary value. For it is only necessary to read from the curve corresponding to the frequency of the sound, the value of  $\phi$  corresponding to the given value of  $P$ .

<sup>1</sup> G. W. Stewart, *PHYS. REV.*, 1914, 4, p. 252.

As the curves give only those values of  $\phi$  lying between  $-90^\circ$  and  $+90^\circ$ , they give only the images located in front of the observer. If extended to  $\phi = 180^\circ$  in each direction the curves would be nearly symmetrical with respect to  $\phi = \pm 90^\circ$ . (The departure from symmetry is due to taking the ears as  $165^\circ$  apart rather than  $180^\circ$ .) This means that any particular phase difference could be produced by a source either in front of or behind the observer and hence we should expect the image to be in either one or both of these positions. As already noted some experimenters find it in one place and some in the other. Inasmuch as the

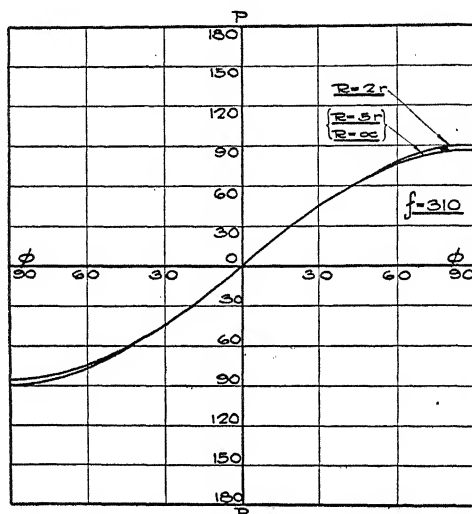


Fig. 1.

majority seems to find the image in front, the discussion will be limited to those quadrants with the understanding that the same general conclusions hold for images at the back.

Consider first the case of the low frequency note of 310 cycles, shown in Fig. 1. For values of phase difference less than about  $86^\circ$ , we may for each ordinate choose an abscissa lying on any one of an infinite number of curves distributed between those of  $R = r$  and  $R = \infty$ . This means that so far as phase alone is concerned there is in this region nothing to fix the distance of the image, while its angular position is determined, within limits, by this distance. The observed fact that the image is generally at a considerable distance is probably to be explained by the equality of intensity. Generally speaking, for any given direction the intensities at the ears become more nearly equal the greater the distance of the source. This then would explain why the image in moving around from the front would stay at a considerable distance till it was



opposite the ear for a phase difference of  $86^\circ$ . For a further increase in phase there is no longer a corresponding abscissa on the curve for  $R = \infty$  so it is necessary to go to the curve for the greatest distance which will still give a point. This means that once the image has reached a position near  $90^\circ$  its direction remains unchanged, but in spite of equal intensity, it moves in toward the ear, reaching it for a phase difference somewhat greater than  $90^\circ$ , being the maximum value of the hypothetical curve for  $R = r$ . This is in agreement with the observations quoted above for the case of continuously changing phase difference. Beyond this point there is no corresponding position for an actual source and hence the curves tell us nothing as to where an image is to be expected. However, it does not follow that such phase differences are foreign to the experience of the listener. They may very easily be produced by the reflections which occur in an enclosed space. Or they may occur when the source is within the head, or when the latter is in direct contact with the sounding body. Most observers describe the image as being located within the head in this case. Bowlker,<sup>1</sup> however, speaks of the image as being spread over a considerable angle on either side of  $90^\circ$ . In any case the definiteness of the sense of location should be much less than for small phase differences. It is, therefore, not surprising that, as Stewart has observed, the position of the image is affected much more by unequal intensities when the phase difference is near  $180^\circ$  than when it is near zero.

Consider next the effect of increasing the frequency. Up to something over 600 cycles (Figs. 1 and 2) the curves are of the same general form and we should expect the same general behavior of the image, as is found to be the case. The main variation is that the phase difference at which the image reaches its maximum lateral displacement increases with the frequency. Thus for 620 cycles (Fig. 2) the sense of lateralness should be a maximum for about  $170^\circ$ , instead of  $90^\circ$  as Rayleigh assumed. In support of this variation in the value of  $P$  for maximum image displacement, may be cited the results obtained by Myers and Wilson<sup>2</sup> using a variable path arrangement. They give curves in which the sensation of lateralness, measured in arbitrary units, is plotted against the path difference, and attempt to show a connection between this apparent displacement and the sine of the phase difference. An inspection of the curves, however, shows that for a frequency of 128, the maximum displacement occurs for a phase difference considerably less than  $90^\circ$ , while above  $90^\circ$  the displacement shows a decided falling off. At 384 cycles, the maximum is more nearly at  $90^\circ$ , while for 512, it is

<sup>1</sup> T. J. Bowlker, *Phil. Mag.* (6), 15, p. 318, 1908.

<sup>2</sup> C. S. Myers and H. A. Wilson, *loc. cit.*

on the side toward  $180^\circ$ . What is perhaps a more striking confirmation is a statement made by Bowlker,<sup>1</sup> in connection with some experiments in which various path differences were obtained by placing tubes of different lengths over the two ears and listening to a distant sound. He says "A wave-length of about 36 inches (frequency about 370) . . . gave an image displaced  $90^\circ$ . Sound with longer wave-length gave a dis-

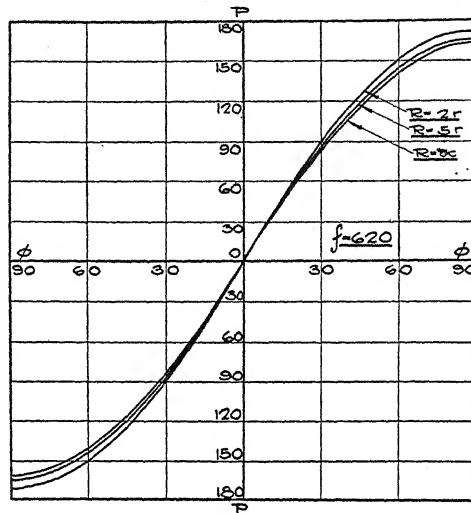


Fig. 2.

placement of  $90^\circ$  before a phase-difference of half a wave-length was reached, and the sound image then seemed to spread over a continually increasing length of arc on each side of  $90^\circ$ ."

As the frequency is still further increased a marked change occurs in the nature of the curves owing to the fact that an actual source may then cause a phase difference greater than  $180^\circ$ . For a frequency of 930 cycles, Fig. 3 shows that for ordinates above  $126^\circ$  there are, on the curve for  $R = \infty$ , two abscissæ, one positive and one negative. This means two images, one on the right and one on the left. In the range between  $126^\circ$  and  $106^\circ$  there can be no image at infinity on the left, but there can be one at some distance greater than  $2r$ . (As before, the equality in intensity should tend to keep the image as far away from the head as is consistent with the phase relations.) For  $P$  less than the critical value corresponding to  $R = r$ , there is only one image as at lower frequencies.

Starting then with  $P = 0$ , the image is in front. As  $P$  increases the image moves rather slowly to the right keeping at a considerable distance.

<sup>1</sup> Bowlker, *loc. cit.*

When  $P$  is somewhat less than  $106^\circ$  it has moved out about  $20^\circ$ , when a second image appears at the left ear. This new image moves quite rapidly outward from the ear, reaching a great distance by the time the first has moved to  $33^\circ$  ( $P = 126^\circ$ ). It then swings around in a circle toward the front, at first rather rapidly and then more slowly, arriving at  $51^\circ$  on the left at the same time that the other image reaches the same position on the right ( $P = 180^\circ$ ). To complete the cycle  $P$  must be varied from  $-180$  to  $0^\circ$ . The image on the right now moves on around the circle with increasing speed till opposite the ear, when it moves in to the ear and disappears. Meanwhile, the left image moves slowly toward

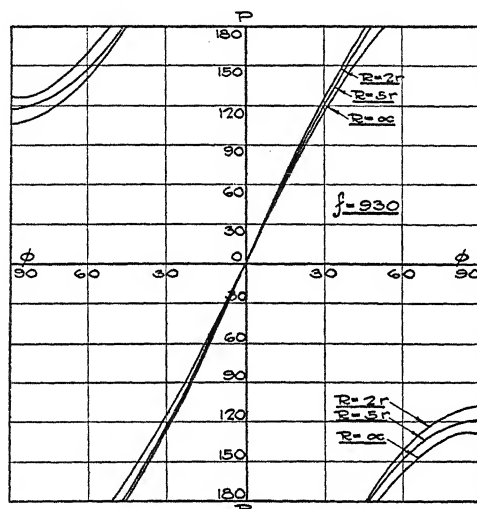


Fig. 3.

the front reaching there for  $P = 0$ . That is to say, the two images follow the same path relative to their respective sides but in opposite directions. The one on the side where the phase-lead is increasing begins in front and disappears at the ear, the one on the other doing the reverse. It will be noticed that the paths followed are exactly the same as for the low frequencies discussed above. The difference lies in the magnitude of the variation of  $P$  required to cover the path.

For still higher frequencies the range on either side of  $180^\circ$  in which two images are present increases progressively (see Fig. 4) until the whole cycle is covered. For higher frequencies still, where the possible phase difference exceeds  $360^\circ$ , there should be a third image for values of  $P$  close to zero, being on the right for small positive values of  $P$  and on the left for negative. Fig. 5 shows a case where this third image is present for most of the cycle. For still further increasing frequencies it follows

from the above that the greatest number of images in the field during any part of a cycle increases by one each time the maximum possible phase difference increases by  $180^\circ$ .

Before the foregoing picture of the sound images for high frequencies can be used to predict the experimental results to be obtained at those frequencies, there must be something known or assumed as to the ability of the listener to perceive and localize a plurality of simultaneous images, giving to each the location assigned to it by the phase relations. It is quite conceivable that the listener would be unable to do this, in which

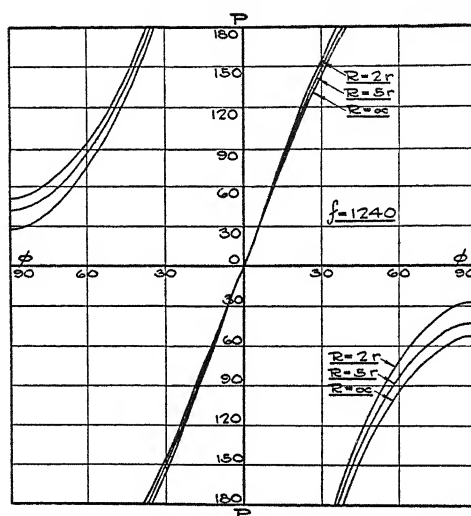


Fig. 4.

case he might either have the sensation of a single image in some sort of average position or he might have no sense of direction at all. In either case his judgment would be much less certain than for a single image. Under these circumstances he would probably be influenced considerably by the equality of intensity, which would tend to cause him to locate the image in the median plane, particularly since at high frequencies the variation of intensity ratio with the position of the source is more marked than at low. This assumption appears to be in agreement with the results of most observers, who find that from about 600 cycles upward the sense of location by phase difference becomes progressively less trustworthy. That the change should appear to be gradual follows from the fact that the region of two images is at first confined to a small part of the cycle and gradually extends over the remainder. So far then, the theoretical deductions are not inconsistent with the experiments, on the assumption that more than one image can not be located separately.

That this is not universally true, however, is shown by a series of experiments by Bowlker,<sup>1</sup> in which he was able to recognize two and even three separate images, in the field at once. Whether this was due to some peculiarity of his apparatus or of his individual sense of hearing is not known, but in either case the experiments are of special interest as being probably the only ones available for testing the above theory regarding multiple images.

The method used was an improvement of that referred to above in which tubes of unequal length were used. In order to eliminate possible

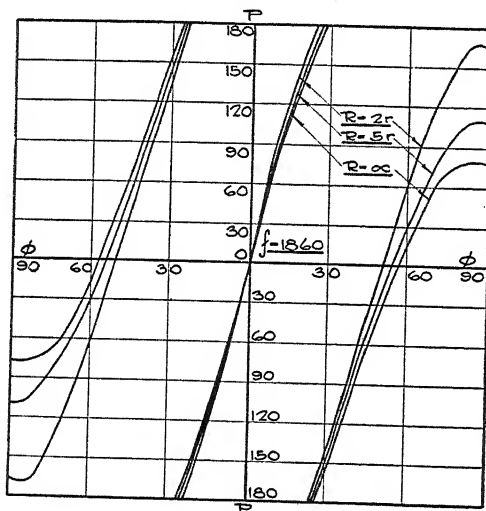


Fig. 5.

inequalities of intensity due to the unequal lengths of the tubes, or to sound shadows cast by the head, he used exactly similar tubes on the two ears, bending them upward so as to make the openings horizontal in a plane well above the top of the head. The difference in path was then secured by facing in various directions relative to the source. The latter consisted of an organ pipe about 30 feet away. The distance  $L$  between the openings was 31 inches. The path difference was given by  $L$  into the sine of the angle turned through from the position facing the source. With this arrangement he was able to locate the direction of the sound image with considerable exactness, especially near the front. Due to some asymmetry either of the tubes or the ears, the image, when facing the source, was not always exactly in front. It was necessary for some frequencies to turn through as much as  $4^\circ$  to bring it in front. This position was then used as the corrected zero. At moderately low

<sup>1</sup> Bowlker, *loc. cit.*

frequencies the image moved around toward the side of leading phase, until at a point corresponding to a path difference of half a wave-length ( $P = 180^\circ$ ), it jumped to a similar position on the left. Continuing the rotation till this new image was straight in front the angle was generally found to agree quite closely with a path difference of a whole wave-length. The position of the image just before and after it crosses to the other side he calls the crossover angle. At frequencies of 263 and 335 he gives this as " $90^\circ$  (wide image)," which agrees with his results for unequal tubes. At higher frequencies it decreases progressively, having the following values; for 485 cycles,  $40^\circ$  to  $50^\circ$ ; 690 cycles,  $36^\circ$ ; 970 cycles,  $18^\circ$ , and 1290 cycles,  $15^\circ$ . He speaks of this cross-over angle as being "near the maximum displacement that phase will produce with the particular wave-length under observation."

At this point he reduced the separation of the openings to 15 inches so as to avoid the necessity of measuring very small angles. Repeating the experiment with 1,290 cycles he found for the cross-over-angle,  $20^\circ$ . He also observed that for all positions corresponding to phase differences between  $180^\circ$  and about  $100^\circ$  both images were present in the field at once. At 1,675 cycles he says: "Two images were evident during nearly the whole range—there were practically always two and sometimes three images evident, though I had some doubts whether the central image was always real or a result of attention to the two side images." In place of the cross-over angle he gives the maximum displacement as  $12^\circ$ .

He then reduced the apertures of the tubes from 2 inches to 0.6 inches. For a note of 2,090 cycles he says, "now three images could always apparently be heard together when facing one of them." The maximum displacement is here given as about  $9^\circ$ . At 2,310 cycles, "the image further to the left seemed stronger and tended to draw off one's attention." At 3,050 cycles, "it was very difficult to determine even approximately the position of an image, the one to the left of the two or three in the field of view seeming the loudest as a rule. I only felt sure that phase was still playing a part in fixing the maxima and minima which gave rise to the centers of the sound images."

The general agreement of these experiments with the theoretical results is obvious. The order of the changes in the images with increasing frequency is in striking agreement. First at low frequencies there is the progressive increase in the value of  $P$  to give maximum lateral displacement, accompanied by a narrowing of the region in which the image appears diffuse. This represents the range of frequency in which the maximum of the  $P - \phi$  curve is less than  $180^\circ$ . Next comes a range in which the displacement of the image for  $P = 180^\circ$  is less than  $90^\circ$  and

decreases progressively with increasing frequency. This corresponds to the diminishing value of  $\phi$  for  $P = 180^\circ$ , over the frequency range in which the values of maximum phase difference exceed  $180^\circ$ . Along with this comes the appearance of a second image on the opposite side, the part of the cycle for which it is present increasing progressively. When it has covered the whole cycle a third image appears, corresponding to the condition where the maximum value of  $P$  exceeds  $360^\circ$ .

The quantitative agreement is, however, not exact. The greatest angular displacement in the case of multiple images is always given as quite small and never reaches  $90^\circ$  as the theory would indicate. Also the frequencies at which the additional images appear are rather higher than the theoretical values. The second of these discrepancies is probably a result of the first. Suppose, for example, that in Fig. 4 the observer could not detect the second image when its displacement exceeded say  $45^\circ$ . The second image ( $R = \infty$ ) would then appear when  $P$  reached  $170^\circ$  instead of  $50^\circ$  as it otherwise would. In order to have it appear at  $50^\circ$  under the assumed conditions it would be necessary to go to a higher frequency. That the second image should not have been detected under certain conditions is not surprising, when it is considered that practically all the other experimenters have failed to observe it at all. Furthermore, the assumptions underlying the theoretical calculations are such that we should not expect a very close agreement with experiment. The head is not a sphere nor is its circumference likely to be exactly 55 cm. The ears do not lie on a great circle and in most cases are not separated by exactly  $165^\circ$ . In fact, the differences, both physical and psychological, among the different observers are so great that there should be wide variations in the experimental results themselves. It should be strongly emphasized, therefore, that although fairly definite quantitative values have been given, it has been done chiefly for the sake of clearness in exposition, and there is no justification for using these values to predict with equal definiteness the results of any given experiment.

RESEARCH LABORATORY OF THE  
WESTERN ELECTRIC COMPANY, INC.

## AN EXTENSION OF THE ELECTRON THEORY OF METALS. I. THERMOELECTRICITY AND METALLIC CONDUCTION.

By A. E. CASWELL.

SYNOPSIS.—Using as a starting-point the simple equations deduced from the electron theory by J. J. Thomson and others for thermo E.M.F., thermoelectric power, Peltier E.M.F. and the Thomson effect, and assuming that the number of free electrons in unit volume of a metal is an exponential function of the temperature, the author shows that (1) the thermoelectric power is a linear function of the temperature, (2) the equations relating thermo E.M.F. and temperature, and Peltier E.M.F. and temperature, represent parabolas having their axes perpendicular to the axis of temperature, and (3) the Thomson effect is different for different metals and may be either positive or negative, but will usually have a positive temperature coefficient. Likewise, using the equations for electrical and thermal conductivity and making the second assumption that the number of positive centers with which the electrons collide changes with the temperature (these centers being atoms, molecules or clusters of molecules), it is found that (4) the electrical conductivity of pure metals decreases with increase of temperature but is not exactly inversely proportional to the absolute temperature, (5) the peculiar behavior of the electrical resistance of alloys can be accounted for, and (6) the thermal conductivity may either increase or decrease with the temperature, the temperature coefficient depending both upon the temperature and the material. A third assumption, viz., that the positive centers take part in the conduction of heat but not of electricity, leads to the conclusion that (7) the usual value deduced for the Wiedemann-Franz-Lorenz ratio is too small, and the variations in the value of this ratio at ordinary temperatures is accounted for. The above theoretical results, especially (1), (2) and (3), are, at least within the ordinary range of temperatures, in substantial agreement with experiment, since the constants involved in the theory can be determined from experimental data, and so the author concludes that (8) the concentration of electrons in a metal is an exponential function of the temperature, and (9) the number of positive centers changes with the temperature, the exact relation being somewhat uncertain. The number, however, must generally increase with the temperature.

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10. The Wiedemann-Franz and Lorenz laws.
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## I. INTRODUCTION.

THE theory outlined in this paper is an attempt to amplify, or expand, the electron theory of metals as applied to thermoelectricity and metallic conduction so as to bring it more nearly into harmony with the experimental facts without at the same time doing violence to such portions of the theory as are not treated here. Three simple assumptions, relating the electrons, molecules and temperature, are made which will be studied in detail later. Certain mathematical forms are employed to express the assumed relations. It is not claimed that these expressions are more than first approximations to the true functions, neither is it claimed that they are even first approximations in the neighborhood of the absolute zero and temperatures near which marked changes in the structure of the material occur. It is claimed, however, that the theory presented here offers, at least within the range of ordinary temperatures, as good an explanation of the experimental facts here treated as previous theories do, and it explains some facts which have been unaccounted for hitherto.

At the outset the writer wishes to state that much of such merit as the present paper possesses is due to the fact that he has had the advantage of constant consultation with his colleague, Professor W. P. Boynton, whose familiarity with the kinetic theory of matter has made his numerous suggestions and criticisms of primary importance.

Throughout this discussion the symbols employed have the following meanings:

$m$  = mass of an electron,

$u$  = "mean square velocity" of the electrons,

$T$  = absolute temperature,

$t$  = Centigrade temperature,

$\alpha$  = the gas constant for a single electron, being defined by the equation  $\frac{1}{2}mu^2 = \alpha T$ ,

$e$  = charge on an electron,

$\lambda$  = mean free path of the electrons,

$N$  = number of free electrons in one c.c. of a metal,

$n$  = number of positive centers in one c.c. of a metal,

$P$  = Peltier E.M.F.,

$Q$  = thermoelectric power,

$E$  = thermo E.M.F.,

$\sigma$  = Thomson E.M.F., or so-called "specific heat of electricity,"

$\kappa$  = electrical conductivity,

$\rho$  = electrical resistivity, and

$k$  = thermal conductivity.

The other symbols used are constants whose significance is obvious from the context. The subscripts 1 and 2 are used to refer to two different metals except where some other convention is specifically stated.

A theory to be completely satisfactory should account for the following facts.

1. The thermo E.M.F. between any two metals can usually be expressed by an equation of the form  $E = At + \frac{1}{2}Bt^2$ .

2. Similarly, the thermoelectric power of any two metals can usually be represented by a equation of the form  $Q = A + Bt$ .

3. The Peltier E.M.F. must satisfy the equation  $P = QT$ .

4. The Thomson effect may be either positive or negative and is different for different metals.

5. The Thomson effect has a temperature coefficient.

6. The electrical conductivity of pure metals at ordinary temperatures is almost inversely proportional to the absolute temperature, or the resistivity is nearly proportional to the absolute temperature. Usually the resistivity increases somewhat more rapidly than the absolute temperature.

7. At ordinary temperatures the electrical conductivity of alloys is frequently nearly independent of the temperature and occasionally increases with increase of temperature.

8. The thermal conductivity of a number of metals is nearly constant at ordinary temperatures and may either increase or decrease as the temperature is raised.

9. The ratio of the thermal to the electrical conductivity at all ordinary temperatures is nearly the same for all metals, the ratio being nearly proportional to the absolute temperature. In the case of pure metals the ratio usually increases somewhat faster than the absolute temperature, but in the case of alloys it usually increases more slowly.

Besides the facts just enumerated which are explained in the present paper there are number of galvanomagnetic, thermomagnetic and electronic emission phenomena which must ultimately be explained. No attempt is made to account for these effects, their treatment being reserved for a subsequent paper. So far as known none of the hypotheses advanced here in any way interferes with the explanation of any of these effects.<sup>1</sup>

<sup>1</sup> For general discussions of the above phenomena, including the subject matter of the present paper, the reader is referred to J. J. Thomson's *Corpuscular Theory of Matter*, p. 49 et seq.; O. W. Richardson's *The Electron Theory of Matter*, p. 407 et seq.; E. Bloch's *La Théorie Electronique des Métaux* (in a series of essays published under the title *Les Idées Modernes sur la Constitution de la Matière*, Villars, Paris); K. Baedeker's *Die Elektrischen Erscheinungen in Metallischen Leitern* (Braunschweig); also N. R. Campbell's *Modern Electrical Theory*, pp. 54-86.

## 2. PREVIOUS THEORIES AND THEIR LIMITATIONS.

The equations deduced from the electron theory by Sir J. J. Thomson, and which are taken as a starting-point in the present discussion, are:

$$P_{12} = \frac{2\alpha T}{3\epsilon} \log \frac{N_1}{N_2}, \quad (1)$$

$$Q_{12} = \frac{2\alpha}{3\epsilon} \log \frac{N_1}{N_2}, \quad (2)$$

$$\sigma = \frac{2\alpha}{3\epsilon} \left\{ T \frac{d}{dT} (\log N) - \frac{1}{2} \right\}, \quad (3)$$

$$\kappa = \frac{\epsilon^2 N \lambda u}{4\alpha T}, \quad (4)$$

and

$$k = \frac{1}{3} \alpha N \lambda u. \quad (5)$$

From equations (4) and (5) we obtain the Wiedemann-Franz law that the ratio  $k/\kappa$  is a constant for all metals at any given temperature and the Lorenz law that this ratio is proportional to  $T$ , which laws together give

$$\frac{k}{\kappa T} = \frac{4}{3} \left( \frac{\alpha}{\epsilon} \right)^2. \quad (6)$$

The thermoelectric equations satisfy the following equations which were deduced from thermodynamical considerations by Lord Kelvin, namely,

$$P = QT, \quad (7)$$

and

$$\sigma_1 - \sigma_2 = T \frac{dQ_{12}}{dT}. \quad (8)$$

The constant multiplying factors on the right-hand side of equations (1) to (5), inclusive, have been given different values by different theorists, but in any case they only differ by simple numerical factors. Thus Drude<sup>1</sup> replaces  $2\alpha/3\epsilon$  by  $4\alpha/3\epsilon$  in equations (1), (2) and (3), and Lorentz,<sup>2</sup> assuming the Maxwellian distribution of velocities among the electrons, finds instead of equation (4) the equation

$$\kappa = \sqrt{\frac{2}{3\pi}} \left( \frac{\epsilon^2 \lambda N u}{\alpha T} \right), \quad (4a)$$

and instead of equation (5) the equation

$$k = \frac{8}{9} \sqrt{\frac{2}{3\pi}} (\alpha N \lambda u). \quad (5a)$$

<sup>1</sup> Ann. Phys., 1, 566, 1900 and 3, 369, 1900.

<sup>2</sup> Lorentz, The Theory of Electrons, pp. 63-67.

From (4a) and (5a) it follows that

$$\frac{k}{\kappa T} = \frac{8}{9} \left( \frac{\alpha}{\epsilon} \right)^2. \quad (6a)$$

The value of the ratio  $k/\kappa T$  thus found is but two thirds of that found from the simple theory by Thomson and Drude. Since these differences in multiplying constants are of only secondary importance in connection with the present discussion they will be disregarded for the present. It may be remarked, however, that experimental data may, with the aid of the theory presented in this paper, enable one to discriminate between the different methods of approach to the problem and to select that one which agrees best with experiment.

It has usually been assumed either that  $N$  remained constant or else was either directly or inversely proportional to the square root of the absolute temperature. The first assumption was made in order to explain the optical properties of metals, the second was made by J. J. Thomson to account for the Thomson effect, and the last was made to account for the fact that at ordinary temperatures the temperature coefficient of resistance of a number of metals is approximately equal to  $1/273$  per Centigrade degree. From equation (4) it follows that if the electrical conductivity is inversely proportional to the absolute temperature, either  $N$  or  $\lambda$  must be inversely proportional to the square root of the absolute temperature, since  $u = \sqrt{2\alpha T/m}$ . We see no reason why  $\lambda$  should vary to any appreciable extent if the number of molecules does not change and the alteration in volume is quite small. Then  $N$  must change with temperature. But from the optical properties of metals it seems that  $N$  does not change.<sup>1</sup> So one has his choice: either  $N$  changes or it does not. But whether  $N$  is constant or is proportional to some power of the absolute temperature, the proportionality factor depending upon the material, certain results follow which are not in harmony with experiment. Some of these discrepancies are:

1. The thermo E.M.F. is a linear function of the temperature.
2. The thermoelectric power is constant for any given pair of metals.
3. The Thomson effect is the same for all metals at all temperatures.
4. The thermal conductivity of any metal is independent of the temperature.
5. The ratio  $k/\kappa T$  is the same for all metals at all temperatures and within the range of ordinary temperatures is usually too small.

On the whole, the most that can be said for the theories so far published is that they show that these properties are to be expected, but they are wholly inadequate when it comes to quantitative measurements.

<sup>1</sup> Drude, Ann. Phys., 14, 951, 1904.

## 3. STATEMENT OF THE NEW THEORY.

The fundamental assumptions upon which this theory is based are

(1) That the number of free electrons in a metal *changes* with the temperature,

(2) That the number of positive centers with which the electrons collide, whether these centers are atoms, sub-atoms, molecules or groups of molecules, *changes* with the temperature, and

(3) That these positive centers take part in the transfer of heat but not of electricity.

The mathematical forms which have been selected to express the relations between the numbers of electrons and positive centers are

$$N = ae^{xT}, \quad (9)$$

and

$$n = be^{yT} \quad (10)$$

where  $a$  and  $b$  are positive constants, and nothing is postulated regarding the signs of  $x$  and  $y$ .

We are led to the first assumption by the fact that unless equations (1), (2) and (3) are entirely wrong,  $N$  must depend upon  $T$ . The exponential form of equation (9) is suggested by the logarithmic form of these three equations. The excellent agreement of the resulting equations with experiment indicates the probability that equation (9) is an accurate representation of the facts.

The assumption that the number of positive centers with which the electrons collide changes with the temperature is somewhat startling.<sup>1</sup> For our purposes it is unimportant whether these centers are atoms, sub-atoms, molecules or clusters of molecules. A great deal of uncertainty exists as to the states of aggregation in solids and liquids, and there is no *a priori* reason why the state of aggregation in a substance should not vary with the temperature of the substance. It is not necessary that all the centers shall be alike. A continual process of disintegration and recombination of the centers may be going on. If a theory based on this assumption should harmonize better with experiment than others, this circumstance would lend color to the assumption.

Remembering that, as in the kinetic theory of gases,  $\lambda = f/nr^2$ , where  $r$  is the radius of the positive centers and  $f$  is a constant, and substituting  $be^{yT}$  for  $n$  in equation (10), we obtain

$$\lambda = ce^{-yT}, \quad (11)$$

where  $c = f/b r^2$ .

When  $T = 0$ ,  $N = a$ ,  $n = b$  and  $\lambda = c$ . But when  $T = \infty$ , then

Cf. Richardson's *The Electron Theory of Matter*, p. 467.

$N$  and  $n$  are both infinite, and  $\lambda = 0$ . In practice, of course,  $T$  never approaches infinity, and since both  $x$  and  $y$  prove to be small, the values of  $N$  and  $n$  at, say,  $2000^\circ \text{C}$ . will probably seldom exceed 50 to 100 times the corresponding values at the absolute zero.

The third assumption seems reasonable from the fact that insulators conduct heat even though they do not conduct electricity, and have very few, if any, free electrons.

The only assumption involved in the theory of thermoelectricity here presented is the first, namely, that the number of free electrons is a function of the temperature. The second assumption is introduced to account for electrical conductivity, namely, that the number of positive centers is a function of the temperature. The third assumption, that the positive centers take part in the conduction of heat, is involved only in those portions of the theory which deal with thermal conductivity.

Equations (1) to (5), inclusive, are taken as the starting-points in the present discussion. This is not to be interpreted as indicating a preference on the part of the writer for one physicist's theory rather than that of another. In the light of our present knowledge, however, it seems that these equations are likely to be as valid as any of the others. Changing the constant factors has no effect upon the qualitative results of the theory presented here, and only to a minor extent upon the quantitative results.

#### 4. PELTIER E.M.F.

Sir J. J. Thomson obtained equation (1) given above for the Peltier E.M.F. by assuming that electrons under the influence of an electric force flow across the boundary between two metals until the electric force is in equilibrium with the opposing pressure gradient at the boundary. The expression for the Peltier E.M.F. on the basis of the present extension of the simple theory is readily obtained by substituting in equation (1) the values of  $N_1$  and  $N_2$  given in equation (9), namely,  $N_1 = a_1 e^{x_1 T}$  and  $N_2 = a_2 e^{x_2 T}$ . This gives

$$P_{12} = \frac{2\alpha T}{3\epsilon} \left\{ \log \frac{a_1}{a_2} + (x_1 - x_2)T \right\}. \quad (12)$$

This equation may be rewritten in the form

$$P = AT + BT^2, \quad (12a)$$

or

$$P = P_0 + (273B + Q_0)t + Bt^2, \quad (12b)$$

where

$$A = \frac{2\alpha}{3\epsilon} \log (a_1/a_2), \quad B = \frac{2\alpha}{3\epsilon} (x_1 - x_2), \quad Q_0 = A + 273B, \quad P_0 = 273Q_0,$$

and  $t = T - 273$ .  $P_0$  is the Peltier E.M.F. at  $0^\circ$  C. and, as we shall see in the next section,  $Q_0$  is the thermoelectric power of the pair of metals at  $0^\circ$  C.

The equations (12), (12a) and (12b) represent a parabola with its axis perpendicular to the axis of temperature. We should expect the Peltier E.M.F. to have a maximum value at the absolute temperature.

$$T' = -A/2B, \quad (13)$$

or in Centigrade degrees,

$$t' = -A/2B - 273 = -(273B + Q_0)/2B. \quad (13a)$$

The value of the Peltier E.M.F. at the temperature  $T'$  should be

$$P' = -A^2/4B. \quad (14)$$

If we knew the experimental form of the parabola it would be possible for us to calculate the values of  $A$  and  $B$ , and from these values we could obtain the value of the ratio  $a_1/a_2$  and also of the difference  $(x_1 - x_2)$ . In practice, however, it will be very much simpler to obtain these values from the thermo E.M.F.

#### 5. THERMOELECTRIC POWER.

The equations for the thermoelectric power are readily obtained from the corresponding ones for the Peltier E.M.F. by dividing by  $T$  (see equation (7)). From equations (12), (12a) and (12b) we thus obtain

$$Q_{12} = \frac{2\alpha}{3\epsilon} \left\{ \log \frac{a_1}{a_2} + (x_1 - x_2)T \right\}, \quad (15)$$

and

$$Q = A + BT, \quad (15a)$$

$$Q = Q_0 + Bt, \quad (15b)$$

where  $A$ ,  $B$  and  $Q_0$  have the same values as in the preceding section. These equations represent quite accurately, by a suitable choice of constants, the thermoelectric power of a thermocouple consisting of almost any pair of metals.

#### 6. THERMO E.M.F.

The expression for the thermo E.M.F. is obtained from that for the thermoelectric power by integrating the latter with respect to  $T$ . If  $E$  is the thermo E.M.F. between two metals 1 and 2, when their junctions are at the temperature  $T_1$  and  $T_2$ , then

$$E = \int_{T_1}^{T_2} QdT,$$

whence

$$E = \frac{2\alpha}{3\epsilon} \left\{ \left( \log \frac{a_1}{a_2} \right) (T_2 - T_1) + \frac{1}{2} (x_1 - x_2) (T_2^2 - T_1^2) \right\}. \quad (16)$$

If we put  $T_1 = 0$  and  $T_2 = T$ , then equation (16) becomes

$$E = \frac{2\alpha}{3\epsilon} \left\{ T \log \frac{a_1}{a_2} + \frac{1}{2} (x_1 - x_2) T^2 \right\}, \quad (17)$$

where  $E$  is the thermo E.M.F. between two junctions at  $0^\circ$  and  $T^\circ$  absolute, respectively. Equation (17) may be rewritten in the form

$$E = AT + \frac{1}{2}BT^2. \quad (17a)$$

Substituting  $t + 273^\circ$  for  $T_2$  and  $273$  for  $T_1$  in equation (16) we obtain

$$E = Q_0t + \frac{1}{2}Bt^2, \quad (16a)$$

where  $E$  is the thermo E.M.F. between two junctions at  $0^\circ$  C. and  $t^\circ$  C., respectively.

Equations (17), (17a) and (16a) represent a parabola with its axis perpendicular to the axis of temperature. The maximum value of  $E$  occurs at the neutral temperature, which is therefore given by the equation

$$T' = -A/B, \quad (18)$$

or

$$t' = -Q_0/B. \quad (18a)$$

The thermo E.M.F. of the couple when one junction is at  $0^\circ$  C. and the other is at the neutral temperature is given by the equation

$$E' = -Q_0^2/2B. \quad (19)$$

Experimentally it is found that the thermo E.M.F. between any two metals is quite accurately represented by a parabola with its axis perpendicular to the axis of temperature, that is, by equations of the type deduced above, where  $A$  and  $B$  may either positive or negative. Thus we see that the equations here deduced for thermoelectric power and thermo E.M.F. are in agreement with the experimental facts. And since equation (7) has been shown to be true both experimentally and theoretically, the equations for the Peltier E.M.F. must also be in agreement with the experimental facts. From our theory we see that  $A$  is positive or negative according as  $a_1$  is greater or less than  $a_2$ . Similarly,  $B$  is positive or negative according as  $x_1$  is greater or less than  $x_2$ .

From the known values of  $A$  and  $B$  obtained from experiment we may calculate the values of  $a_1/a_2$  and  $(x_1 - x_2)$  by means of the following equations:

$$a_1/a_2 = e^{3\epsilon A/2\alpha}, \quad (20)$$

and

$$x_1 - x_2 = 3\epsilon B/2\alpha. \quad (21)$$

It may be remarked in passing that if  $N$  were a constant different for different metals, or if  $N$  were proportional to any power of the absolute



temperature, the proportionality factor being different for different metals, then the thermoelectric power would be constant and the thermo E.M.F. would be proportional to the difference between the absolute temperatures of the two junctions. These conclusions, however, are contrary to experiment.

#### 7. THOMSON EFFECT.

When we substitute  $ae^{xT}$  for  $N$  in equation (3) we obtain

$$\sigma = \frac{2\alpha}{3\epsilon} (Tx - \frac{1}{2}). \quad (22)$$

Since for practically all metals the absolute value of  $\sigma$  is considerably less than  $\alpha/3\epsilon$ , it follows that  $x$  must be positive. Hence, we may conclude that *in order to account for the Thomson effect it is necessary to assume that the number of free electrons in a metal increases with the absolute temperature*. So long as  $x$  is positive,  $\sigma$  will have the same sign as  $(Tx - \frac{1}{2})$ , and the absolute value of  $\sigma$ , when it is negative, cannot be greater than  $\alpha/3\epsilon$ . On the other hand,  $\sigma$  may have any positive value when  $x$  is positive. In the case of certain alloys<sup>1</sup> it appears that  $\sigma$  is negative and greater than  $\alpha/3\epsilon$ . In such cases  $x$  must be negative.

Differentiating equation (22) with respect to  $T$ , we obtain

$$\frac{d\sigma}{dT} = \frac{2\alpha x}{3\epsilon}. \quad (23)$$

From equation (23) it appears that *the Thomson effect has a temperature coefficient, and that this temperature coefficient is normally positive*. Qualitatively at least, this result is in harmony with some of the best experimental results such, for example, as those obtained by Berg<sup>2</sup> on copper. In the case of the alloys mentioned above, which have abnormally high negative values of the Thomson effect, Laws found that the temperature coefficients were negative. This is in harmony with our theory.

From the value of the Thomson effect at any temperature it is possible to calculate the value of  $x$ . Solving equation (22) for  $x$  we obtain

$$x = \frac{3\epsilon\sigma + \alpha}{2\alpha T}. \quad (24)$$

Generally speaking,  $Tx$  cannot be very far different from  $\frac{1}{2}$  at ordinary temperatures, since  $\sigma$  is usually quite small in comparison with  $\alpha/3\epsilon$ . This being so, let us assume that  $Tx = \frac{1}{2}$  and that  $T = 300^\circ$  absolute ( $27^\circ$  C.). Then  $x = 0.001667$ . Substituting this value of  $x$  in equation

<sup>1</sup> Laws, Phil. Mag., 7, pp. 560-578, 1904, and Caswell, Phys. Rev., N.S., XII., pp. 231-237, 1918.

<sup>2</sup> Ann. Phys., 32, pp. 477-519, 1910.

(9), we find the ratio of the value of  $N$  at any temperature  $(T + 1)$  to its value at the temperature  $T$  to be 1.0017. Or, to use a somewhat larger temperature interval, the ratio of  $N$  at  $(t + 100)^\circ \text{C.}$  to its value at  $t^\circ \text{C.}$  is 1.181. For a temperature interval of  $1000^\circ \text{C.}$  the ratio is 5.294. Assuming 0.002 as a probably maximum value of  $\alpha$  for a pure metal, we find the corresponding values of the ratio for a temperature interval of one degree Centigrade to be 1.0020, for a  $100^\circ \text{C.}$  interval 1.2214, and for a  $1000^\circ \text{C.}$  interval 7.389.

If our theory is correct, the Thomson effect is a linear function of the temperature which approaches the common value for all metals at the absolute zero of  $\sigma = -\alpha/3\epsilon$ . In case we should have used  $4\alpha/3\epsilon$  instead of  $2\alpha/3\epsilon$  in equations (1), (2) and (3), then  $\sigma$  should have a different limiting value. The same is true if any other factor is used instead of  $2\alpha/3\epsilon$ . If this limiting value of  $\sigma$  can be determined, then we have a means of selecting the best from among the different equations proposed.

It follows at once from equation (22) that

$$\sigma_1 - \sigma_2 = \frac{2\alpha T}{3\epsilon}(x_1 - x_2), \quad (25)$$

or

$$\sigma_1 - \sigma_2 = BT, \quad (25a)$$

these equations being in the form of equation (8). It is obvious that the metal having the more positive value of  $x$  will have the more positive value of  $\sigma$ , and *vice versa*. Consequently, if we were to arrange a series of metals in the order of the slope of the corresponding lines in a thermoelectric power diagram, the metals should also be arranged in the order of their Thomson effects at any given temperature.

## 8. ELECTRICAL CONDUCTIVITY.

In the preceding sections the only assumption which has been employed is that the number of free electrons is a function of the temperature, and the satisfactory agreement between the theory and experiment seems to prove the wisdom of selecting the expression  $ae^{xT}$  to represent this number at any temperature  $T$ . No assumption was made as to the sign of  $x$ , but we have found that to account for the Thomson effect in most substances  $x$  must be positive. To this assumption we now add a second, namely, that the number of positive centers is a function of the temperature, and, in consequence, the mean free path of the electrons is a function of the temperature. Any discrepancies between theory and experiment which appear in this section must, therefore, arise not from the first assumption, but from the second.

Substituting the value of  $N$  from equation (9), the value of  $\lambda$  from equation (11), and  $u = (2\alpha T/m)^{1/2}$  in equation (4), we obtain as the value of the electrical conductivity

$$\kappa = \frac{\epsilon^2 ac}{2 \sqrt{2\alpha m}} e^{(x-y)T} \cdot T^{-1/2}, \quad (26)$$

which we shall rewrite in the form

$$\kappa = C e^{-zT} \cdot T^{-1/2}, \quad (26a)$$

where  $C = \epsilon^2 ac/2(2\alpha m)^{1/2}$ , and  $z = y - x$ . Both  $C$  and  $z$  are constants for any given metal.

Since the electrical conductivity of pure metals within the range of ordinary temperatures is roughly inversely proportional to the absolute temperature, it follows from equation (26a) that the exponential factor on the right-hand side of the equation must decrease with increase of temperature. In order that this shall be so,  $z$  must be positive or  $(x - y)$  must be negative. But since  $x$  is positive in the case of pure metals,  $y$  must also be positive and greater than  $x$ . Hence, we conclude that *in the case of pure metals both the number of free electrons in the metal and the number of positive centers with which they collide increases with the temperature, the number of positive centers, in general, increasing faster than the number of electrons.*

In the case of alloys the electrical conductivity cannot be said to bear any general relation to the temperature. In fact in the case of some alloys, such as manganin, the temperature coefficient is exceedingly small. In such cases the exponential factor may increase with the temperature. That is, if  $x$  is positive, then  $y$  is either positive and less than  $x$  or else  $y$  is negative, but if  $x$  should happen to be negative, as we have seen is probable in the case of some alloys, then  $y$  is also negative but numerically greater than  $x$ . The case when both  $x$  and  $y$  are positive seems the most likely to occur, since this is the rule for pure metals. In any case we may conclude that *in some substances, particularly alloys, the number of free electrons increases with the temperature faster than the number of positive centers with which they collide.*<sup>1</sup>

The electrical conductivity of carbon, which though not a metal conducts metallically, increases rather rapidly with the temperature. This means that  $z$  for carbon has a comparatively large negative value. This probably indicates a large positive value of  $x$  with a smaller positive, or possibly negative, value of  $y$ . An abnormally large value of  $x$  corre-

<sup>1</sup> A good account of the electrical properties of alloys is given in Baedeker's *Elektrischen Erscheinungen in Metallischen Leitern*. Lord Rayleigh (*Scientific Papers*, Vol. IV., p. 232) has suggested that the resistance of alloys is unduly high owing to a "false resistance" arising from the Peltier effect between non-homogeneous parts of the metal.

sponds to an abnormally large positive value of the Thomson effect.

Since  $\rho$  is the reciprocal of  $\kappa$  it follows that

$$\rho = \frac{1}{C} e^{zT} \cdot T^{1/2}, \quad (27)$$

whence

$$z = \frac{1}{T} (\log C + \log \rho - \frac{1}{2} \log T). \quad (28)$$

If  $\rho$  is proportional to  $T$ , as is assumed but is not strictly true to fact, then

$$z = \frac{1}{T} (\log (CD) + \frac{1}{2} \log T), \quad (28a)$$

where  $\rho$  is put equal to  $DT$ .

From equation (28a) it appears that either  $\rho$  is not proportional to  $T$  or else  $z$  is not constant as assumed, but is a function of  $T$ . Whenever the value of  $z$  is greater than the right-hand side of equation (28a) the resistivity increases faster than the absolute temperature, but when the reverse is true the resistivity increases more slowly than the absolute temperature.

From equation (27) it is obvious that the resistivity of a metal approaches zero at the absolute zero, and increases continuously with the temperature providing  $z$  is positive. In case  $z$  is negative the resistivity should have a maximum value when  $T = - (1/2z)$ .

Since the factors  $a$  and  $c$ , which are contained in the value of  $C$ , are as yet undetermined, we cannot use equation (28a) to calculate the value of  $z$ . But since we know how  $\rho$  varies with  $T$  we may write

$$\frac{\rho_1}{\rho_0} = e^{z(T_1 - T_0)} \left( \frac{T_1}{T_0} \right)^{1/2}, \quad (29)$$

where  $\rho_1$  and  $\rho_0$  are the resistivities at  $T_1^\circ$  and  $T_0^\circ$  absolute, respectively. Putting  $T_1 = T_0 + 1$ , and solving for  $z$ , we obtain

$$z = \log \frac{\rho_1}{\rho_0} - \frac{1}{2} \log \left( 1 + \frac{1}{T_0} \right). \quad (29a)$$

If we denote the temperature coefficient of resistance at  $T_0$  by  $\beta$ , we may rewrite equation (29a) thus:

$$z = \log (1 + \beta) - \frac{1}{2} \log \left( 1 + \frac{1}{T_0} \right), \quad (29b)$$

or as a first approximation which gives a result about one half per cent. too high

$$z = \beta - \frac{1}{2T_0}. \quad (29c)$$

We find the temperature coefficient of resistivity of copper given as 0.00428 at 18° C., *i. e.*,  $T_0 = 291$ , and so  $z$  is found to be 0.002545 if we use equation (29*b*), but is found to be 0.00256 if we use equation (29*c*).

From the ratio of the resistivities of two metals at any temperature  $T$  we may determine the ratio  $c_1/c_2$ , thus:

$$\frac{c_1}{c_2} = \frac{\rho_2 a_2}{\rho_1 a_1} e^{(z_1 - z_2)T}. \quad (30)$$

Since all the quantities on the right can be determined the ratio  $c_1/c_2$  can be calculated. It should be noted, however, that the numerical values which we obtain for  $a_1/a_2$ , and consequently for  $c_1/c_2$ , depend upon whether we use  $2\alpha/3\epsilon$  or some other multiplying factor in equations (1), (2) and (3).

#### 9. THERMAL CONDUCTIVITY.

We now proceed to introduce the third assumption, namely, that the molecules or, more generally, the positive centers, whether these are atoms, sub-atoms, molecules or groups of molecules, take part in the transmission of heat, although they have no part in the transmission of electricity in solid bodies. In the simple equation (5), namely,  $k = \frac{1}{3}\alpha N\lambda u$ ,  $N$ ,  $\lambda$  and  $u$  refer specifically to the electrons. A fundamental assumption of the electron theory is the equi-partition of energy among the electrons and molecules, but there are certain reasons why we are at present unable to assign values to the molecular quantities corresponding to  $N$ ,  $\lambda$  and  $u$ . Indeed for our present purpose it is unimportant whether such values can be assigned. Obviously, there is the probability that heat is transmitted by the molecules, or positive centers, by radiation as well as by conduction. What fraction of the total heat transmitted is transmitted by radiation we are unable to determine. For the present it remains an open question whether the equi-partition of energy law applies to the molecules or to the positive centers.

It is rather obvious that the mean free paths and the velocities of the positive centers, or of the molecules, will vary in the same directions as  $\lambda$  and  $u$ , respectively. We shall assume, for simplicity, that  $\lambda_1$  is directly proportional to  $\lambda$ , and that  $u_1$  is directly proportional to  $u$ , where  $\lambda_1$  is the mean free path of the positive centers, or molecules, as the case may be, and  $u_1$  is the corresponding mean square speed. We may then rewrite equation (5), substituting  $N + n$  for  $N$  and introducing a constant  $\omega$  which includes the proportionality factors connecting  $\lambda$  and  $\lambda_1$ , and  $u$  and  $u_1$ , and also the ratio of the total amount of heat transmitted by the molecules to that conducted by them. We thus obtain

$$k = \frac{1}{3}\alpha u\lambda(N + \omega n). \quad (31)$$

Equation (31) simply means that in view of our ignorance of the behavior of these positive centers, we feel justified in assuming that the fraction of the total heat transmitted by them is proportional to their number.

Replacing  $N$  by  $ae^{xT}$ ,  $n$  by  $be^{yT}$ ,  $\lambda$  by  $ce^{-yT}$  (see equations (9), (10) and (11)), and  $u$  by  $(2\alpha T/m)^{1/2}$ , equation (31) reduces to

$$k = \frac{\sqrt{2\alpha}}{3\sqrt{m}} ac \left( e^{(x-y)T} + \frac{\omega b}{a} \right) T^{1/2}. \quad (32)$$

It is at once apparent that in the case of pure metals the exponential factor in the first term on the right-hand side of equation (32) tends to make  $k$  decrease as  $T$  increases, since  $(x - y)$  is negative, while the factor  $T^{1/2}$  tends to make both terms increase as  $T$  increases. So  $k$  may either increase or decrease with increase in temperature.

#### 10. THE WIEDEMANN-FRANZ AND LORENZ LAWS.

We have already seen that the Wiedemann-Franz law coupled with that of Lorenz leads to the relation that  $k/\kappa T = a \text{ constant}$ . Substituting the values of  $k$  and  $\kappa$  from equations (32) and (26), respectively, we obtain

$$\frac{k}{\kappa T} = \frac{4\alpha^2}{3\epsilon^2} \left( 1 + \frac{\omega b}{a} e^{zT} \right). \quad (33)$$

When  $\omega = 0$ , this equation reduces to equation (6) as a special case. Equation (33) leads us to expect that the experimentally determined value of the ratio will always be greater than that calculated from the simple theory. Assuming the validity of equation (6), the experiments of Jaeger and Diesselhorst<sup>1</sup> on thirteen metals and three alloys gave values in every case, except that of aluminum, greater than we should expect from the simple theory; but if we assume the validity of equation (6a) the discrepancy is much greater, even aluminum being about thirty per cent. too high. It has been objected that the amount of heat transmitted by the positive centers, whatever they may be, must be much too small to account for the discrepancies between Jaeger and Diesselhorst's experiments and the simple theory. But this objection is based on the assumption that the heat so transmitted must be approximately the same as that transmitted by an insulator. This does not necessarily follow. If there are practically no free electrons in an insulator and there are large numbers of them in conductors, is it not reasonable to suppose that there may be more freedom of movement among the molecules in a conductor than among those in an insulator?

The minimum value which the right-hand side of equation (33) can have is  $4\alpha^2(1 + \omega b/a)/3\epsilon^2$ . This seems out of harmony with the results

<sup>1</sup> Sitzungsber. Berlin, 1899, p. 719.

obtained by Meissner<sup>1</sup> at very low temperatures, since he found that for copper at 20° absolute the value of the ratio  $k/\kappa T$  was but one seventh of its value at 0° C., the value at 0° C. being only slightly greater than  $4\alpha^2/3\epsilon^2$ .

If the subscripts 1 and 2 refer, respectively, to the temperature  $T_1$  and  $T_2$ , then

$$\frac{k_2/\kappa_2}{k_1/\kappa_1} = \frac{\left(1 + \frac{\omega b}{a} e^{zT_2}\right) T_2}{\left(1 + \frac{\omega b}{a} e^{zT_1}\right) T_1}. \quad (34)$$

According to the simple theory this should be

$$\frac{k_2/\kappa_2}{k_1/\kappa_1} = T_2/T_1. \quad (34a)$$

Jaeger and Diesselhorst, working between the temperatures of 18° C. and 100° C., *i. e.*, between 291° and 373° absolute, found instead of the theoretical value of the ratio on the left-hand side of equation (34a), *viz.*,  $T_2/T_1 = 1.28$ , that eight of the thirteen metals tested gave values greater than 1.28, two of them, platinum and palladium, giving values of 1.35, two gave values of 1.28, one of 1.27, one of 1.26 and one, bismuth, of 1.12. The three alloys tested gave results less than 1.28. Whenever  $y$  is greater than  $x$ , as we have seen is the case in pure metals, equation (34) leads us to expect the ratio to be greater than 1.28. In the majority of cases this seems to be true. But in the case of an alloy, such as manganin, which has a very small temperature coefficient of resistance,  $x$  is greater than  $y$ , and equation (34) leads us to expect the ratio to be less than 1.28. This is actually the case. In the case of manganin the ratio is 1.21.

## II. DISCUSSION OF THE THEORY.

In this section the author wishes to make some general observations concerning the theory here presented. In the first place, we see that, since the equations are of the proper form and the values of the constants involved can be determined from experimental data, the agreement of the theory with the phenomena of the Peltier and Seebeck effects is not only qualitative but quantitative as well. Not only do the values of the constants so determined indicate that, in general, the number of free electrons increases with the temperature, but this is what we might expect, since the kinetic energy and, consequently, the agitation of the molecules increases with the temperature.

In a previous paper the author has shown that either (1) the electrons

<sup>1</sup> Deutsch. Phys. Gesell., Verh. 16, 5, pp. 262-272, 1914.



which take part in thermoelectric phenomena are different from those which take part in electric conduction, or (2) the mean free path of the electrons in an alloy is noticeably different from that in a pure metal, one of the constituents of the alloy. The former statement seems unreasonable, while the latter is plausible if the formation of an alloy involves the regrouping of the atoms or molecules. Thus the writer was led to the conclusion that in some way a change is brought about in the mean free path of the electrons. The introduction of the exponential value of  $N$  from equation (9) into equation (4) makes the resulting change in the electrical conductivity in the opposite direction to that in which it actually occurs. So we have adopted the hypothesis that the mean free path of the electrons as well as their number is a function of the temperature. The change in the mean free path may be thought of as being brought about in either of two ways, always assuming that the number of collisions between electrons is negligible in comparison with the number between electrons and the positive centers. Either the size of the positive centers may change or their number. Assuming that the size changes but not the number we are led to conclusions which do not agree as well with experiment as do the conclusions which we arrive at by assuming that the number of positive centers is a function of the temperature. The author is inclined to believe that the exponential form of the function which has been employed in this paper is a fair representation of the facts, if temperatures near which marked changes in the structure of the metal occur, such as eutectic-points and melting-points, are excluded. In this connection it should be noted that the Peltier E.M.F. and the thermo E.M.F., both of which we assume depend only upon the concentration of the electrons, are not discontinuous functions of the temperature at the melting-point, but it appears that the electrical conductivity of a metal changes markedly as it passes from the solid to the liquid state. In the case of a number of metals, *e. g.*, cadmium, lead, potassium, sodium, tin and zinc, the electrical conductivity in the solid state at the melting-point is about one half the value in the liquid state at the same temperature. Exceptions are bismuth and antimony. In both of these the conductivity is approximately doubled as the substances passes from the solid to the liquid state. The usual case, that of decrease in conductivity in passing from the solid to the liquid state, is apparently due to a sudden increase in the number of positive centers. This is what one might expect since there is, apparently, a reduction in the forces of cohesion.

It seems probable that at low temperatures the molecules of a metal are gathered in clusters which act as single positive centers and that



as the temperature is raised the increased activity of the molecules causes these clusters to break up slowly. The complete theory should, therefore, take account of the change in the cross-sections of the clusters as they break up as well as the change in their number. What the change in the cross-section is must depend upon the number and arrangement of the molecules in each cluster, and any general expression must represent a mean rate of change in the cross-sections. Since the number of positive centers must increase faster than their average cross-section decreases, the change in the former is more important than that in the latter.

In the theory as developed in this paper no allowance has been made for the expansion of the metal as the temperature is raised. The complete theory ought to take account of this expansion. The correction, however, is likely to be quite small.

It is obvious that an exhaustive study of the quantitative relations of this theory and experiment ought to be made. We have shown that qualitatively it is capable of explaining all of the phenomena considered if we exclude very low temperatures. It is also capable of explaining each of these phenomena separately quantitatively, but it remains to be seen to what extent the values of the constants involved are identical when different phenomena are compared. The writer is now carrying out the necessary computations to put the theory on a quantitative basis and hopes to publish the results of this work in the near future.

PHYSICAL LABORATORY,  
UNIVERSITY OF OREGON,  
February, 1919.

## THE AUDION AS A CIRCUIT ELEMENT.

BY H. W. NICHOLS.

SYNOPSIS.—The functional equations for the three-element audion are used to deduce the actions taking place in any circuit containing the audion. The equations for small variations in the general network containing audions are developed and simple equivalent circuits to replace the tube are shown. Next the effects of the capacities between elements are worked out and also some examples of amplifiers and oscillation generators.

THE three-element thermionic tube has come into such general use in radio telegraphy and telephony as well as in wire telephony and other fields that it seems worth while at the present time to publish some of those relations, holding between its characteristics and those of the connected electrical circuits, which are used in the design of amplifying, oscillating and detecting systems.

In this paper will be considered only those actions which may be sufficiently well described by supposing the variations in plate and grid currents and voltages to be small. In this discussion the methods adopted in a previous paper<sup>1</sup> will be shown in their application to vacuum tube circuits, in which application, as a matter of fact, their greatest practical use has been found.

The first figure shows a conventional circuit drawing of an audion in which the grid and plate potentials, with filament potential as zero, are denoted by  $v$  and  $V$  respectively, while the corresponding currents are denoted by  $i$  and  $I$ . A discussion of the operation of this device is unnecessary, since several papers have appeared describing the construction and operation in considerable detail. It has been found that, as long as there is an excess of electrons supplied by the filament so that temperature saturation is maintained, the plate current is represented with considerable accuracy by a function of a *single* variable, thus:

$$I = F(V + \mu v + c),$$

in which  $c$  is a constant which depends partly upon the contact differences of potential of the elements in the tube and  $\mu$  is another constant which is characteristic of the structure. In many forms which the device may take the plate current is very approximately proportional to the square

<sup>1</sup> PHYS. REV., Aug., 1917.

of the variable above, but for the purpose of this discussion it is not necessary to so limit it.

The grid current depends also upon the two potentials, but its functional form is not known. For this discussion it is sufficient to recognize that it is such a function and write it:

$$i = f(V, v).$$

The problem to be considered in this paper is now the following. An audion is connected into an electrical network in any manner, that is, its three terminals are joined by any system of impedances; certain grid and plate voltages are impressed upon it with the result that certain currents, determined by the functional equations above, flow to plate and grid; and then it is required to describe the small variations about

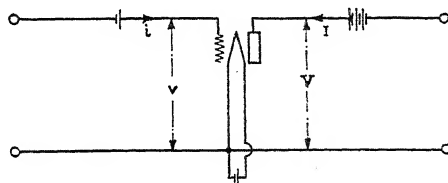


Fig. 1.

this steady state which will take place when the system is acted upon by arbitrary electromotive forces or otherwise disturbed. This is the problem of amplifiers and oscillation generators with the limitation that the amplitude of oscillation shall be small. While this limitation is serious in some cases, for example if it is desired to find the maximum power output of the device, still a great deal of information concerning the operation of the actual device may be obtained by thus studying the ideal case, which, in fact, is the only one which lends itself to simple and general treatment from the point of view of electrical networks.

In Fig. 2 is shown the audion connected into the network. In this figure the impedances  $Z_1, Z_2, Z_3$ , may be of any form whatever and are not restricted to single conducting paths as indicated. They are supposed only to be such circuits that it is possible to write an equation of the form  $ZI = e$ , connecting the current entering the terminals and the E.M.F. producing this current when the circuit is isolated. Certain electromotive forces,  $E$ , are supposed acting in each branch and the three circuits are coupled to one another by mutual impedances,  $M_{jk}$ , between impedance  $j$  and impedance  $k$ . These mutual impedances are defined, as usual, by the statement that a unit current in branch  $j$  induces an E.M.F.  $M_{jk}$  in branch  $k$ . Both self and mutual impedances are in the

general case operators, functions of  $p = d/dt$ , as used in the paper above cited.

If in this network the three currents are chosen as shown, we may write down the network equations in terms of them as below:

$$Z_1(I_1 + I_3) + v = E_1 + M_{12}(I_2 - I_3) - M_{13}I_3$$

$$Z_2(I_2 - I_3) + V = E_2 + M_{12}(I_1 + I_3) + M_{23}I_3$$

$$Z_3I_3 + V - v = -M_{13}(I_1 + I_3) + M_{23}(I_2 - I_3).$$

These equations are of course exact and hold for all conditions; however, because they are not linear, their solution is somewhat complicated and it is preferable to obtain the first and approximate idea of the be-

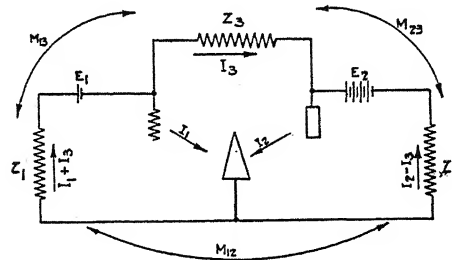


Fig. 2.

havior of the network by considering only small changes in the currents from their steady or mean values. This amounts to considering current variations over a part of the characteristic curve so small that it may be taken to coincide with the tangent at the given operating point and consequently the solutions obtained will not describe any effects which depend upon the curvature of the characteristic, such effects, for example, as the detection of signals.

Suppose then that in the departure from the steady values  $I_1$  changes to  $I_1 + x_1$ , etc., these changes being small and produced by superposed electromotive forces  $e_1$ , etc. We shall then have

$$\delta v = \frac{\partial v}{\partial I_1} x_1 + \frac{\partial v}{\partial I_2} x_2, \quad \text{etc.}$$

Substituting these values in the network equations, there will result a set of linear differential equations, with independent variable  $t$ , of the type:

$$Z_1(x_1 + x_3) + \frac{\partial v}{\partial I_1} x_1 + \frac{\partial v}{\partial I_2} x_2 = e_1 + M_{12}(x_2 - x_3) - M_{13}x_3$$

which set may be conveniently represented in the following tabular form:

$x_1$	$x_2$	$x_3$	
$Z_1 + \frac{\partial v}{\partial I_1}$	$-M_{12} + \frac{\partial v}{I_2 \partial}$	$Z_1 + M_{12} + M_{13}$	$e_1$
$-M_{12} + \frac{\partial V}{\partial I_1}$	$Z_2 + \frac{\partial V}{\partial I_2}$	$-Z_2 - M_{12} - M_{23}$	$e_2$
$-M_{13} + \frac{\partial V}{\partial I_1} - \frac{\partial v}{\partial I_1}$	$-M_{23} + \frac{\partial V}{\partial I_2} - \frac{\partial v}{\partial I_2}$	$Z_3 + M_{13} + M_{23}$	$e_3$

These are the network equations for small disturbances but it is convenient to throw them into another form which takes account of the physical meaning of some of the terms. To do this, note that  $\partial v / \partial I_1$  is evidently the resistance,  $r$ , of the grid circuit under the given operating conditions, and  $\partial V / \partial I_2$  is the resistance,  $R$ , of the plate circuit under the same conditions. From the characteristic equations we find by the usual methods, the further relations:

$$\frac{\partial v}{\partial I_1} = \frac{F'}{J} = r, \quad \frac{\partial v}{\partial I_2} = \frac{-1}{J} \frac{\partial f}{\partial V},$$

$$\frac{\partial V}{\partial I_1} = -\frac{\mu F'}{J} = -\mu r, \quad \frac{\partial V}{\partial I_2} = \frac{1}{J} \frac{\partial f}{\partial v} = R,$$

where

$$J = F' \left( \frac{\partial f}{\partial v} - \mu \frac{\partial f}{\partial V} \right).$$

Now since  $\partial V / \partial I_1$  is the E.M.F. induced in the plate circuit through the tube by unit current in the grid circuit and  $rx_1$  is the corresponding increment,  $\delta v$ , in grid potential, it follows from the above relations that one action of the tube is to introduce into the plate circuit an effective E.M.F. equal to  $\mu$  times the grid voltage variation.

Also, since  $\partial v / \partial I_2$  is the E.M.F. introduced into the grid circuit through the tube by unit current in the plate circuit, we find that another action of the tube is to introduce the voltage

$$\frac{\partial v}{\partial I_2} x_2 = -\frac{1}{J} \frac{\partial f}{\partial V} x_2 = -R x_2 \frac{\partial f / \partial v}{\partial f / \partial V} = -\delta V \cdot \frac{\partial f / \partial V}{\partial f / \partial v}$$

into the grid circuit. Thus the two mutual impedances of the tube are in general different in the two directions, which accounts for the amplifying properties of the device as explained in the paper already mentioned. This reverse action, from plate circuit to grid circuit, is often ignored in discussions of the audion and it is assumed that, since the plate current is a function of  $V + \mu v$  only, the action of the device is summed up in the statement that the grid is  $\mu$  times as effective as the plate in pro-

ducing current changes. This action is, however, the only one if, for example, the grid current is independent of plate potential: there is then no reaction upon the grid circuit (neglecting capacity from grid to plate, which will be considered later) and the tube is a purely unilateral device. Since in practice this condition often holds very approximately, it is often legitimate to neglect  $\partial v / \partial I_2$  and consequently to consider the tube as a device which, for small current changes, simply introduces into the

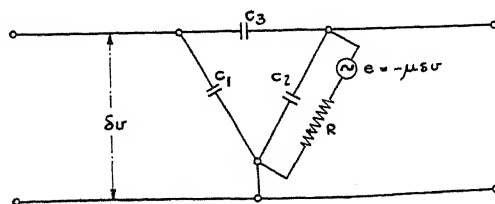


Fig. 3.

plate circuit (just where will develop later) an E.M.F. equal to  $\mu$  times the change in grid potential. When this condition holds the last equations are still further simplified into:

$x_1$	$x_2$	$x_3$	
$Z_1 + r$	$-M_{12}$	$Z_1 + M_{12} + M_{13}$	$e_1$
$-M_{12} - \mu r$	$Z_2 + R$	$-Z_2 - M_{12} - M_{23}$	$e_2$
$+M_{13} - (\mu + 1)r$	$-M_{23} + R$	$Z_3 + M_{13} + M_{23}$	$e_3$

*Equivalent Circuit of the Tube.*—When the grid current does not vary appreciably with plate voltage it is possible to use a very simple equivalent circuit for the audion itself. This circuit is shown in Fig. 3 in which the resistance  $R$  of the plate circuit is shown between plate and filament and the three condensers,  $c_1$ ,  $c_2$ ,  $c_3$  represent respectively the internal capacities between grid-filament, plate-filament and grid-plate. These capacities must obviously be taken into account at sufficiently high frequencies. Under the given conditions  $r$  is effectively infinite. Located in the impedance of the plate circuit is an alternator whose E.M.F. is equal to  $\mu$  times the voltage,  $\delta v$ , produced across the grid-filament terminals. If this circuit is worked out it will be found that it is the exact equivalent of the actual tube under the given conditions. The feature of the circuit is the location of the fictitious E.M.F. between filament and plate, and the network will serve for the solution of tube problems as long as the electromotive forces impressed upon the network are small enough not to violate the conditions. The use of this equivalent avoids the necessity of setting up the complete equations.

It is obvious that, as a closer approximation, an equivalent circuit may be drawn in which another alternator is used to represent the reaction upon the grid circuit, but for practical use it has been found that this complication is in most cases unnecessary. Except at the very highest frequencies this network may be still further simplified by omitting the condenser  $c_2$  since its capacity is but a few micro microfarads and it is shunted by the resistance  $R$ . The other condensers are, how-

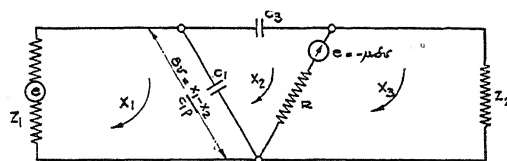


Fig. 4.

ever, very important, even at moderate frequencies, and this will be shown by two examples.

1. *The Input Impedance of an Audion Amplifier.*—Fig. 4 represents the equivalent circuit of an audion for frequencies below a few million and with the grid maintained negative with respect to the filament so that the input resistance is infinite. The output circuit is closed through an impedance  $Z_2$ . If there were no connection between the plate and grid circuits the impedance of the amplifier, as measured from the input terminals, would be simply that of the condenser  $c_1$ . The grid-plate capacity, however, modifies this result greatly. This impedance will be worked out and to do this the network equations, in terms of the mesh currents  $x_1, x_2, x_3$  are given below with an E.M.F.  $e$  in the input circuit.  $p$  represents  $d/dt$  as usual and the fictitious E.M.F. in the tube resistance is  $\mu$  times the voltage drop,  $(x_1 - x_2)/c_1 p$ , over the condenser  $c_1$ . The sign is such that a positive grid tends to increase the plate current.

$x_1$	$x_2$	$x_3$	
$Z_1 + \frac{1}{c_1 p}$	$-\frac{1}{c_1 p}$	0	0
$-\frac{1 + \mu}{c_1 p}$	$\frac{1 + \mu}{c_1 p} + \frac{1}{c_3 p} + R$	$-R$	0
$\frac{\mu}{c_1 p}$	$-R - \frac{\mu}{c_1 p}$	$R + Z_2$	$e$

If  $D$  is the operational determinant of this system and  $D_{11}$  the minor of the element (1, 1) the input impedance is

$$Z = D/D_{11} \text{ with } Z_1 = 0,$$

and this works out to be

$$Z = \frac{1}{c_1 p} \frac{1 + Wc_3 p}{1 + Wc_3 p + \frac{c_3}{c_1} \left( 1 + \mu - \mu \frac{W}{Z_2} \right)},$$

in which  $W$  is equal to  $RZ_2/(R + Z_2)$ , the impedance of  $R$  and  $Z_2$  in parallel. Note that if the grid-plate capacity is zero, this reduces to  $1/c_1 p$  which is the impedance of the grid-filament condenser. This impedance is in the operational form; to find the impedance to steady alternating currents of frequency  $n/2\pi$ ,  $p$  is to be replaced by  $in$  where  $i$  is the imaginary unit.

To save space only a few consequences will be noted, since it is easy to draw conclusions from the formula. In the first place, even at low frequencies (when the terms containing  $p$  as a factor are negligible) the impedance is charged into

$$\frac{1}{c_1 p} \frac{1}{1 + \frac{c_3}{c_1} \left( 1 + \mu - \mu \frac{W}{Z_2} \right)}; \quad p \text{ small.}$$

In some cases this impedance only remotely resembles a capacity reactance; for instance if the output impedance,  $Z_2$ , is inductive, the impedance may have a negative resistance component. This means that the amplifier is likely to sing under these conditions, and in fact the tendency of a tube to oscillate "through its internal capacities" has often been observed.

At extremely high frequencies the formula shows that the impedance again approaches a capacity reactance, as it obviously should do since then the condenser  $c_1$  practically short-circuits the current entering the input terminals. It is instructive to work out the impedance under various simplifying assumptions, and in the case of amplifiers which sing the solution of the difficulty will often be found in this manner.

2. *Amplification at High Frequencies.*—As a second example of the use of the equivalent circuit of the tube, suppose the circuit of Fig. 4 is to be used to produce in the impedance  $Z_2$  an amplified current under the influence of the generator  $e$  in the input circuit. It is clear from inspection that for very high frequencies the condenser  $c_3$  will have such a low impedance that the amplifying action of the tube will be seriously impaired. To get a quantitative measure of the effect of varying the frequency, consider what is meant by amplification. If the tube were a perfect unilateral amplifier a current  $x_1$  in mesh 1 would produce a certain E.M.F.,  $e_3$ , in mesh 3, while a current  $x_3$  in mesh 3 would produce no E.M.F. in mesh 1. On the other hand, if the network were a simple



transforming one (no amplification), the same E.M.F.'s would be produced by the same currents in either direction through the network. Hence, defining the two mutual impedances in the usual way as

$$M_{13} = \frac{e_3}{x_1}, \quad M_{31} = \frac{e_1}{x_3},$$

it is clear that for a perfect unilateral amplifier  $M_{31}/M_{13} = 0$ , while for a perfect transforming network this ratio is unity. The above ratio of the two mutual impedances is therefore a measure of the failure of the tube as an amplifier.

From the network determinant, already used, we find this ratio to be

$$\frac{M_{31}}{M_{13}} = \frac{D_{31}}{D_{13}} = \frac{1}{1 - \frac{\mu}{c_3 R p}},$$

or for simple harmonic currents of frequency  $n/2\pi$ :

$$\frac{M_{31}}{M_{13}} = \frac{1}{1 + \frac{i\mu}{Rc_3n}}.$$

It will be seen that for  $c_3 = 0$  this ratio is zero, as it should be, and for  $c_3$  infinite it is unity. Also, since the frequency appears in the ratio, the amplification will be different for different frequencies, so that the tube will distort and tend to eliminate higher frequencies.

With certain tubes and for certain frequencies for which the ratio  $\mu/Rc_3n$  is comparable with unity, the amplification is greatly impaired. This difficulty may, however, be avoided at any given frequency by the device of shunting the grid-plate capacity with an inductance of suitable value to make the impedance between grid and plate infinite at the given frequency. In doing this it is of course necessary to put in series with the inductance a large condenser to avoid conductive connection of grid and plate. It will be found that this method permits greater amplification at one frequency but it is open to the objection that variation of frequency is not convenient. Of course care must be taken not to couple this antiresonant mesh with others.

*The Network as an Oscillation Generator.*—In my PHYSICAL REVIEW paper the general theory of oscillation generators was given and it was shown that the possibility of sustained free oscillation of a network depends upon the existence of certain types of unsymmetrical terms in the characteristic determinant. The vanishing of the determinant then determines both the frequencies of all the possible modes of free oscillation and also the relations which must hold among the circuit

constants in order that the oscillations shall be sustained (or have given damping). Since the solution for any oscillation generator requires only the ability to write down the network equations and find the roots of the determinant, it is hardly necessary to go into the detailed solution for any special case. As an example, however, the network of Fig. 5, purposely simplified to avoid long equations, will be handled by the method of the simplest equivalent circuit. The plate-grid capacity is

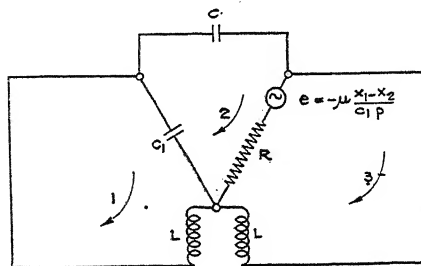


Fig. 5.

included in the capacity  $C$  and the mutual inductance of the two coils is  $m$ .

Choosing the mesh currents as shown, the network equations are

$x_1$	$x_2$	$x_3$	
$\frac{1}{c_1 p} + Lp$	$-\frac{1}{c_1 p}$	$-mp$	0
$-\frac{\mu + 1}{c_1 p}$	$\frac{\mu + 1}{c_1 p} + \frac{1}{Cp} + R$	$-R$	0
$mp + \frac{\mu}{c_1 p}$	$-R - \frac{\mu}{c_1 p}$	$Lp + R$	0

Note that these are the same as the last set with the addition of the transformer coupling. Now if we require that the oscillations be sustained,  $p$  must be taken equal to  $in$ , in which  $n$  is  $2\pi$  times the frequency of oscillation. Then the vanishing of the determinant, which is complex, will require that the real and imaginary parts separately shall vanish, that is, that the even and odd powers of  $p$  shall separately be zero. As explained in the paper above cited, this determines both the frequencies of all the modes of oscillation and the circuit adjustments which must be made to permit the oscillations.

It will be noticed that in this determinant all the even powers of  $p$  appear in the coefficient of  $R$ ; consequently by combining rows and columns until  $R$  appears in but one element, the even powers of  $p$  will be contained in the minor of that element, which minor, equated to

zero, will determine the frequencies of sustained oscillation. This process leads to the frequency equation:

$$(L^2 - m^2)p^4 + \left(\frac{L}{C} + 2\frac{L - m}{c_1}\right)p^2 + \frac{1}{Cc_1} = 0,$$

and there are therefore two possible frequencies of sustained oscillation, corresponding to the two values of  $p^2 = -n^2$ . Whether or not these oscillations will actually take place depends upon whether the circuit constants are adjusted to make the odd powers of  $p$  vanish. This second equation, obtained by equating to zero the imaginary part of  $D = 0$ , is:

$$(L^2 - m^2)\left(\frac{c_1}{C} + \mu + 1\right)p^2 + \frac{L + \mu m}{C} = 0.$$

If, when one of the values of  $p^2$  obtained from the first equation is substituted in this, it is satisfied, the oscillation corresponding to that value of  $p$  will take place. This fact explains a peculiarity of this kind of circuit, namely the common experience that in changing the frequency of an oscillator circuit the frequency may suddenly jump to some entirely different value for a small change in the constants. The reason is that in the process of changing the frequency the circuit constants are so changed that the relation holding among them becomes more nearly correct for another oscillation which would not before satisfy that relation. For example, in circuits of the type above, the most stable frequency is, for most ordinary adjustments, approximately that determined by the transformer and condenser  $C$  alone; but for some relations of  $L$ ,  $m$ ,  $\mu$ , etc., the circuit will not maintain this frequency but will pass to the other. It is possible to design a network in which, as a constant is varied, the frequency will jump discontinuously from one value to others. The converse problem is to design an oscillation circuit in which the condition for sustaining a given oscillation holds for all variations of the element which is used to vary the frequency. Such a circuit will be stable over the whole frequency range.

Returning to the particular circuit above, it will be seen that if the capacity  $c_1$  is very small the frequency of sustained oscillation will be that for which

$$n^2 = \frac{1}{2(L - m)C},$$

namely that corresponding to the single resonant circuit of the transformer and condenser  $C$ . The second equation then shows that this oscillation will be sustained if  $\mu = 1$ . If the inductances of the two windings had not been taken equal,  $\mu$  would have turned out to be a

function of  $L_1$ ,  $L_2$ ,  $m$ . It is worth noting that this circuit will oscillate even if there is no mutual inductance provided the circuit constants are properly adjusted. It then becomes identical with the amplifier, discussed above, with an inductive load.

It is probably unnecessary to mention the fact that the solution for all three-mesh oscillation generators may be derived directly from the determinant appropriate to this one by simply inserting the proper impedances. The solution for any case is theoretically just as simple, but practically the algebra is usually troublesome when there are more than three degrees of freedom. An exception is the case of so-called "iterative networks" which have periodic structures and certain other symmetrical networks.

As has been said, these methods of treating the audion in a circuit are approximate only, because of the limiting assumptions made, but considerable use of them in connection with experimental circuit work has shown that they are useful. The problem of finite amplitudes cannot be handled so generally, but when it is necessary to consider such refinements special methods are available which, it is hoped, may be given in another paper.

RESEARCH LABORATORIES OF THE  
WESTERN ELECTRIC CO., INCORPORATED,  
February, 1919.

## THERMIONIC AND PHOTO-ELECTRIC PHENOMENA AT THE LOWEST ATTAINABLE PRESSURE.

By C. F. HAGENOW.

SYNOPSIS.—The object of these investigations was fourfold:

1. To study the photo-electric effect of tungsten at the lowest possible pressures, especially after the illuminated plate had been denuded of occluded gases by continued electronic bombardment of such energy as to maintain it at a white hot temperature.

The pressures attained were as low as  $3.5 \times 10^{-7}$  mm. of mercury. Upon denuding a tungsten plate of its occluded gases by raising it to a bright yellow heat by electronic bombardment, the photo-current rose to many times the value obtained before such denuding and further heating produced no further change.

2. To find, by direct observation, the short wave-length limit for tungsten and to observe how this changes, if at all, with the removal of occluded gases.

The upper wave-length limit of tungsten was located in the region between 2100 and 2300 Å. According to Einstein's equation, the corresponding values of the work necessary to free an electron from the metal would then be between 5.7 and 6.3 volts. These are larger than the corresponding values in the case of thermionic emission.

3. To observe, with the aid of the electric vacuum gauge, recently devised by O. E. Buckley, the progress of the clean-up effect, particularly at extremely low pressures.

Under the conditions employed it was not found possible to reduce the pressure by means of the clean-up alone lower than  $10^{-5}$  mm., which was not as low as could be attained with the pump itself.

4. To study pressure and other changes accompanying the appearance of the "blue haze."

A gradually increasing potential difference was applied between the tungsten plate and a glowing filament until the blue haze appeared. *No pressure change whatever was observable at the moment of its appearance.* Certain current and voltage discontinuities, which occur under the experimental condition just described, were found to have characteristics similar to those observed by O. W. Richardson and Charles B. Bazzoni in the case of mercury vapor. The clean-up of the blue haze is very rapid and shows a striking increase in its rate of absorption as the pressure approaches the final minimum. This behavior is in marked contrast to the clean-up with a lighted filament, but without a field, and is quite similar to that observed by S. Brodetsky and B. Hodgson in their experiments with a vacuum tube discharge under the condition of abnormal cathode fall. At the pressures employed in these trials, about .01 mm. of mercury, this final pressure remained unchanged in the presence of the field, and did not change further even in those cases in which a trace of the blue glow remained in the tube.

## I. EFFECTS OF ELECTRONIC BOMBARDMENT UPON PHOTO-CURRENTS FROM TUNGSTEN.

THE question of the part played by occluded gases in photo-electric phenomena has received the attention of many observers. Among those who have contended that the photo-electric effect is essentially dependent upon the presence of occluded gases may be mentioned Paech,<sup>1</sup> Fredenhagen,<sup>2</sup> Küstner<sup>3</sup> and Wiedemann and Hallwachs.<sup>4</sup> In particular, Küstner finds a total absence of the effect in the case of zinc when gases were removed. The zinc was scraped in vacuo and all "reacting" gases were removed by a spark discharge over potassium.

These conclusions are contradicted by experiments made by Pohl and Pringsheim,<sup>5</sup> Hennings,<sup>6</sup> Piersol,<sup>7</sup> Dushman,<sup>8</sup> Millikan and Souder,<sup>9</sup> and Welo.<sup>10</sup> For example, Piersol finds an increased sensitiveness in a number of metals when the occluded gases are removed by heating the electrode with an electric current. He also observed a superposed maximum effect "dependent on the activity of the gaseous surface," which disappeared after the highest temperature was reached.

This author states that a vacuum of .0002 cm. of mercury was used

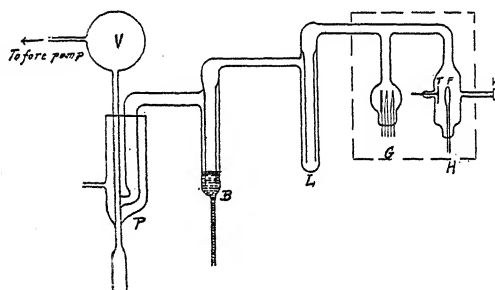


Fig. 1.

and does not mention the use of liquid air, so that the degree of exhaustion which he attained was not of the order aimed at in this experiment.

As the technique for the removal of the occluded gases and production of higher vacua has been improved, the evidence has become more and

<sup>1</sup> G. Paech, *Ann. der Phys.*, 43, 135, 1913.

<sup>2</sup> K. Fredenhagen, *Phys. Zeit.*, 15, 65, 1914.

<sup>3</sup> H. Küstner, *Phys. Zeit.*, 15, 68, 1914.

<sup>4</sup> G. Wiedemann and W. Hallwachs, *Ber. d. D. Phys. Ges.*, 16, 107, 1914.

<sup>5</sup> R. Pohl and P. Pringsheim, *Ver. d. D. Phys. Ges.*, 16, 336, 1914.

<sup>6</sup> A. E. Hennings, *Phys. Rev.*, 4, 228, 1914.

<sup>7</sup> R. J. Piersol, *Phys. Rev.*, 8, 238, 1916.

<sup>8</sup> See I. Langmuir, *Phys. Zeit.*, 15, 524, 1914.

<sup>9</sup> R. A. Millikan and W. H. Souder, *Proc. Nat. Acad. Sci.*, 2, 19, 1916.

<sup>10</sup> L. A. Welo, *Phys. Rev.*, 4, 251, 1918.

more conclusive for the contention that the photo-electric effect is an intrinsic property of the pure metal. This paper furnishes further convincing support of this point of view.

The apparatus (Fig. 1) was exhausted by the mercury diffusion pump *P* (kindly furnished by the Western Electric Company), working against a fore-vacuum of .01 mm. of mercury, or less, produced by a Töpler pump. Once established, no further pumping was necessary, as the volume, *V*, was quite large compared with that of the rest of the apparatus. A barometer tube, *B*, served as a valve to shut off the part to the right of it. *L* is the liquid air trap, which was kept continually in liquid air while the apparatus was set up. At no time was there any evidence of mercury vapor in the tube *H*. For measuring the pressures the extremely sensitive ionization gauge, *G*, due to Buckley,<sup>1</sup> was used. The photo-electric tube, *H*, contained the tungsten plate,<sup>2</sup> *T*, opposite which, at a distance of about a centimeter, the tungsten filament, *F*, and, in line with both, a tube sealed with a quartz window, *W*; the last being fastened with De Khotinsky wax. The connecting tubing had a bore of about 2.5 cm. and the whole apparatus was made as compact as possible. The source of illumination was, for the most part, a spark between zinc terminals. To avoid all electromagnetic disturbances, the spark gap, together with the induction coil, storage cell and Leyden jar, were all enclosed in an iron box.<sup>3</sup> A Dolazelek electrometer served for measuring the photo-currents, a sensibility of about 300 mm. per volt being usually employed. An electric oven could be slipped over a region indicated in the figure by the dotted line.

The ionization gauge, calibrated by Buckley against both a McLeod and a Knudsen manometer, gives the pressure in millimeters of mercury according to the formula (for air),

$$p = .05 \frac{I_G}{I_A},$$

where  $I_A$  is the current, in amperes, set up by a coated filament in a field of 200 volts and  $I_G$  the resulting flow of positive ions. The former can be read on a milli-ammeter, at least for the larger values, and the latter by means of a galvanometer. In order to read the smaller currents for  $I_A$  at the higher pressures more accurately, the writer employed the following null method of measuring these small currents. Fig. 2 shows the connections. A known resistance,  $R_1$  is placed in series with the

<sup>1</sup> O. E. Buckley, Nat. Acad. Sci. Proc., 2, 683, 1916.

<sup>2</sup> This was a thorium free sample kindly furnished by Dr. Langmuir and of the same sort used by him in the determination of Richardson's "b."

<sup>3</sup> W. H. Kadesch, Phys. Rev., 3, 367, 1914.

circuit whose current it is desired to measure. Connected to one end,  $A$ , of this resistance, is a circuit containing a variable resistance,  $R_2 + R_3$

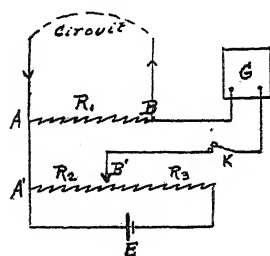


Fig. 2.

and a cell, whose E.M.F. need be known only to the degree of accuracy of the gauge itself. A Leeds and Northrup portable galvanometer was put at  $G$ . The adjustment consists simply in finding a point  $B'$  so that on closing the key,  $K$ , no deflection resulted.  $B$  and  $B'$  being then at the same potential, we have, calling the currents in  $R_1$  and  $R_2$  respectively  $C_1$  and  $C_2$ ,

$$C_1 R_1 = C_2 R_2.$$

Also

$$C_2 = \frac{E}{R_2 + R_3},$$

Therefore

$$C_1 = \frac{R_2 E}{R_1 (R_2 + R_3)}.$$

As actually used, the resistances were set in a fixed relation and the heating current of the glower of the gauge regulated for zero deflection of the galvanometer. Since  $I_A$  need not be changed very often (*i. e.*, only for a large variation in the range of pressure) a few previously calculated values of  $R_1$ ,  $R_2$  and  $R_3$  enabled one quickly to set on any desired current.

The limit of this instrument to indicate low pressures is set only by the possibility of being able to measure the current  $I_C$ . On the other hand, it was found that a maximum reading was obtained at .019 mm. This was located by plotting the values of  $I_C$  against time as the apparatus was gradually exhausted by a Gaede mercury pump.

The photo-currents measured were the saturation currents for a timed duration of the spark. The plate  $T$  (Fig. 1) was connected to one of the quadrants of the electrometer, the other being earthed. A positive potential was applied to the filament,  $F$ , the other pole being earthed, and the liberated electrons thus removed from the tube. A field of 10 to 20 volts was sufficient for saturation.

With the apparatus as first set up and no further exhaustion than simply pumping with the diffusion pump, the following readings were taken.

The photo-currents are expressed in millimeters deflection on the basis of 300 mm. per volt sensibility and 2-second illumination when the electrometer alone is used. (Sometimes an auxiliary condenser had to be connected.) These are given under  $C$  in the last column.



TABLE I.

Date.	No.	$\phi$ .	C.	Date.	No.	$\phi$ .	C.
May 10	1	$4.8 \times 10^{-3}$	9.6	May 13	8	$7.7 \times 10^{-3}$	9.8
11	2	"	7.2	14	9	$5.2 \times 10^{-3}$	8.7
12	3	"	7.1	"	10	$3.8 \times 10^{-4}$	7.7
"	4	"	6-6.8	"	11	$1.4 \times 10^{-4}$	8.1
"	5	"	8.2	"	12	$8.0 \times 10^{-5}$	8.9
13	6	$5.0 \times 10^{-3}$	7.9	15	13	$5.1 \times 10^{-5}$	7.8
"	7	$3.2 \times 10^{-4}$	8.7	"	14	$8.5 \times 10^{-5}$	7.3

As is to be expected from all preceding photo-electric work there is here no evidence for a relation between pressure and photo-currents. The table includes every run that was made until the apparatus was opened. On one or two occasions the writer has noticed fluctuations in excess of those here recorded, but the foregoing are representative of the sort of constancy that readings, taken days or weeks apart under apparently the same conditions, would show.

It may be mentioned, parenthetically, that readings were sometimes taken at zero fields, the filament  $F$  being earthed. Both  $T$  and  $F$  being of the same metal there should be no contact potential present. The results were very variable in this case as might be expected under the conditions. The number of electrons caught by the filament would depend on their particular distribution at any particular time, and this would be influenced by the charging of the inner surface of the glass tube. It was observed that illuminating the plate repeatedly, even when it was earthed at the time, would cut down the ensuing current greatly, doubtless because of such accumulation of charge on the walls. The effect would disappear after a short while. This phenomenon practically ceased at the saturation voltage, as it should do if the foregoing explanation is correct.

The effect upon the photo-currents,  $C$ , of denuding the tungsten plate of gases is shown in Table II. The headings have the same significance as in Table I. The denuding was done by glowing the filament while the negative potential indicated in the last column was applied to it.

So far no parts of the apparatus had been baked out or the metal parts heated, except the illuminated plate as described.<sup>1</sup>

No further readings were taken for four months. A new tungsten plate was mounted, though from the same original piece. Then the data contained in Table III. were taken.

The increase in no. 3 is not as large as was expected in view of the

<sup>1</sup> The ionization gauge and the tungsten filament in the photo-tube had been renewed since the readings in Table I. were taken.

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TABLE II.

Date.	No.	$p$ .	$C$ .	Remarks.
June 9	1	$1.5 \times 10^{-3}$	1.4	Just after set up.
"	2	"	187.6	After first bombardment, for 10 minutes up to 960 volts D.C. No observable heating of the plate.
	11	$1.9 \times 10^{-3}$	133.0	No further treatment since last observation.
	12	$2.0 \times 10^{-3}$	433.0	After bombardment with 1,500 volts, about 30 minutes.
	13	$1.9 \times 10^{-3}$	449.0	No further treatment since last observation.
		(Apparatus opened.)		
July 6	6	$2.6 \times 10^{-3}$	94.0	Note decrease after exposure to the air.
"	7	$2.0 \times 10^{-3}$	1885.0	After bombarding at 1,500 volts. Not visibly heated but surface slightly altered in appearance.
"	8	$2.0 \times 10^{-5}$	1900.0	After pumping down to 1/100 value of preceding pressure. Practically no effect on the photo-current.
	7	$1.6 \times 10^{-3}$	1715.0	Day after a bombardment up to 1,400 volts, about 30 minutes. First time plate at red heat in spots.
		(Apparatus opened.)		
	18	$2.9 \times 10^{-3}$	4.0	Decrease after exposure to air.
"	11	$4.8 \times 10^{-4}$	968.0	After bombardment at 3,500 volts A.C. about 30 minutes. Plate bright red in central portion, duller at edges.

effect of other cases of bombardment. Also the spontaneous increase in no. 6 is hard to account for, unless the fact mentioned in the last column may have had some bearing on it. However in view of the results of all these observations, it appears that there is an enormous increase of photo-sensitiveness after the occluded gases are removed. No. 7 shows the final reading at the lowest pressure attained for photo-electric observations in the course of these experiments. The tube and gauge had been baked out, the metal parts all glowed (the collector plate of the gauge had shown no visible heat during the bombardment, but a subsequent bombardment that caused it to glow did not influence the pressure attained). It can then fairly represent the conditions of a pure metal in a very low pressure thoroughly out-gassed. The close agreement of the final values of  $C$  in Tables II. and III. is probably largely accidental. On account of a rearrangement of the set up, necessitated by the addition of a monochromatic illuminator, the intensity of illumination of the plate in Table III. was slightly changed.

According to Table II. the maximum photo-current occurs after only

TABLE III.

Date.	No.	<i>p.</i>	<i>C.</i>	Remarks.
Nov. 30	1	$2.2 \times 10^{-3}$	50	
Dec. 1	2	"	93	After a slight bombardment to try a new transformer.
Jan. 1	3	$6.2 \times 10^{-5}$	112	After bombarding 15 minutes at 3,500 volts A.C. Plate at red heat all over.
	2	$9.0 \times 10^{-4}$	106	No further treatment since last observation.
	3	$1.1 \times 10^{-5}$	111	After baking tube and gauge for one hour at 300° C.
	7	$1.2 \times 10^{-4}$	407	This was after the tungsten filament had been brightly glowed for one hour for a clean up experiment.
	8	$3.5 \times 10^{-6}$	964	After bombarding 15 minutes at 2,200 volts A.C. Plate at a bright orange yellow heat. Now appears for the first time a bright metallic lustre.

a moderate heating of the plate. This maximum does not appear in Table III. and is probably due to some accidental surface condition in the first case. Piersol,<sup>1</sup> however, has observed a similar phenomenon in the case of a number of other metals. When new trials along this line were undertaken some weeks later, troubles unfortunately developed with the mountings of the tungsten plate that made further tests of this kind impossible.

Pressure readings were made with this apparatus down to  $3.5 \times 10^{-7}$  mm., with the photo-tube taken off. This is below the lowest pressure which it was possible to maintain for photo-electric observations. The De Khotinsky wax, used to hold the quartz window, may account for this difference. Also, as has been mentioned above, it was impossible to bake out every part of the apparatus. An attempt to heat the parts not inclosed by the oven with a Bunsen burner, while the diffusion pump was in operation, proved a failure. With the gauge and tube in the electric oven, and the diffusion pump going, mercury vapor diffused over into the tube.

It was also found that, though taking off the liquid air caused the pressure to rise rapidly, the vacuum could be as quickly restored by replacing it, with the pump in operation. Thus in one case the pressure rose almost immediately from  $5 \times 10^{-6}$  to  $1.2 \times 10^{-2}$  mm. and returned to  $5 \times 10^{-6}$  mm. It may be remarked that the low pressures mentioned just above were read after mercury vapor had diffused through the tubing as described.

<sup>1</sup> *Loc. cit.*

## II. DIRECT OBSERVATION OF LONG WAVE-LENGTH LIMIT OF TUNGSTEN.

To determine the long wave-length limit of sensitivity of the tungsten a Hilger monochromatic illuminator was set up before the quartz window; a Heraeus quartz mercury lamp being the source of light. As used with this apparatus, this lamp with direct illumination of the tungsten plate, gave a scale deflection about three times as large as that obtained previously with the zinc spark, *i. e.*, about 1,000 mm. in two seconds of illumination when no condenser was used. When the light was passed through the monochromator it was found impossible to obtain any deflection at all with any wave-length which would pass through the instrument. However some idea was gained of the limits of sensitiveness by means of two absorption media.

A solution of methyl alcohol placed in the path of the direct beam from the source<sup>1</sup> cut out the effect entirely. The first trials were made after the runs no. 2 in Table III. Other trials after no. 3 gave the same results. Finally after the last run, no. 7, a small deflection was observed. This was less than one thirteen hundredth of the deflection caused by the unfiltered light and about twice that which could still be observed with certainty. Since this sensitiveness was not attainable before the plate was thoroughly out-gassed, any conclusions as to a real change of the wave-length limit based on the absence of any observable effect under those conditions would be correspondingly doubtful.

The empty absorption cell itself, by virtue of the absorption of its fused quartz windows, cut down the deflection to only one tenth of its value, both before and after the plate was out-gassed.

Mt. M. J. Kelly, at this laboratory, found, photographically, that methyl alcohol had a sharply defined absorption region beginning with decreasing wave-length, at 2370 Å, and that the fused quartz windows had a similar absorption region beginning at 2160 Å. Now the above experiment showed that about nine tenths of the total light effect must have been due to wave-lengths shorter than 2160 Å, and all but one part in thirteen hundred to wave-lengths less than 2370 Å. The question arises whether this residual effect is due to active wave-lengths longer than 2370 Å or to a very small amount of light of shorter wave-length that was not entirely absorbed by the methyl alcohol. The latter is the more natural assumption, so that the indication of this experiment is that the long wave-length limit of tungsten is between 2100 and 2300 Å.

We can compare these results with the minimum frequency for which tungsten is light-sensitive, as calculated from Einstein's equation,

$$hv_0 = w_0$$

<sup>1</sup> The zinc spark was also used at times.

where  $w_0$  is the work done by an escaping electron against the forces which tend to retain it in the metal. If we assume that  $w_0$  has the same value as in thermionic emission, we can use its value as obtained from thermionic experiments. Lester<sup>1</sup> has made some experimental verifications of the identity of the "work function,"  $\phi$ , of an electron escaping from a metal and " $b$ " in Richardson's equation,

$$i = a\theta^{1/2}e^{-b/\theta}.$$

The relation is

$$\phi e = bk,$$

where  $e$  is the charge on an electron,  $k$  the gas constant for one molecule.  $\phi$  was calculated from experiments made on the "cooling effect." Now if we assume that  $w_0$  in Einstein's equation has the same value as in thermionic emission, using Lester's value for tungsten,  $\phi = 4.478$  volts, we have

$$\nu_0 = \frac{\phi e}{h} = \frac{4.478 \times 4.774 \times 10^{-10}}{300 \times 6.547 \times 10^{-27}} = 1.088 \times 10^{15},$$

or

$$\lambda_0 = 2757\text{\AA}.$$

On the assumption that the experiment with the methyl alcohol light filter does really indicate a total absence of light sensitiveness to wave-lengths longer than  $2370\text{\AA}$ , the difference is seen to be at least  $400\text{\AA}$ . This is much larger than the variations in either the values of  $\phi$  as observed by Lester, or the values of  $b$  obtained by Langmuir when proper precautions were taken to have all occluded gases removed.

It may be mentioned that Lester's value of  $b$  ( $= \phi e/k$ ) is 52,130. This agrees well with the weighted mean value,  $b = 53,130$ , which that writer has calculated from some measurements by Smith<sup>2</sup> and Langmuir.<sup>3</sup>

Lester<sup>4</sup> also found that the effect of residual gases was to increase the value of  $\phi$ . This he tested by sealing off a tube containing a tungsten filament and allowing the glowing filament<sup>5</sup> to clean up the active gases present. Inert gases do not affect  $\phi$ . If this fact has any direct bearing on the present experiment it would mean that the occluded gases responsible for the higher value of  $\phi$  were such that they were not driven out by the treatment the plate received as described above.

It is, of course, not established that the work function is the same in photo-electric as in thermionic emission, or that these phenomena deal with the same type of electron. Concerning the case of platinum

<sup>1</sup> Horace Lester, Phil. Mag., 31, 197, 1916.

<sup>2</sup> K. K. Smith, Phil. Mag., 29, 802, 1915.

<sup>3</sup> I. Langmuir, Phys. Rev., 2, 450, 1913.

<sup>4</sup> Loc. cit.

<sup>5</sup> I. e., the same filament which was the subject of investigation.

Hughes<sup>1</sup> makes the following comparison: Richardson<sup>2</sup> gives 5.34 volts as representing the work done (per unit charge) when an electron escapes from hot platinum. But from photo-electric observations, using the most probable value of  $\lambda_0$ , viz., 2910 Å,  $V_0$  comes out 3.86 when calculated from  $V = k\nu - V_0$ . When instead of  $k$  the corresponding value from the quantum theory,  $h/e$  is used,  $V_0$  becomes equal to 4.32 volts.

The results of the present experiments indicate a value of the work function, calculated from Einstein's equation, as lying between 5.7 and 6.3 volts. The work required to remove an electron from tungsten photo-electrically thus seems to be larger than that required in the case of thermionic emission.

### III. CLEAN-UP EFFECTS.

The fact that an incandescent filament tends to improve the vacuum has been noted by many observers. It has been subjected to detailed study by Langmuir<sup>3</sup> for a number of gases.

The present attempt offers nothing new, except as it might serve to make a comparison with another sort of clean-up to be discussed later in connection with the blue haze experiments.

The mercury valve,  $B$  (Fig. 1), was closed and the filament glowed a little more brightly than in the ordinary Mazda lamp. The following readings were then taken.

TABLE IV.

Time (Minutes).	$p$ .	Absorption Cu. mm. $\times 10^3\%$ .	Per Minute.
0	$1.2 \times 10^{-4}$		
5	$6.8 \times 10^{-5}$	6.84	1.37
10	$3.5 \times 10^{-5}$	4.55	.91
20	$1.9 \times 10^{-5}$	2.20	.22
30	$1.6 \times 10^{-5}$	.40	.04
40	$1.4 \times 10^{-5}$	.26	.026
50	$1.3 \times 10^{-5}$	.13	.013
60	$1.25 \times 10^{-5}$	.07	.007

The absorbed air is given in cu. mm. at atmospheric pressure per cu. mm. of the volume,  $v$ , of the apparatus used. The last column contains the average rate per minute in the same units.

The next day but one following this run the pressure was found to have risen to  $2.8 \times 10^{-5}$  mm., when the filament was again glowed at a higher temperature, bringing the pressure down to a little below  $10^{-5}$  mm. The graph is shown in Fig. 3.

<sup>1</sup> A. L. Hughes, Photo-electricity, Cambridge University Press.

<sup>2</sup> O. W. Richardson and K. T. Compton, Phil. Mag., 24, 576, 1912.

<sup>3</sup> I. Langmuir, J. Am. Chem. Soc., 34, 1310, 1912; 35, 107, 1913; 35, 931, 1913.

Table IV. shows that the rate of clean-up, as shown in the last column, diminishes to 1/200th of its initial value while the pressure drops to 1/10th of its original amount. According to Langmuir<sup>1</sup> the rate of clean up for oxygen is proportional to the pressure. For nitrogen<sup>2</sup> it is different according to the range of pressure experimented with, but is in-

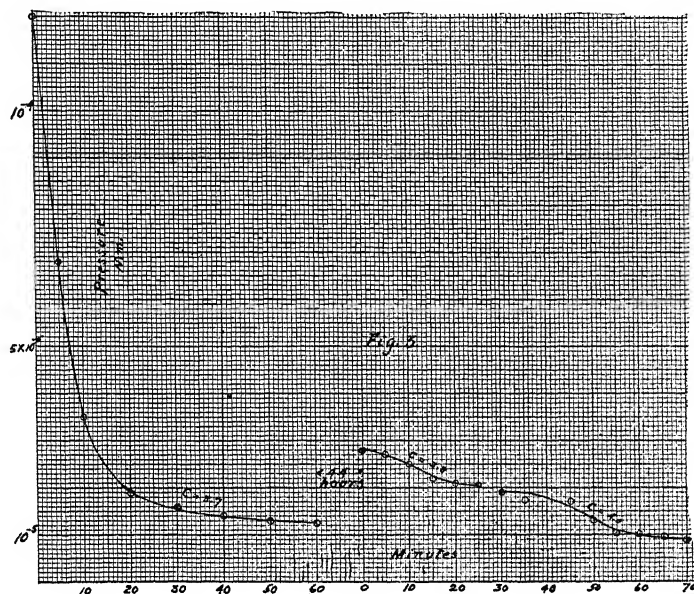


Fig. 3.

dependent of the pressure in the interval from .0065 mm. to .0001 mm. of mercury.

The present experiment indicates that the rate of clean up of air decreases at a greater rate than the pressure and that the latter reaches a fairly well defined limiting value, which is manifestly not due alone to the inert gases. Langmuir found a "fatigue effect" in the case of hydrogen, so that a continual treatment of the glowing wire in that gas brought the clean-up practically to a standstill. In view of these experiments and the fact that the rate of clean up depends on the temperature of the filament in a way widely different for the different gases, no general conclusions can be drawn from experiments on a mixture such as air.

#### IV. PHENOMENA ACCOMPANYING THE APPEARANCE OF THE "BLUE HAZE."

The bluish glow, known as the "blue haze" has often been remarked.<sup>3</sup>

<sup>1</sup>I. Langmuir, J. Am. Chem. Soc., 35, 107, 1913.

<sup>2</sup>Loc. cit., 35, 931, 1913.

<sup>3</sup>W. R. Whitney, Proc. Am. Inst. El. Eng., 31, No. 1, 921, 1912. Saul Dushman, Phys.



It can be seen, at a certain state of exhaustion, in the ordinary incandescent lamp. Whitney<sup>1</sup> observed that it "cleaned up" gases and was accompanied by a disintegration of the lighted filament.

The present investigation may be grouped under three heads: (A) Pressure changes and general observations; (B) Current-voltage relations; (C) Clean-up effects.

(A) *Pressure Changes and General Observations.*

To attempt to study the pressure changes preceding and accompanying the appearance of the glow, the following procedure was adopted.

The valve *B* (Fig. 1) was closed, the filament *F* glowed brightly and a positive potential applied to the plate *T*; the filament being the negative pole. The indications of the gauge were observed while the potential was gradually increased.

The first experiments were performed at a comparatively high field voltage of 900 to 2,100 volts A.C. and at pressures varying from  $6 \times 10^{-4}$  to  $5 \times 10^{-5}$  mm. of mercury. No change in the pressure was observed, even at the instant the blue haze appeared and the plate was red hot with the bombardment (showing incidentally how thoroughly the metal had been out-gassed). It was noticed that the gauge current  $I_A$  was always diminished when the field was applied to the tube *H*. But  $I_C$  was found to diminish in the same ratio, so that as far as the gauge indicated, there was no change of pressure.

In order to bring the necessary potential within lower and more accurately measurable limits, it was decided to use a pressure as high as .01 mm. A potentiometer supplied the field up to the available 220 volts, D.C.

It was then observed that the blue haze appeared very sharply, accompanied by a sudden fluctuation in the voltage downward and a break upward in the electronic current. This phenomenon has been described by Richardson and Bazzoni,<sup>2</sup> in their experiments on mercury vapor.

At the first one of such fluctuations a blue glow appears directly behind the anode; it can be made to appear and disappear within a variation of potential too small to be observed on a Weston 30-volt voltmeter, with which a change of one fifth volt could be easily detected. On increasing the field the glow spreads into the tube and, a little later, at a voltage which seemed to be constant for all pressures tried, a second "kick" occurred. Here the glow suddenly spread further into the tubing above

REV., 4, 123, 1914. A. Wehnelt and E. Liebreich, Phys. Zeit., 15, 557, 1915. O. W. Richardson and Charles B. Bazzoni, Phil. Mag., 32, 426, 1916.

<sup>1</sup> *Loc. cit.*

<sup>2</sup> *Loc. cit.*



the bulb  $H$  and also increased greatly in intensity. At this point the glow was very sharply defined and a column of it could be pushed along the tubing at will by controlling the field.

The gauge showed no change in the pressure at any time, except that after some minutes it would begin to register the falling pressure due to the above-mentioned clean-up effect. The blue glow then began gradually to recede into the tube  $H$  and finally to disappear altogether. The rate of disappearance depends on the pressure of the absorbable gases present, and for the lower pressure, the rapid disappearance of the glow, once it begins to withdraw, is very striking. As shown at the end of the paper, this disappearance was accomplished at times in an interval of a second. In these experiments also, as in the first trials with A.C. fields, it was observed that the gauge current  $I_A$  was always diminished when the field was applied in the tube  $H$ . But  $I_C$  diminished in the same ratio, so that the gauge recorded no change in pressure.

(B) *Current-Voltage Relations.*

In order to maintain the pressure as nearly constant as possible during a set of observations the mercury seal  $B$  was left open. A constant P.D. of seven volts was maintained across the terminals of the filament. To the apparatus mentioned above was added a milliammeter. A typical set of readings is shown in Table V.  $V_1$  is the first "kick" voltage, when the blue color first appears behind the anode;  $C_1$  the corresponding current change. In this set the initial current was not readable on the scale used.  $V_2$  is the voltage drop at the second kick, already referred to above. The voltages are measured at the positive end of the glowing filament, the average would thus be 3.5 volts greater. Under  $C_2$  are given the current readings corresponding to the voltages,  $V_2$ .

TABLE V.

No.	$p$ .	$V_1$ .	$C_1$ .	$V_2$ .	$C_2$ .
1	.0101	18 $\rightarrow$ 14	0 $\rightarrow$ .005	26 $\rightarrow$ 22 30 $\rightarrow$ 25	.130 $\rightarrow$ .133 .135 $\rightarrow$ .140
2	.0086	22 $\rightarrow$ 16	$\rightarrow$ .006	22 $\rightarrow$ 20	.113 $\rightarrow$ .115
3	.0068	24 $\rightarrow$ 16	$\rightarrow$ .007	22 $\rightarrow$ 20	.118 $\rightarrow$ .122
4	.0047	33 $\rightarrow$ 18	$\rightarrow$ .013	25 $\rightarrow$	
5	.0023	54 $\rightarrow$ 24	$\rightarrow$ .026		
6	.0010	92 $\rightarrow$ 44	$\rightarrow$ .045		

Owing to the fact that even with the considerable volume in use the pressure would slowly diminish, it was usually impossible to duplicate any reading during a run. The kicks are rather hard to catch accurately

and rapid fluctuations are apt to occur at these points of instability, especially at the lower pressures. Thus in case of no. 6 the voltage and current made the jump back and forth twice, finally remaining at the "before the kick" value. This meant, of course, that the clean-up had progressed so far that the higher potential of  $V_1$  was not sufficient to start the blue glow again.

Fig. 4 is the plot from the above data. No. 1 seems to have three points of instability, though that may well be the result of the manipulation of the potentiometer. It did not occur in the other sets. The

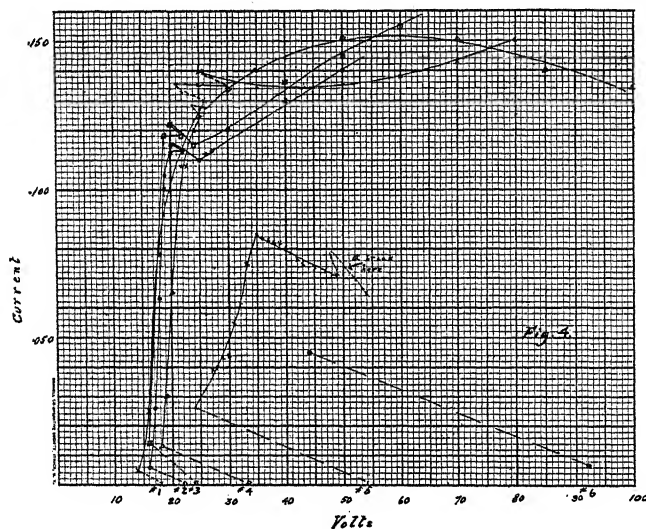


Fig. 4.

currents before the first break are not shown in the plot; they increase gradually from zero until the break. Thus the whole phenomenon resembles that described by Richardson and Bazzoni<sup>1</sup> in the case of mercury, even to the order of magnitudes involved. These observers mention also the second point of instability, but are of the opinion that it is due to the "discharge wandering in the tube" and did not regard it as of as much interest as the first. Coming as it does at approximately the same voltage for all the pressures would suggest that it is a function of the apparatus rather than of the other conditions. This would remain to be investigated.

Referring again to the curves, it seems as if in no. 5 the first break had overtaken the second, being just at that voltage; it shows also another break later. But the conditions at that stage were too variable to make

<sup>1</sup> *Loc. cit.*

any general statement. This is also evident from an inspection of the progress of the currents, after the first very rapid increase immediately after the first break. No. 6 consisted of only the first jump.

A second table is added to show the duplicability of the phenomenon. The seventh column gives the ratio of the currents, before and after the first kick. This was used by Richardson and Bazzoni as a measure of the magnitude of the kick. It shows a maximum as found by them.

TABLE VI.

No.	$p$ .	$V_1$ .	$G_1 \times 10^4$ .	$V_2$ .	$C_2$ .	Ratio of $G_1$ .
1	.0120	17 $\rightarrow$ 14.5	.75 $\rightarrow$ 5.9	26 $\rightarrow$ 24	.104 $\rightarrow$ .107	7.9
2	.0113	19. $\rightarrow$ 16	1.5 $\rightarrow$ 12.0	28 $\rightarrow$ 22	.136 $\rightarrow$ .148	8.0
3	.0070	26 $\rightarrow$ 16	1.8 $\rightarrow$ 40.0	22 $\rightarrow$ 21	.105 $\rightarrow$ .108	22.2
4	.0026	60 $\rightarrow$ 26	2.7 $\rightarrow$ 50.0	28 $\rightarrow$ 26		18.5
5	.0013	114 $\rightarrow$ 42	45 $\rightarrow$ 600			13.3
6	.0009	206 $\rightarrow$ 105	110 $\rightarrow$ 1270			11.5

The behavior of the voltage  $V_1$ , also, is similar to that found by those observers for mercury. Thus, an attempt to lower the voltage after the first "kick" results in a sudden drop to the initial part of the curve; *i. e.*, to a smaller current and larger voltage.

As is evident from Tables V. and VI., the minimum potential necessary to maintain the glow (which is practically the lower reading in the column under  $V_1$ ) approaches a limiting value as the pressure increases. In these experiments the minimum value for  $V_1$  was never below 14 volts, with a pressure up to .02 mm., the highest readable on the gauge. Taking account of the potential difference across the terminal of the glowing filament, the average potential would be 17.5 volts. Compared to the ionizing potential for nitrogen (using 7.7 volts), this is roughly in the same ratio as the corresponding quantities Richardson and Bazzoni found for mercury, *viz.*, 11.5 volts for the glow as compared to minimum ionizing potential of 4.9 volts. However, in the light of some recent experiments by Hebb,<sup>1</sup> who has shown that pure mercury vapor will arc at 4.9 volts, the above comparison may not have any significance. The present trials were necessarily complicated by the fact that they were made on a mixture of gases subject to a sorting-out effect due to the clean-up. A detailed study of this phase of the ionization problem on pure gases would seem to be a profitable undertaking.

The writer does not lose sight of the fact that where there is very probably a selective absorption of the gases present in air, the gauge constant would be subject to change. Nevertheless, such a variation

<sup>1</sup> T. C. Hebb, *PHYS. REV.*, 11, 170, 1918.

would not obliterate the general trend of the phenomenon in question. This also applies to the experiments on clean-up effects, which follow.

(C) *Clean-Up Effects.*

The clean-up effect of the blue haze was very marked, as was the wearing away of the lighted filament. The walls of the tube became covered with a shiny brown coating, showing interference colors. Langmuir<sup>1</sup> refers to a "clear brown" coating as being  $WN_2$ . This clean-up is characteristically different from that produced by the lighted filament without a field between the filament and a neighboring electrode. As will be shown more in detail later, the rate of clean up is quite different and increases with decreasing pressure just before the blue glow disappears. Also it operates at a considerably higher pressure than the ordinary clean-up. Thus it was always possible to evacuate with the blue haze at .02 mm. pressure, though a glowing filament, without the field, had absolutely no effect at this pressure.

In order to accelerate the progress of the phenomenon a smaller volume was used, *B* being closed. The first observations were taken immediately after the experiments described in part "B" above were completed; that is, after the air in the apparatus had presumably been deprived of its more easily absorbed constituents.

The filament being glowed as before and the maximum field applied (not over 220 volts) pressure readings were taken at intervals of a minute, during the first two runs. As soon as the minimum pressure was attained, *B* was opened a moment, thus equalizing the pressure throughout the apparatus, and *B* again closed for the next run.

The tube and about half of the connecting tube to the gauge was filled with intense purple glow which turned to a pure blue as the exhaustion proceeded. After a time the glow would begin to recede into the tube and, usually, disappear altogether. In the instances where a glow remained there was no difference in the pressure observed. The rapid decrease in the time required for glow to disappear as run after run was made was very striking. Table VII. gives a summary of these trials. The initial and final pressures are denoted by  $p_1$  and  $p_2$  respectively;  $t$  is the time taken for the glow to disappear, which was also the interval between the two pressure readings, as nearly as they could be taken. No great accuracy is claimed for the time in the shorter intervals, but they serve sufficiently to indicate the progress of the phenomenon. The quantities in the last two columns are the same as the corresponding ones in Table IV.

<sup>1</sup> I. Langmuir, J. Am. Chem. Soc., 35, 931, 1913.

TABLE VII.

No.	$p_1$ .	$p_2$ .	$t$ , Seconds.	Absorption Cu. mm. $\times 10^{10}$ v.	Absorption Per Min.
1	.0110	.0015	360	12.5	2.10
2	.0076	.0011	240	8.55	2.14
3	.0072	.0010	30	8.16	16.3
4	.0065	.0009	20	7.37	22.1
5	.0062	.0015	4	6.2	93
6	.0058	.00061	2	6.8	205
7	.0053	.00068	1	6.0	360
8	.0049	.00099	—	5.4	—
9	.0046	.0011	—	4.6	—

Thus as the proportion of absorbable gases decreases with each run, the rate of clean-up increases. The total pressure at the beginning of the final run is only a little less than half that at the commencement of the experiment.

The matter was next tested by admitting a fresh lot of air into the apparatus. The valve *B* was left open and the entire apparatus evacuated in the above manner. Readings were taken every minute or two until a steady state was reached at a pressure of .0008 mm. at the end of 45 minutes.

The results are best represented in a graph, shown in Fig. 5. The

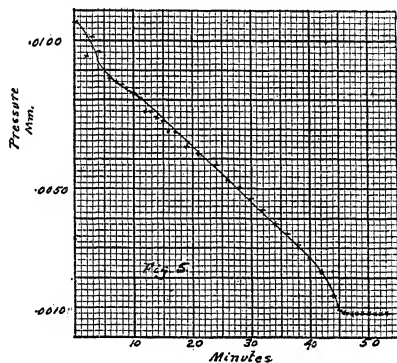


Fig. 5.

curve is practically a straight line up to a pressure of .0022 mm., when there is a rapidly increasing acceleration until the minimum pressure is attained. It is at that point the glow begins to recede and disappear.

The field and current strength were recorded. Both remained at a fairly steady state during the greater part of the time, *i. e.*, until the bend in the curve was reached, when the voltage rose and the current fell rapidly.

In the present case we are dealing with a mixture of gases, hence, since the rate of absorption increases with the diminution of the total pressure, it must vary still more rapidly with the partial pressure of the more absorbable constituents.

According to Langmuir<sup>1</sup> the oxygen atoms collide with electrons in the tungsten filament by virtue of the energy of agitation of the former, which then take on a negative charge and combine with positive tungsten atoms. A field might conceivably aid or retard this process; negative oxygen ions being repelled before chemical combination could take place; or, positive ions of the gas impelled to the negative filament.

In an investigation on the absorption of gases in vacuum tubes, Brodetsky and Hodgson<sup>2</sup> have given evidence for a mechanical theory of the phenomenon. Glass vacuum tubes were used, the current being obtained from a high potential battery. A curve for air shown by them has the general characteristics of Fig. 5.

According to their work the absorption, together with the disintegration of the cathode, is conditioned on the presence of an abnormal cathode fall: neither of the two effects appearing when the cathode fall is normal. The fact is cited (supporting the mechanical theory) that the inactive gases helium and argon are also readily absorbed. Hodgson<sup>3</sup> shows curves, similar to the above, for helium. Whatever may be the mechanism of the process, the absorption in the case of the blue haze is certainly of the same nature as the vacuum tube discharge. Just how selective it is and how it may vary with the field, which affects the ionization, remains a subject for further investigation. In all cases of absorption with the blue haze in the present experiments the minimum pressure was about 10 per cent. of the initial one. This is of course too large to be accounted for by a residue of the inert gases alone.

A number of attempts were made to obtain some information of the nature of the process by spectroscopic means.

A Schmidt and Haensch direct vision spectrometer (after Hoffmann) was employed. As has been stated above, the color of the glow changes from a sort of purple to a pure blue toward the end of the exhaustion. The spectrum of the blue haze at .01 mm. pressure showed the brighter nitrogen and oxygen lines and some of the nitrogen bands of the first and second Deslandres series as described by Kayser. No hydrogen lines were seen. After a clean-up, whenever there remained a visible glow, it was found that the nitrogen bands of the first group had disappeared and some of the second.  $H_\alpha$  and  $H_\beta$  were observed.

<sup>1</sup> I. Langmuir, J. Am. Chem. Soc., 35, 107, 1913.

<sup>2</sup> S. Brodetsky and B. Hodgson, Phil. Mag., 31, 478, 1916.

<sup>3</sup> B. Hodgson, Phys. Zeit., 13, 595, 1912.

It was thought that the removal of a great deal of the nitrogen and oxygen might allow other lines to become more prominent, such as those of argon. In order to make possible a more careful and detailed study of the spectrum, a table was constructed showing the observed lines and the most prominent lines of the common and the noble gases in parallel columns. A number of new lines did appear that corresponded to those of argon within the limits of accuracy of the spectroscope, but they were not among the prominent ones, and could usually be duplicated by either nitrogen or oxygen, though these latter had not been seen before the clean-up. Thus it was practically impossible to draw any safe conclusions from the character of the spectrum. Also the feeble luminosity, in the bright glare of the lighted filament, made observation difficult.

In conclusion the writer wishes to take this opportunity to thank Dr. Millikan, at whose suggestion these experiments were undertaken, for his very kind assistance and numerous helpful suggestions.

## RESIDUAL GASES AND VAPORS IN HIGHLY EXHAUSTED GLASS BULBS.

BY J. E. SHRADER.

SYNOPSIS.—The investigation of high vacua has been extended by making observations on the effect of the sealing off of highly exhausted glass bulbs, and noting the subsequent change in pressure with time over an extended period. On sealing off the bulbs and gauges by melting the glass at a constriction there is a comparatively large evolution of gases and vapors which can be partly removed by pumping during the time of sealing off, but the pressure in the bulbs is always four or five times as large as the pressure before sealing off.

BY the use of the mercury diffusion pump and the Knudsen absolute gauge pressures of the order of  $1 \times 10^{-8}$  mm. Hg have been produced and measured.<sup>1</sup> These pressures have been measured during the operation of the pump and it has not come to the attention of the author that any data have been obtained showing what pressures can be maintained in bulbs which have been pumped to this degree of exhaustion and sealed off. To secure information on this subject four glass vessels were exhausted to a high vacuum and sealed off and their change in pressure with time was observed. The system for this investigation consisted of a mercury diffusion pump with liquid air trap, a sensitive Knudsen absolute gauge of about 500-c.c. capacity and the bulb to be tested of about 1,500-c.c. capacity. A narrow constriction was made in the tube connecting the bulb and gauge to the pump so that they could be sealed off readily by heating a limited amount of glass. The gauge and bulb were so disposed that they could be heated by an electric oven to a short distance beyond the constriction. At the same time all the glass connecting tubing was heated with a bunsen flame to release occluded gases and vapors.

It has been recognized that the vacuum obtainable with the diffusion pump is limited practically by the heat treatment of the glass, though it is true that the greater the speed of the pump the lower the pressure that can be reached. Equilibrium will be established when the speed of the pump is equal to the rate at which gases and vapors are given off from the glass. If the escape of the gases and vapors is accelerated by

<sup>1</sup> Shrader and Sherwood, Production and Measurement of High Vacua. *PHYS. REV.*, Vol. XII., No. 1, July, 1918.



heat treatment, this equilibrium is reached sooner and the ultimate pressure is lower. The effect of heat treatment on the vacuum obtainable after pumping until equilibrium was reached at that temperature is shown by the following results:

Temperature.....	20	100	200	300	300	500
Pressure in mm. Hg..	$1 \times 10^{-5}$	$1.9 \times 10^{-6}$	$1.7 \times 10^{-7}$	$1.2 \times 10^{-7}$	$2 \times 10^{-7}$	$2.4 \times 10^{-8}$

While these results are only relative the importance of heat treatment is clearly shown.

The cleaning of the glass is very important in high vacuum production. It was found that the removal of grease by potassium bichromate solution followed by a prolonged washing with boiling water to remove soluble salts proved most effective.

The electric oven in which the gauges and bulbs were placed was kept at a temperature of  $500^{\circ}\text{C}$ . while the system was pumped for two hours.

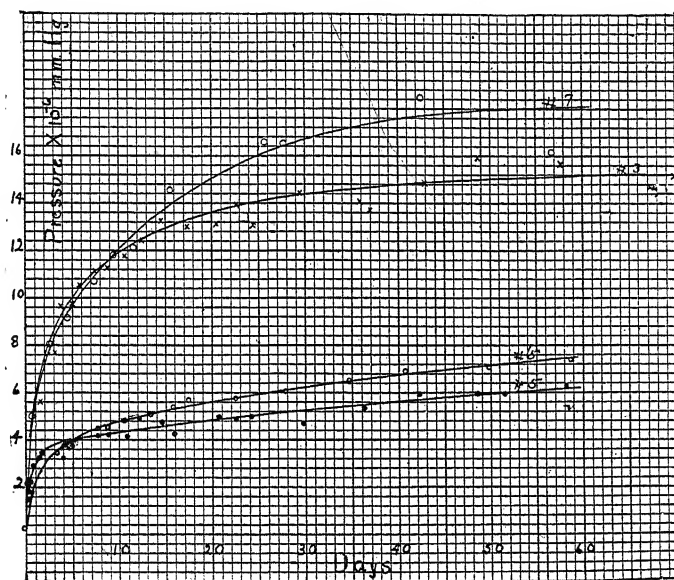


Fig. 1.

The glass would easily stand this temperature though it would collapse at  $550^{\circ}\text{C}$ . This margin of temperature was kept for safety. After the two hours heating and pumping the electric oven was removed and the bulbs and gauges were allowed to cool. After the pressure had been observed the sealing-off process was accomplished by heating the constriction with the pointed flame of a blowpipe. When the glass was near the melting point the gauge indicated a considerable increase in pressure.

The glass was kept heated near the melting point for about ten minutes while the pump continued to operate. The pressures before and immediately after sealing are given in the table below for four bulbs and gauges which had been given identical treatment.

	No. 3 (702-P).	No. 5 (702-E.J.).	No. 6 (702-P.).	No. 7 (702-E.J.).
Pressure before sealing off...	$2.7 \times 10^{-7}$	$3 \times 10^{-7}$	$8.5 \times 10^{-8}$	$9.7 \times 10^{-8}$
Pressure after sealing off....	$1.1 \times 10^{-6}$	$1.5 \times 10^{-7}$	$3.4 \times 10^{-7}$	$8.6 \times 10^{-7}$

These bulbs and gauges were securely mounted and the changes in pressure with time were observed. These results are shown in Fig. 1. It is noted that in all cases there is a rapid initial rise of pressure with time followed by a slower and more uniform change. No conclusions can be drawn from these data on the different grades of glass as to its suitability for vacuum work. Bulbs 3 and 6 were of Corning glass 702-P. while 5 and 7 were of 702-E.J. or Pyrex. It is observed that 5

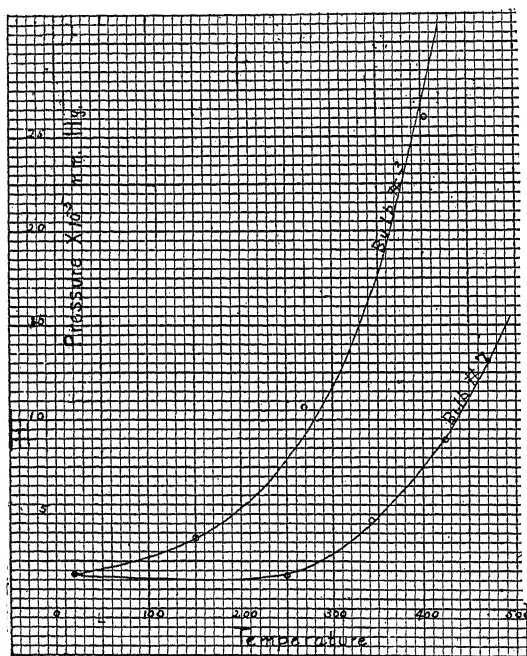


Fig. 2.

and 6 and 3 and 7 almost duplicate each other. It is also observed that bulbs 6 and 7 were pumped to the highest degree of exhaustion yet the pressure in no. 7 is the highest of all after a few days. It is noticed, however, that 5 and 6, which had the lowest pressure after sealing off, still have the lowest pressure after sixty days.

To determine the effect of subsequent heating, two of the bulbs and gauges were replaced in the oven and heated for one hour at increasing successive temperatures and the increased pressure resulting from each heating was determined. These results for bulbs 3 and 7 are shown in Fig. 2. It is seen that heating 7 to 250° did not change the pressure while for 3 the pressure increased with all increases of temperature. The changes of pressure at the lower temperatures have always been observed to be less rapid than at the higher temperatures.

This investigation shows that the vacuum in sealed vessels deteriorates with time, rapidly at first and then more slowly, and that subsequent heating even at temperatures lower than the heat-treating temperature results in increase of pressure due to further liberation of gases and vapors from the glass. No connection between different samples of the same glass or different glass can be established. It is quite probable that there are variations in the properties of different samples of the same glass quite as great as the variations between different glasses of about the same grade, for example, Corning 702-P and Pyrex.

## THE JOULE-THOMPSON EFFECT FOR AIR AT MODERATE TEMPERATURES AND PRESSURES.

By L. G. HOXTON.

**SYNOPSIS.**—In the experiments as described air was pumped first through a set of tubes designed to remove carbon dioxide and moisture, then through the throttling device or "porous plug" and finally back to the pump. The plug and tubing immediately leading to it were immersed in an oil bath thermostat. The bath temperatures, for the most part, lay between 0° and 100° C., the mean pressures, from 4.5 to 6.4 meters of mercury, while the pressure-drop varied between 0.25 and 0.80 meters of mercury. The temperature-drop was measured by platinum resistance thermometers differentially connected. The plug was of the radial flow type originated by Regnault, roughly cylindrical in form, through the walls of which the air flowed toward the axis, escaping finally through one end. The accidental errors were apparently of an order under one per cent.

The Joule-Thomson coefficient ( $\mu$ ) was found to have a decreasing linear variation with increasing mean pressures. Its dependence upon both temperature and pressure are embodied in either of two empirical formulas, namely:

$$\mu = +0.3935 - 0.00691p - 0.001835t + 0.00000134t^2 + 0.0000325pt$$

$$\mu = -0.2599 + 182.0 \frac{1}{\tau} - 552.4 \frac{p}{\tau^2},$$

all pressures being measures in meters of mercury,  $t$  in degrees C., while  $\tau = 273 + t$ .

These formulas are applied to calculate the temperature of the ice point on the thermodynamic scale, to reduce to the same scale the readings of the constant pressure air thermometer and to discuss the variations of  $c_p$  with pressure. Good agreement with the results of other work is found in the second and third cases, while in the first, the result, in common with that found from other Joule-Thomson data, is higher than the result found from the characteristic gas equations.

The paper includes a brief historical summary of previous experimental work, a discussion of several theoretical points and of sources of error, especially heat leakage.

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### INTRODUCTION.

A NEED for new determinations of the Joule-Thomson effect has long been recognized by students of thermodynamics, and it has found expression occasionally in the literature touching this subject.

Such data, as is well known, still preserve their original importance for the study of molecular attractions in gases and for the establishment of the thermodynamic scale of temperatures. More recently they have acquired interest for the engineer, namely for the computation of steam tables, ammonia tables and the like. The latter aspect has been discussed by Davis.<sup>1</sup>

Aside from the desirability of experimenting over a wide range, increased accuracy is obviously one of the first improvements in the data that is needed. The published results have been considered too uncertain on the whole, and in many cases too difficult of interpretation to be of great value. The present paper is devoted largely to the latter phase of the problem and is therefore somewhat limited in scope.

At first it was planned to work with several gases and cover a considerable extent of temperatures and pressures. However, a year's experimentation at the Johns Hopkins University with carbon dioxide disclosed many difficulties in the way of obtaining results at all consistent with one another. It was decided, therefore, to confine the work for the time being to an attempt to develop a reliable method and to proceed to this end by ways that seemed the most direct. Side problems were to be avoided wherever possible. Hence the range of temperatures and pressures were restricted within the limitations of the resources already at hand. Air was chosen as the gas to be experimented upon, chiefly because of its availability and the small amount of attention to contamination and slow leaks in the apparatus that its use would entail. Results from pure gases are, of course, to be preferred in general, yet those obtained from air are not without interest as regards thermometry and for comparison with work on air in this and related fields.

It is hoped that in the experiments to be herein described, most of the larger sources of error have been overcome. The accidental errors are small for work of this kind, being of an order considerably under one per cent. Perhaps there is a systematic error outstanding, indicated possibly by the relatively large value computed for the thermodynamic temperature of melting ice. The sources of such error, if existent, have so far not been found. Further experiments with other gases should throw light upon this matter, especially those gases whose expansion coefficients have been determined with great care by Chappuis, namely, nitrogen, hydrogen and carbon dioxide. The author would welcome, in the meantime, any suggestions occurring to the minds of those interested in this subject.

*Historical Outline.*—Although a historical summary of experimental work down to 1905 has been given by Kester, in a paper to be taken up

later, a complete outline would seem desirable here, first to supply a few omissions and secondly to include a considerable amount of new material.

After the original classical experiments of Joule and Thomson,<sup>2</sup> carried out in the years 1852 to 1862 on air, nitrogen, oxygen, carbon dioxide and hydrogen, whose work is too well known to need further comment, one of the early experimenters to enter this field was Regnault.<sup>3</sup> His work has often been overlooked. The most interesting feature of his apparatus is the ingenious design of its porous wall or throttling partition, which was not in the form of a "plug." It was a hollow enclosure through the wall of which the gas flowed from the outside to the interior. A hole in the wall was provided for the continuous escape of this gas. The obvious purpose of this design was to provide thermal protection. In this respect it seems greatly superior to the plug form used by Joule and Thomson. Yet Regnault abandoned the work as futile with an expression of regret at the waste of time spent on a research so "sterile." He found that the temperature readings within the enclosure were too uncertain to have any meaning. In the light of subsequent advances in methods of measurement, several possible causes for these troubles might be assigned, but, especially since his descriptions are not full in detail, their discussion here might not be profitable.

The forms of throttling device or "plug" introduced in these two investigations are typical. Most of the subsequent experimenters have employed the form used by Joule and Thomson, possibly because the work of Regnault, as far as it went, has had little attention directed to it. In 1910 Burnett and Roebuck,<sup>4</sup> in a paper entitled "On a Radial Flow Porous Plug and Calorimeter," proposed a plug on the Regnault principle adding improvements in detail. Their work, however, was entirely independent. Later Roebuck<sup>5</sup> used apparatus of this type on liquid water in order to determine the mechanical equivalent of heat. Only two other experimenters so far have published work with similar apparatus with gases, namely, Trueblood<sup>6</sup> and the present author. Following the usage of Trueblood the term "axial flow" will be applied to the plug form used by Joule and Thomson to distinguish it from the other or "radial flow" type.

One should not fail to note in this connection that certain protective features of the radial flow principle have come into use more or less generally in continuous flow calorimetry, as for example, in the work on  $C_p$  by Swann<sup>7</sup> and by Scheel and Heuse.<sup>8</sup>

Hirn<sup>9</sup> carried out throttling experiments upon steam, using an orifice. Work of others on steam, namely, Griessmann,<sup>10</sup> Grindley,<sup>11</sup> Peake<sup>12</sup>

and Dodge,<sup>13</sup> has been discussed by Davis.<sup>14</sup> As Davis states, the first named used a porous plug very much like that of Joule and Thomson, while the others used a throttling calorimeter, their work being undertaken for the "purpose of determining the variation of the specific heat of steam with pressure and temperature, an investigation which has since been more satisfactorily accomplished in other ways."

Cazin<sup>15</sup> in 1870 published an account of experiments which were essentially modifications of the Gay-Lussac free-expansion experiment. His apparatus was complicated, and, as he admits, his results were qualitative.

Natanson<sup>16</sup> in 1887 described porous plug experiments on carbon dioxide at 20° C. only. His pressure-drop was about one atmosphere and his highest absolute pressure 25 atm.; and between the lowest and highest pressures he found a decided dependence of the cooling effect upon pressure, whereas Joule and Thomson had not noted (up to 6 atm.) any such effect. This dependence does not agree with that found later by Kester and is in the opposite sense from that found for air in the present paper.

Mention is made here of work reported by Witkowski,<sup>17</sup> who carried out experiments on air at high pressures, passed through a succession of porous plugs. Only a brief abstract of his paper has been available, which contains no quantitative results. There is no obvious theoretical advantage to be gained by employing a succession of plugs. All the data could be had from a single plug by properly varying the pressures and bath temperatures.

Kester<sup>18</sup> in 1905 extended the work of Natanson on carbon dioxide to temperatures between 0° and 100° C. finding also a dependence upon pressure. The results of the two observers are not in agreement, Kester's results at the same temperature (20° C.) being independent of pressure up to the highest pressure employed at that temperature, namely, 41 atm., while the work of Natanson yielded a positive pressure coefficient. At one pressure, 26.4 atm., Kester found an indication of a positive coefficient.

Olszewski<sup>19</sup> performed experiments (using not a porous plug but a throttle valve) on hydrogen in order to determine its "inversion temperature" as bearing upon the problem of its liquefaction. The high pressure of the experiment varied from 117 to 170 atm. and the low pressure was uniformly one atmosphere. The one datum of this experiment was - 80.°5 for the inversion temperature under these conditions. The effect observed was, of course, an integrated one and leaves much to be desired for application of thermodynamic equations.



Five years later<sup>20</sup> (1907) he did the same thing for nitrogen and air, finding a series of inversion points for magnitudes of the pressure-drop varying from 160 atm. to 30 atm.

Rudge<sup>21</sup> carried out (1909) a few experiments on carbon dioxide supplied on the market in the form of "sparklets" for making carbonated water. The duration of each experiment was but a few minutes and their author does not attach great importance to the results, but offers them in view of the scanty known data. It would seem that the ideal conditions for continuous-flow methods could hardly be realized in so short a time.

Bradley and Hale<sup>22</sup> in the same year carried out experiments upon "the nozzle expansion of air at high pressure," *i. e.*, from initial pressures of 70 to 200 atm. to final pressures of 1 atm. Their temperatures extended from 0° C. to - 120° C. approximately.

Their apparatus was a liquefier of the Hampson type and the air expanded through a specially constructed "nozzle." Their particular problem was the process of liquefaction; and their results were in qualitative agreement with those of Joule and Thomson. These experiments would seem subject to error from heat leakage.

Again in the same year (1909) Dalton<sup>23</sup> carried out at the Leyden laboratory similar nozzle experiments in air at 0° C. only and for pressure drops from about 5 to 40 atm. His results agree with the Joule and Thomson figures at 0° C., and he notices no pressure effect, although his curve would indicate an extremely slight negative coefficient.

Vogel,<sup>24</sup> whose original papers have been unavailable, carried out porous plug experiments in the Laboratory of Technical Physics of the Technische Hochschule in Munich. He was looking for a pressure effect and found a negative coefficient. The temperature of his experiments was 10° C.

Noell,<sup>25</sup> whose original paper likewise has been unavailable but an abstract<sup>26</sup> of which is at hand, continued the work of Vogel in the same laboratory. His pressure-drops were 6 and 8 atmospheres while the arithmetic mean of the absolute high-side and low-side pressures varied from 25 to 150 atm. His high-side temperatures varied from - 55° C. to + 250° C. He worked with air only. He found a pressure as well as a temperature effect which he embodied in the interpolation formula:

$$\frac{\Delta T}{\Delta p} = \frac{50.1 + 0.0297 p}{T} + \frac{14830 - 1.674 p}{T^2} + \frac{366000 - 19093 p}{T^3} - (0.122 + 0.0000157 p).$$

This is evidently an empirical expansion in descending powers of  $T$ , whose coefficients are linear functions of  $p$ .



The pressure units seem to be kg. per cm.<sup>2</sup> in the left-hand member and atmospheres on the right. The difference in any case would not be more than about 3 per cent., but the author does not make it clear. The deviations of his observed values from those calculated by this formula vary from 2 per cent. to 6 per cent. at the lower temperatures, to 50 per cent., and over at the highest temperatures. Nevertheless his work seems to be more systematic and satisfactory than that of his predecessors, certainly for air.

Cuts of the apparatus of Vogel and Noell may be found in Jellineks' *Lehrb. d. Ph. Chem.*, Vol. I., 1914. They show two forms of plug, the earlier being of the axial flow type, the later, a combination of the axial and radial flow types in which an inner axial flow plug was surrounded by a jacket containing the inflowing high side air at bath temperature.

Buckingham<sup>27</sup> has proposed a modified form of the porous plug experiment in which the process is to be rendered isothermal by supplying heat, say electrically, to the plug in just the sufficient amount to prevent the temperature-drop. If there should be a heating effect, as with hydrogen, there would have to be devised some way of abstracting heat from the plug. No extended experiments by this method have been published.

Trueblood<sup>6</sup> carried out experiments at about the same time as those of the present author, which are described in a paper entitled "The Joule-Thomson Effect in Superheated Steam; I. Experimental Study of Heat Leakage." He limited the work to a single temperature and pressure and carried out a thorough and effective study of the heat leakage. Extensive tests, under varying conditions, were made upon a number of different plugs of the axial and radial flow types. He concluded the latter to be the superior and that it is capable of yielding reliable results. A remarkable form of indirect heat leak was found for the radial flow type, which he named the "regeneration effect." This matter will come up later under a separate head.

The experimental work in this field, thus briefly summarized, lends itself only in small measure to the application of thermodynamic equations owing to the difficulty or impossibility of attaching a definite meaning to a large part of it. Thus the only work found available for the calculation of the thermodynamic scale of temperatures hitherto published<sup>28, 29, 30, 31</sup> has been that of Joule and Thomson principally, with later computations including the results of Natanson, Kester and, in one instance, of Olszewski.

It has seemed to the author that in some cases the programs of experimentation have been, perhaps, too ambitious in the direction of extreme

temperatures and pressures with the result that an oversight of vital details affecting accuracy was quite possible. The intention to limit the present work in the way mentioned at the beginning was confirmed by considerations of this sort.

*General Conditions of Present Experiment.*—The range of temperatures was confined approximately to the fundamental interval—actually from 15° C. to 90° C., although experiments had been started at 110° C. when the apparatus called for repair. The term “pressure” in the title of this paper refers to the arithmetic mean of the absolute pressures on the two sides of the plug. The range lay from about 6 to 8½ atmospheres, or more precisely, from 4.5 to 6.4 meters of mercury. The pressure-drop ranged from 0.25 to 0.80 meter of mercury. Pressures throughout this paper will be expressed in meters of mercury. For the immediate purpose of these experiments outlined above, the obvious gas to be chosen was, of course, air.

#### REMARKS ON THEORY.

The general theory of the plug experiment has been given so often in treatises and in journal articles that a full treatment of it here would be out of place. As much, however, should be given as is sufficient to define the necessary symbols and to develop such special formulæ as are needed in the computation of results.

The function  $u + pv$ , the “total heat” or so-called “enthalpy” of Kamerlingh Onnes, is supposed to be the same in the regions of equilibrium above and below the plug. Let us call it  $H$ . Then the Joule-Thomson coefficient ( $\mu$ ) is defined as the limiting value of the ratio of the temperature-drop to the pressure-drop, when these each approach zero, under the condition  $H = \text{const.}$ , that is,

$$\mu = \left( \frac{\partial T}{\partial p} \right)_H^1 \quad (1)$$

Graphically represented this is the slope of the tangent to the curve  $H = \text{const.}$  on the  $(p, T)$  diagram. From (1), through the application of the second law of thermodynamics, follows the general relation always associated with the porous plug experiment, namely,

<sup>1</sup> It may be worth while to note, in passing, that inasmuch as one often finds confusion of statement between the free expansion or Gay-Lussac effect and the Joule-Thomson effect, the distinction between the two may be brought out by expressing the two as

$$\left( \frac{\partial T}{\partial p} \right)_u \quad \text{and} \quad \left( \frac{\partial T}{\partial p} \right)_H$$

respectively.

$$T \left( \frac{\partial v}{\partial T} \right)_p = v + \mu c_p. \quad (2)$$

It will be shown later that  $\mu$  was found experimentally to be a function of temperature and pressure. In the search for an empirical formula to express this relation and one which would have a little more rational basis than a mere power series, resort was had to the equation of van der Waals. While this equation does not strictly fit the measurements on isotherms, yet the author is in accord with Berthelot<sup>32</sup> in thinking of it as a valuable index to broad features of the properties of gases and as being something more than an empirical interpolation formula.

Applying to van der Waals' equation, namely

$$\left( p + \frac{a}{v^2} \right) (v - b) = RT,$$

the relation (2) above, we get, keeping only terms of the *second order* in the small quantities  $a$  and  $b$ , the relation

$$\mu c_p = -b + \frac{2a}{RT} - \left( \frac{6ab}{R^2 T^2} - \frac{4a^2}{R^3 T^3} \right) p. \quad (3)^2$$

<sup>1</sup> Thus calling  $s$  the entropy

$$Tds = du + p dv = dH - v dp,$$

or, putting  $T = \text{constant}$ ,

$$T \left( \frac{\partial s}{\partial p} \right)_T = \left( \frac{\partial H}{\partial p} \right)_T - v.$$

But

$$\left( \frac{\partial s}{\partial p} \right)_T = - \left( \frac{\partial v}{\partial T} \right)_p \text{ ("thermodynamic relation")}$$

and

$$\left( \frac{\partial H}{\partial p} \right)_T = - \left( \frac{\partial H}{\partial T} \right)_p \left( \frac{\partial T}{\partial p} \right)_H = -c_p \mu.$$

$$\therefore T \left( \frac{\partial v}{\partial T} \right)_p = v + \mu c_p. \quad (2)$$

A short proof quite similar to this has been given by Callendar.<sup>29</sup>

<sup>2</sup> Expanding van der Waals's equation,

$$pv + \frac{a}{v} - bp - \frac{ab}{v^2} = RT,$$

and applying the operator  $(\partial/\partial T)_p$  to both sides,

$$\left[ p - \frac{a}{v^2} + \frac{2ab}{v^3} \right] \left( \frac{\partial v}{\partial T} \right)_p = R.$$

$$\begin{aligned} \therefore \mu c_p &= T \left( \frac{\partial v}{\partial T} \right)_p - v = RT \left[ p - \frac{a}{v^2} + \frac{2ab}{v^3} \right]^{-1} - v \\ &= \left[ RT - pv + \frac{a}{v} - \frac{2ab}{v^2} \right] \left[ p - \frac{a}{v^2} + \frac{2ab}{v^3} \right]^{-1} \\ &= \left[ -b + \frac{2a}{pv} - \frac{3ab}{pv^2} \right] \left[ 1 - \frac{a}{pv^2} + \frac{2ab}{pv^3} \right]^{-1}, \end{aligned}$$

on substituting the value of  $RT - pv$  from the expanded equation of van der Waals and

Now the variation in  $c_p$  within the limits of the present investigation is small or at most may be considered a linear function of  $p$ . If then we neglect the term in  $T^3$ ,  $\mu$  should be a function of the form

$$\mu = A + \frac{B}{\tau} + p \frac{C}{\tau^2}, \quad (4)$$

where  $\tau = 273 + t$ .

The first two terms are those of the complete formula of Rose-Innes.<sup>30</sup> It will be noted that the pressure effect is linear and even then only comes in if we retain second order terms. Later this formula as well as a five-constant power series linear in  $p$  and of the second degree in  $t^\circ$  C. will be fitted to the observations. Obviously the second power of  $\tau$  in (4) could be replaced by the first power and the adjustment be empirically just about as good within the small range of observed temperatures, but the second power is retained to keep closer to the original basis.

#### DESCRIPTION OF APPARATUS.

*The Air Circuit.*—The air-circuit and the disposition of the various parts are shown in Fig. 1. Air is drawn in by the compressor at the intake  $A$  which is kept open all the time in order to replenish the supply of air lost by slow leakage. The air then passes upward through the driers  $D$  and  $D'$ , the former containing a column of about a meter of calcium chloride, the latter about half a meter of soda-lime to remove carbon dioxide, followed by a half meter of calcium chloride to remove the water put in by the reaction of the carbon dioxide with the soda-lime. Water trapped in the lower end of  $D$  from the air under compression is

factored out  $p$ . Let us now expand the right hand bracket and in all expressions retain no order in  $a$  and  $b$  higher than the second. Then

$$\begin{aligned} \mu c_p &= \left[ -b + \frac{2a}{pv} - \frac{3ab}{pv^2} \right] \left[ 1 + \frac{a}{pv^2} - \frac{2ab}{v^3} + \dots \right] \\ &= -b + \frac{2a}{pv} - \frac{4ab}{pv^2} + \frac{2a^2}{p^2v^3} + \dots \end{aligned}$$

Now, in the second order terms,  $pv$  may be replaced by  $RT$  alone. In the first order term we must put  $pv = RT - a/v + bp$ .

$$\begin{aligned} \therefore \frac{2a}{pv} &= \frac{2a}{RT} \left[ 1 - \frac{a}{vRT} + \frac{b}{RT} p \right]^{-1} \\ &= \frac{2a}{RT} + \frac{2a^2}{vR^2T^2} - \frac{2ab}{R^2T^2} p + \dots \\ &= \frac{2a}{RT} + \frac{2a^2}{R^3T^3} p - \frac{2ab}{R^2T^2} p + \dots \end{aligned}$$

$$\therefore \mu c_p = -b + \frac{2a}{RT} - \frac{6ab}{R^2T^2} p + \frac{4a^2}{R^3T^3} p \dots$$

Q. E. D.

drawn off by the valve *B*. This was done usually once during a run which lasted some four or five hours.

From the driers the air is carried through a quarter-inch gas pipe to the plug apparatus proper in a neighboring room. Here it passes first through a series of throttling needle-valves *t*, then through a third drying tube *D''* containing phosphorus pentoxide. From this it passes

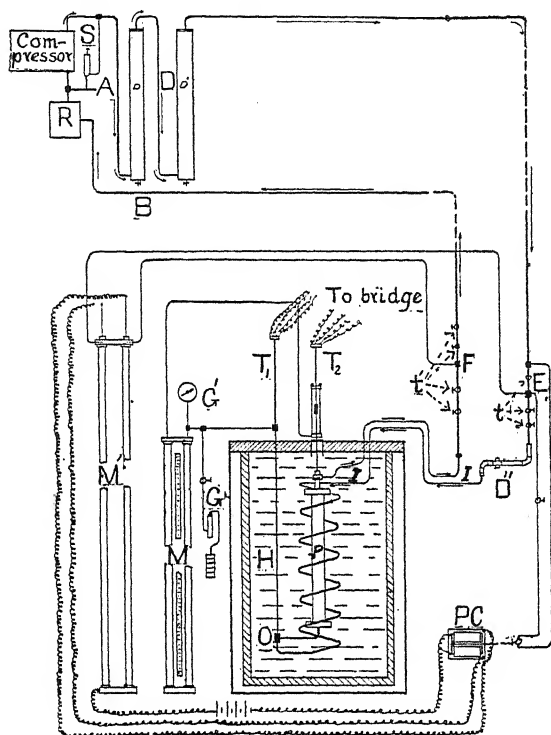


Fig. 1.

through copper "automobile tubing" of about 5 mm. outside diameter, into the bath, under the surface of which the automobile tubing continues for about two meters, connecting thence with a coil of larger pipe of brass and about one cm. internal diameter and 6 meters in length. This coil terminates in the plug casing *P*. After passing through the porous plug, the air leaves the casing by way of another length of automobile tubing much shorter than the first and soldered parallel with it for a purpose stated below.

*Interchanger.*—For a length *I, I* of about a meter the tubes are soldered together side by side, forming thus a heat interchanger, so that when the bath and the room were at considerably differing temperatures the

air leaving the bath would be brought to approximately room temperature and the air entering the bath would be brought to approximately the bath temperature in advance. This device was found to be necessary for two reasons: first, it conserved the heat of the bath and improved its temperature regulation, and secondly, it prevented the throttles  $t'$  in the return line from progressively warming, a condition which was found seriously to interfere with pressure regulation. This warming of the valves would gradually change their opening and hence change the rate of flow of air through them.

After leaving the low pressure throttles  $t'$  the air returns to the compressor by way of a receiver  $R$  of about 10 liters capacity. Curiously enough, this receiver, when placed on the low-pressure side, was quite effective in reducing the pulsations in the air-circuit produced by the intermittent action of the compressor while no receiver was needed at all on the high pressure side.

*Pressure Regulation.*—It is obviously necessary to provide means to keep constant the pressures at all points, and especially the pressure-drop in passing through the plug. The heat of compression liberated here might quite mask the Joule-Thomson effect. It was observed, for instance, on one occasion, that a fluctuation in the pressure-drop of 0.1 per cent. resulted in a fluctuation of about 5 per cent. in the temperature-drop and consequently by 5 per cent. in the observed Joule-Thomson effect. Special precautions therefore were necessary to provide good pressure regulation. Further, this had to be automatic on account of the necessity for the observer to work alone. Fluctuations of 20 per cent. in the power service had to be cared for.

Rough pressure regulation was attained as a preliminary by placing an adjustable spring safety-valve  $S$  (Fig. 1) in a by-pass from the high pressure to the low pressure lines near the compressor. This was usually set so that the difference on the two sides of the valve would be from 80 to 160 lbs., gauge reading, depending upon the pressure drop at the plug.

Superposed upon this was a finer method of regulation which was designed to control the pressure drop between the points  $E$  and  $F$ . This was carried out by means of an electrically controlled valve mechanism actuated through contacts in a mercury manometer  $M'$ , which was connected in at the points  $E$  and  $F$ . The valve mechanism is indicated at  $PC$  which is connected as a by-pass to the first or upper throttle  $t$  so that when opened the flow of gas is increased.

The two electrical contact-points of the manometer  $M'$  are set at any desired difference of level such that the tops of the mercury columns play just under them when the device is functioning properly. If for

some reason the pressure difference decreases, the lower contact will be made, and this will actuate one of the electromagnets at *PC*, which, through setting in motion a train of gears will open the by-pass needle-valve at *PC* very slowly, thus supplying an increased flow of air and bringing the pressure difference back to its proper value. A similar action in the reverse direction will take place if the pressure drop exceeds the limits set by the positions of the contact points. When the pressure is at its proper value, the valve at *PC* is at rest.

There were, of course, small fluctuations in pressure remaining, even though this device gave much finer regulation than did the safety-valve *S*. These residual fluctuations were practically damped out by the throttles *t* and *t'* lying between the points *EF* and the plug. The greatest variations in the pressure drop at the plug were of the order of 0.1 mm. of mercury and usually too small to be observed.

The throttles *t* and *t'* together with the safety-valve *S* served, in addition, as a means of setting the absolute pressures in the system to any desired values.

*Mounting of Thermometers.*—*T*<sub>1</sub> and *T*<sub>2</sub> (Fig. 1) are brass tubes housing the stems of platinum resistance thermometers, one of which is situated in the elbow *O*, on the high-pressure side of the plug, and the second in the interior of the plug casing *P*. Emerging from the tops of these tubes and passing through sealed joints in appropriate hard rubber insulation are the leads which connect through heavy lamp-cords to the bridge. Into the latter they are connected differentially as will be described later. Thermometer tube *T*<sub>2</sub> has a telescoping joint indicated in the diagram whose purpose is to permit one to shift vertically the thermometer and so to explore the region on the low-pressure side of the plug in order to study the distribution of temperature therein.

*The Differential Manometer.*—From the sides of the stems of the thermometers *T*<sub>1</sub> and *T*<sub>2</sub> lead two small tubes connected to the manometer *M* as shown in Fig. 1. The latter is thus put in communication with the spaces occupied by the thermometer coils themselves and is intended to measure the pressure-drop between the high and low pressure sides of the plug. The manometer was somewhat over a meter in height and was furnished with a brass meter-bar of the "H" form made by the Société Genèveise. This was placed between the tubes and read by a micrometer telescope from a distance of about one meter, the tube and scale being in the same field of view at one time. The crosshair was leveled by setting on the image of the level surface of mercury placed in a glass cell with plane sides at the same distance from the telescope as the manometer. The heights of meniscus were read off in order to

supply data for the capillary correction which never exceeded 1 mm., and if omitted, would not have introduced an error exceeding 1/500 of the observed pressure drop, and even that would be within the accuracy demanded, for errors somewhat less than one per cent. in the Joule-Thomson effect came in from other sources. The corrections, however, were applied making use of the tables given in Kohlrausch, "Physical Measurements," 3d edition (English), p. 440. The internal diameters of the tubes varied from 5.0 mm. to 5.7 mm. along the portions where readings were made.

The readings were likewise reduced to 0° C., making use of tables given in Landolt and Börnstein's "Tabellen," 3d Aufl., pp. 34 and 35. Pressures throughout this paper are expressed in meters of mercury at 0° C. at the University of Virginia. The value for  $g$  (980 cm./sec<sup>2</sup>) is needed only in final computations.

*Pressure Gauges.*—To the high-pressure side of the differential manometer were connected a piston gauge and a dial gauge,  $G$  and  $G'$  (Fig. 1) respectively. The latter was used for setting to the desired absolute pressures; while the former yielded their correct measure. The piston was of  $\frac{3}{8}$ " drill rod mounted in a snugly fitting brass cylinder provided with a leather packing at the top, so curved as to become tighter as the pressure increased. This also insured that the effective area of the piston was its own cross section and not a compromise between this and that of the bore of the cylinder. Provision was made for rotating the piston, in a manner similar to that carried out by Amagat, in order to eliminate the static friction.

The diameter of the piston measured by two micrometer calipers came to 0.9536 cm. giving an area of 0.7142 sq. cm. Thus 1 kg. on the piston corresponded to a pressure of 1.030 meters of mercury at 0° C. and at the same locality.

The weights were adjusted by a set of platinum plated weights furnished by Rüprecht of Vienna, and brought to within 0.02 per cent. of accuracy or about one fiftieth of the errors of reading of the gauge when in use. Great accuracy was not needed, for it will be seen that the effect of pressure on the Joule-Thomson coefficient is small.

*Bath and Thermostat.*—The bath was about 90 cm. deep and 30 cm. in diameter with triple walls of sheet brass. The space between the middle and inner walls was filled with asbestos, and that between the middle and outer was designed for the circulation of steam, but not so used. The outer wall was lagged with asbestos. The cover was lagged with about 10 cm. thickness of cotton waste which has been all that was necessary for temperatures up to 130° C. "Renown engine oil" made



by the Standard Oil Co. was used to fill the bath and, on account of its great thermal expansion, a siphon overflow had to be provided. Rapid circulation was provided by two motor-driven propellers on one vertical shaft rotating in a well. The stirrer and heating coils are not shown in the diagram.

Heating was applied electrically from a 220-volt D.C. circuit, through four coils totaling a capacity of one kilowatt. These could be cut in independently in series or in parallel, various combinations being used, including auxiliary resistances, according to the temperature of the bath.

Thermostatic control was obtained by keeping a steady current in some of the resistances and varying the current in the remainder. This was done automatically by the wheatstone bridge type of thermostat similar in principle to that described by Darwin<sup>33</sup> and by Randall<sup>34</sup> and later made use of by the Leeds and Northrup Co. Its performance was quite good, keeping the bath, under the best working conditions,

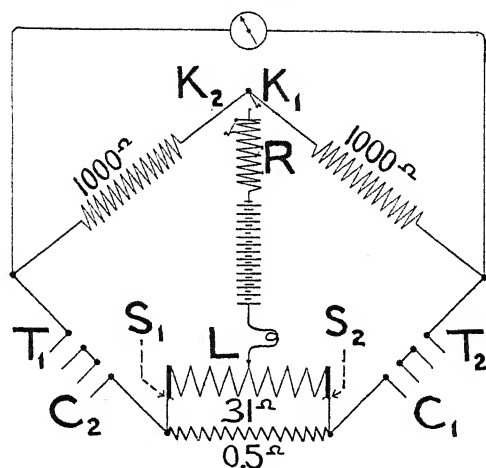


Fig. 2.

constant to  $0^{\circ}.001$  C.; and in but few instances was the fluctuation more than  $0^{\circ}.004$  C. It is a remarkably convenient form of apparatus for setting at any desired temperature.

The use of a 220-volt D.C. supply is perhaps open to criticism; but with care in insulation, trouble from leakage through the galvanometers from this source was avoided. It was, moreover, the only kind of power-supply at hand.

*The Wheatstone Bridge.*—Resistance measurements were made in terms of a decade resistance box manufactured by the Leeds and Northrup Co. of Philadelphia. The coils needed (from 30 ohms down) were cali-

brated twice, two years apart, and found to remain constant to well within the required accuracy.

The thermometers on the high and low-pressure sides of the plug were connected into the bridge differentially as shown in Fig. 2, where  $T_1$  and  $T_2$  represent the wires leading to the thermometer coils and  $C_1$  and  $C_2$  the compensation leads which will be described below.

The quantity desired is the difference in the effective resistances of the two thermometers. The Carey-Foster<sup>35</sup> method was employed because it measures directly the difference between resistances and eliminates all fixed contact- and lead-resistances. The mercury commutator for effecting the transposal of the resistances to be compared, which in this case are  $T_1 + C_2$  and  $T_2 + C_1$  is omitted from the diagram.

It will be noted that the arrangement of battery and galvanometer is not that usually published in the text-books. By putting, as indicated, the battery instead of the galvanometer in series with the sliding contact and by having no key in series with the galvanometer the error of thermal E.M.F.'s was eliminated. This arrangement follows the practice of the Bureau of Standards. It has worked admirably and obviated the necessity of reversing the battery current.

The slide wire  $S_1LS_2$  was a "Students' Potentiometer" manufactured by W. G. Pye & Co., of Cambridge, England, and consists of a wire wound in a helix on a slate drum and with sliding contacts  $S_1S_2$  at the ends. It has 10,000 divisions from end to end and a resistance of 31 ohms approximately. This being too high for direct use, it was shunted by a resistance of about half an ohm made of manganin wire. With this arrangement and by repeated tests at different dates the value of one division was determined to be 0.00005666 ohms or, in terms of the particular thermometers used, equivalent to about 0.0005° C. The precise conversion of bridge readings into degrees will be given later.

The errors due to the variability of the sliding contact resistances  $S_1$  and  $S_2$ , being connected in series with a resistance as high as 31 ohms, proved to be negligible. For, by test, the uncertainty was not over 0.12 division (0.00006° C.) while, in the final readings, only the nearest whole division was taken into account. All other junctions in the four arms were either amalgamated or soldered. The wire  $S_1LS_2$  was calibrated several times and was found uniform to a small fraction of a division.

For resistance thermometry, and with the battery connected between the thermometers as shown, the more sensitive arrangement is to have high-resistance ratio-arms; hence a high value for these, namely, 1,000 ohms, was employed.

The key  $K_1$  opened and closed the battery circuit while  $K_2$  in conjunction with the resistance  $R$ , permitted the observer to vary the battery current for the purpose of obtaining corrections due to the heating effect of the measuring current in each thermometer.

*The Platinum Resistance Thermometers.*—These were of the four-lead compensated type introduced by Callendar.<sup>36</sup> The ends of the compensating leads were united by a short length, about 1 cm., of the same sort of wire as that used for the thermometer coils. The thermometers were connected differentially into the bridge, thermometer No. 1 and compensating-lead No. 2, being connected in series on one side, while thermometer No. 2 and compensating-lead No. 1 were in series on the other side. These connections (Fig. 2) are lettered  $T_1$ ,  $C_2$  on the left,  $T_2$  and  $C_1$  on the right. In differential connections, the need for the compensation leads is very much reduced, but, in the present instance, they were kept in to be on the safe side, especially in view of the fact that the depth of immersion of the two thermometers was not quite the same.

The wire for these thermometers was obtained from the Cambridge Scientific Instrument Co., Ltd., very pure and recommended by them for thermometer work, in which they make a specialty. The diameter of the wire was about 0.03 mm., and its length such as to give a change in resistance of about 11 ohms for a change in temperature from 0° C. to 100° C. It was wound upon the usual crossed mica frames in the form of a compact cube approximately 5 mm. each way. The fineness of the wire and the compactness of its mounting was for the purpose of allowing exploration of the space on the low-pressure side of the plug.

The four leads were sealed into the ends of a glass tube as appears at  $T_1$  or  $T_2$ , Fig. 3, and passed thence up the tube and out through a wax-sealed joint into the room whence they were connected to the bridge by heavy No. 14 lamp-cord about 3 meters long each.

The thermometer coils were exposed to the flowing air in some of the preliminary experiments, but later, copper foil caps indicated by the dotted lines at  $T_1$  and  $T_2$  were slipped on. This made the galvanometer considerably steadier and did not introduce an objectionable lag because the copper caps should take the temperature of the gas before the walls of the enclosure.

The glass tubes containing the leads, after emerging from the air channels, were enclosed in larger brass tubes of  $\frac{1}{8}$ " pipe size (the brass tube for  $T_1$  shown in the diagram is larger than natural size), which came in contact with the bath and allowed air-connection to the differential manometer. The immersion of  $T_1$  was about 44 cm. and that of  $T_2$

about 34 cm. under the bath. The latter immersion was extended by a lagged air-space above the bath's surface by about 12 cm. This was thought sufficient to eliminate the effect of heat conduction along the leads inside the glass tubes in spite of the stagnant air-space by which the leads were surrounded.

The electrical insulation proved sufficient at all times. Repeated tests were made; in fact, the bridge wiring was arranged so that the thermometer insulations between pairs of leads could be coupled in parallel and the combination thrown in series with the galvanometer and a battery of 30 volts. This test, as a matter of precaution, was made a part of the routine of observation. In dry weather the galvanometer was not sensibly deflected, while in one or two instances in damp weather a film seemed to form on the outside insulation whose resistance was of the order 20 megohms. This obviously would not affect the measurements.

Several tests were made for a possible effect of pressure on the resistance of the thermometers but none was found.

*Application of Callendar's Formulæ.*—These were adapted to the work in hand, as embodied in equation (8) below, in the following manner, beginning with the formulæ themselves, namely:

$$t_p = 100 \frac{R - R_0}{R_{100} - R_0} \text{ (defining the "platinum scale")} \quad (5)$$

and

$$t - t_p = \delta \left( \frac{t}{100} - 1 \right) \frac{t}{100} \text{ (correcting to the hydrogen scale),} \quad (6)$$

where  $\delta$  is a quantity to be determined by experiment for the sample of platinum wire used.

Now, if  $\Delta t$  is a small temperature change, we get from (6)

$$\begin{aligned} \Delta t &= \Delta t_p \left\{ 1 - \frac{2\delta}{10000} (t - 50) \right\}^{-1} \\ &= \Delta t_p \{ 1 + 0.0003(t - 50) \} \end{aligned} \quad (7)$$

to the first order of approximation, if we put  $\delta = 1.50$ . This value, given by the Cambridge Scientific Company, who furnished the wire, was accepted as sufficiently accurate. For it is evident that its omission altogether could not alter the right hand member of (7) by more than 1.5 per cent. in the range from  $0^\circ$  to  $100^\circ$  C. covered in these experiments.

If  $\mu_{tp}$ , therefore, is the value of the Joule-Thomson coefficient observed in terms of the platinum scale and  $\mu$  in terms of the hydrogen scale, then we have

$$\mu = \mu_{tp} \{ 1 + 0.0003(t - 50) \}, \quad (8)$$

where  $\mu_{tp}$  is the observed Joule-Thomson coefficient in terms of platinum degrees, and  $\mu$  this coefficient calculated in terms of the gas-scale. (The distinction between the hydrogen and nitrogen scales need not be drawn for these measurements.)

*The values of the fundamental intervals, i. e.,  $R_{100} - R_0$ :*

	Thermometer T <sub>1</sub>	Thermometer T <sub>2</sub>
May 12, 1915.....	10.952 ohms.	10.930 ohms.
" 14, " .....	10.963 "	10.932 "

Accurate knowledge of the fundamental interval of thermometer  $T_2$  is all that is necessary, a matter that will come up later. The weighted mean for  $T_2$  is 10.932 ohms.

The above tests were made with currents of air flowing past the coil at two speeds and likewise with the air stagnant. No variation of consequence occurred. The thermometers, moreover, showed themselves more than sufficiently stable during tests covering a year previous. The actual data then obtained cannot be used, however, on account of subsequent slight alteration in the thermometers.

*Correction for Heating Due to Current.*—The bridge current, of course, heats to some extent the thermometer coils themselves and this must be allowed for. Since the liberation of heat is proportional to the square of the current, the resulting small rise in temperature should follow the same law. Callendar's method is to double the bridge current and subtract one third the difference of the two corresponding settings from the first setting. The author found it a little more convenient to increase the bridge current in the ratio  $1 : \sqrt{2}$ . Thus the correction is immediately the difference in the two bridge settings.

Moreover, in order to make certain that this rule held under the present conditions a direct test was made with bridge currents in the ratio  $1 : 2 : 2.5 : 4$ . The linear relation between the bridge readings and the square of the current was satisfactory. This held also in varying currents of air. Thus, where the rates of flow were 0, 2.9, 19, 36, 80, 132 and 250 cc./sec. the corrected bridge readings were 0.°016, 0.°018, 0.°015, 0.°015, 0.°015, 0.°015, 0.°014 respectively (reckoning from an arbitrary zero).

The order of magnitude of the heating correction in the final runs was 0.°01 C. The lower value of the battery current was 6 milliamperes.

Incidentally, the above data show that the length of the coil of pipe in the bath was sufficient to give the gas the temperature of the bath before it reached the plug.

*Conversion of Bridge Readings to Degrees.*—After the heating correction has been applied, the differential reading may be converted into degrees by a multiplying factor which was obtained as follows:

The unit for reading off settings was taken as one revolution of the slide wire drum and is called the "bridge unit" hereafter. Repeated standardizations of the wire at four different dates in terms of the cali-

brated ohm gave the value 0.005666 ohms per bridge unit.

Combining this with the figure just given of 10.932 ohms per  $100^{\circ}$  C. we get the factor 0.05178 degrees per bridge unit.

The values for  $\mu$  obtained from the observations in terms of bridge units per meter of mercury are convertible into their proper values in terms of degrees per meter of mercury by the application of first this factor and then formula (8).

*The Porous Plug.*—The form of plug finally adopted was suggested by the research of Regnault already alluded to. Its longitudinal section is shown in Fig. 3 (dotted shading) the material being earthenware—a Pasteur filter. As may be seen, the flow of gas through the plug is radial toward an axis, and after passing through the plug wall its outflow takes place parallel to the axis. The purpose of this

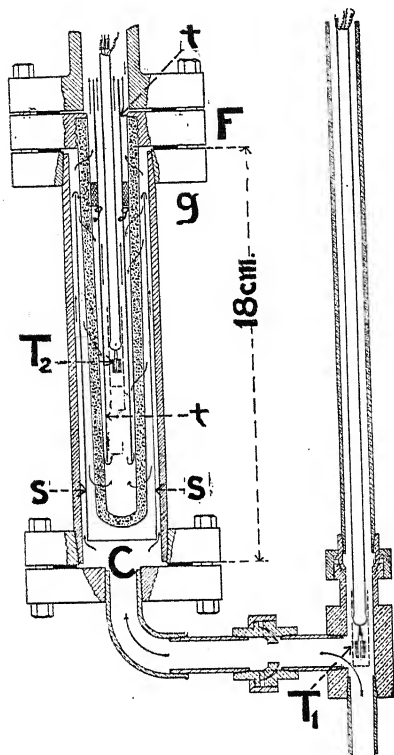


Fig. 3.

construction, as already pointed out, is to furnish thermal protection.

The whole apparatus shown in Fig. 3 is immersed in the bath, whose surface is 44 cm. above the point  $T_1$ . Air after traversing about 8 meters of pipe, in the bath, passes  $T_1$ , enters the plug casing at  $C$  and travels upward between the wall of the casing and a bright tinned radiation shield  $ss$ , as is indicated by the arrows. After passing over the top of the radiation shield it fills the space immediately outside the wall of the plug, through which it flows in two streams, divided by the "guard ring"  $gg$ . Within the plug is a glass tube  $tt$  which constrains the lower of these two streams to pass, all of it, past the thermometer  $T_2$  while the upper stream is kept from passing the thermometer. Both streams pass out finally at the top.

The purpose of the guard ring is to avoid errors arising from conduction of heat along the material of the plug from its mounting  $F$  and ultimately reaching the thermometer inside.

The temperature of the plug in general is different from that of the casing. Radiation between the two would introduce an error. The radiation shield is to prevent this. As will be seen later, the radiation shield proved to be unnecessary but it was retained as a precaution, and is free from objection.

The construction of the plug casing (Fig. 3) permits the use of the principle of interchangeable parts. It may be seen that different plugs may be bolted in and that different sizes of casing as well may be employed. This proved a valuable feature in the preliminary experiments.

#### PRELIMINARY EXPERIMENTS.

*Early Experiments with the Axial Flow Plug.*—A number of experiments were carried out at the Johns Hopkins University in 1905-6, on carbon dioxide with a straight-flow plug of the type used in the Joule-Thomson experiments. Platinum resistance thermometers were used and so arranged as to be movable. The pressure-drops were of the order of one atmosphere. Plugs of various materials were tried, eider down, cotton, silk and paper. A vacuum-jacket was attempted and, what seemed better, the guard ring principle contained in a suggestion by Buckingham<sup>27</sup> was carried out. The porous plug was coaxial with a second annular plug designed to eliminate lateral temperature gradients.

The last arrangement improved things somewhat, giving one result in agreement with those of Joule and Thomson, but the general characteristic of the experiments was their lack of reproducibility.

On displacing the low-side thermometer, for instance, a steep temperature gradient along the axis was observed. The observed cooling effect continually fell off as the thermometer was further withdrawn. A shift, for instance, from 1 to 4 and 6 cm. distances cut the effect to about one half and one third respectively.

With varying rates of flow also the effect varied, but with a tendency toward a limit for the higher speeds. Natanson observed a variation with flow also. Perhaps we can thus account for the partiality of observers for high flows and plenty of gas. This feature and the ample dimensions of the apparatus of Joule and Thomson undoubtedly contributed to the accuracy of their results.

In addition, an enormous amount of time had to be consumed before a steady state could be attained, amounting to an hour and a half before a reading could be taken. Apart from other considerations, this would lead to a large chance for error unless the pressure regulation were watched very closely. A damped oscillation in the cooling-effect similar to that in the Joule-Thomson experiments was also observed.



It was decided, then, as already mentioned, to narrow the field and to concentrate on developing a method for the attainment of reliable results. What was to be desired was thermal protection, fine pressure regulation, and good temperature control. These problems have been taken up in the manner already described. The later experiments have been carried out in the Rouss Physical Laboratory of the University of Virginia.

*Preliminary Experiments with Radial Flow Plug—Exploration.*—Referring to Fig. 3, it will be remembered that the thermometer  $T_2$  was capable of being shifted throughout the entire length of the plug. In the initial explorations the inner tube  $tt$  had not been introduced. The temperature distribution was fairly uniform except above the middle, and the correction for the heating of the thermometers due to the bridge current was much larger at the bottom of the plug, where the air-flow was slow, than at the top where the air-flow was faster. This led to the introduction of the tube and guard ring with the result that it not only reduced this heating correction but rendered the temperature distribution along the tube uniform to the order of  $0.001^\circ\text{C}$ . Closer tests than this were not pressing nor possible, for time-fluctuations due to bath and pressure fluctuations were of this order of magnitude.

The final arrangement of the plug with tube and guard ring is almost identical with the arrangement published by Burnett and Roebuck (*loc. cit.*).

*Rate of Flow.*—This was separately measured by collecting the air over a pneumatic trough. Roughly the linear speed was found to be directly proportional to the pressure drop and independent of the absolute pressures, and hence of density. The result can therefore be simply expressed by a single number, namely 2.3 cm. per sec. per cm. drop of mercury. In the final runs this would mean a maximum speed within the tube  $tt$  of 180 cm. per sec., a little more than this at the bottom of  $tt$  (Fig. 3) and about double this around the thermometer caps. In the coil leading into the plug-housing, the speed was about the same.

*Velocity Cooling.*—The upper limit of the possible errors arising from the change in kinetic energy involved in the above variation of speeds was computed to be of the order of magnitude of  $0.004^\circ\text{C}$ . Actually none was observable. For, in a run with the plug removed and with speeds of flow in the ratios 50 : 100 : 200 : 330 the bridge readings (with reference to an arbitrary zero) were  $0.037$ ,  $0.038$ ,  $0.038$ ,  $0.038$  respectively. The fastest flow exceeded the greatest speed of the air in the final runs. This run, moreover, was taken with the thermometer caps on, when the error from velocity cooling should be a maximum, as



is plain from a glance at Fig. 3. For, when the caps are removed, the speed of air past the thermometers would be less.

The formula used in this computation, namely,

$$c_p(T_1 - T_2) = \frac{1}{2}(w_2^2 - w_1^2) + g(h_2 - h_1)$$

was not strictly applicable, for, as in all deductions of this sort the temperatures could be given only by thermometers *moving with the gas*. Stationary thermometers would be expected to indicate smaller temperature differences. The preliminary experiments of Joule and Thomson, as well as those just outlined, are in agreement with this idea.

*Aspirator Effect.*—This is an effect cognate with velocity cooling. For to measure the pressure of a stream of fluid the manometer likewise should be moving with the fluid, strictly speaking. A test for error arising from this cause was made by taking readings on the differential manometer with the plug removed while the air was run through at velocities higher than those employed in the runs. The manometer showed zero reading throughout.

*Effect of Varying Rates of Flow upon the Observed Temperature Drop.*—These observations are depicted graphically in Fig. 4. Curve 2 was taken with the plug whose characteristic rate of flow has just been given,

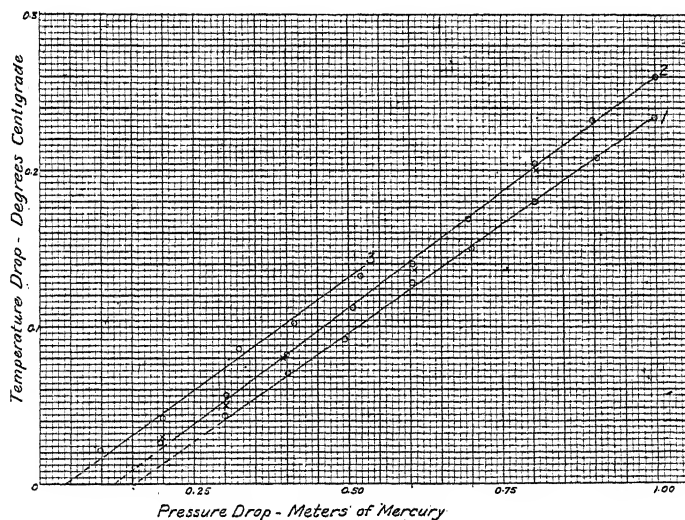


Fig. 4.

Preliminary Experiments, Varying Rate of Flow and Position of Guard Ring.

Curve 1: Relative Flow 0.7—Slope 0.28

" 2: " " 1.0— " 0.30

" 3: " " 3.0— " 0.29

Crosses: Guard Ring shifted 4 cm.—flow as in 2.

namely, 2.3 cm. per sec. per cm. of Hg pressure drop. Curve 1 applies to the same plug wrapped with rubber bands till its flow was 0.7 as great, while curve 3 is for a plaster of Paris plug whose flow was three times as great.

The first essential feature of these curves is that they are straight lines within the error of experiment, except near the origin. (This is true not only in the preliminary but in the final runs.) Near the origin the points were so irregularly distributed that they have been omitted from the diagram.

The second essential feature is that the more rapid the flow the nearer the curve approaches to a limiting line passing through the origin. A third feature to be noted is that these lines are sensibly parallel. These features will be taken up later, in discussing the effects of a residual heat leakage.

*Heat Leakage by Conduction.*—The nature of this residual heat leakage was sought in conduction, radiation and convection. Conduction through the gas does not seem probable. The only remaining path would be along the material of the plug. This point was tested by shifting

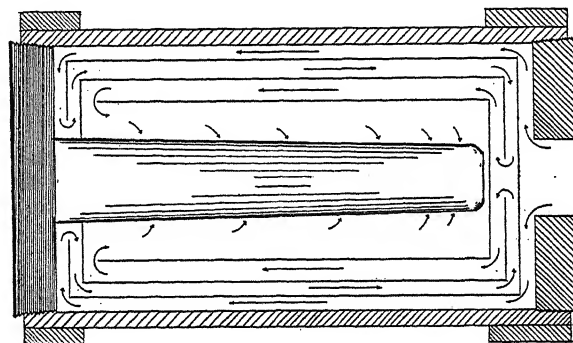


Fig. 5.

the guard-ring from a position 1.5 cm. from the plug flange *F* (Fig. 3) to a point 5.5 cm. therefrom. The results as shown by the rings and crosses (Fig. 4) rule out this source of error, the curves (No. 2) in these two cases being indistinguishable.

*Heat Leakage by Radiation.*—Since the surface of the plug must be at a different temperature (lower in case of air) from that of the walls of the plug casing, heat may pass by radiation across the annular space between them and thus reach the interior without first being absorbed by the gas itself, and so an error may be introduced. This was first tested by comparing runs with the dark iron casing walls lined with

bright tin and then without the tin. There was no systematic difference. Again a trial was made with three radiation shields on the one hand, disposed as in Fig. 5, in a large casing and on the other, with no radiation shields at all (the casing being that shown in Fig. 3). Fig. 6 shows that

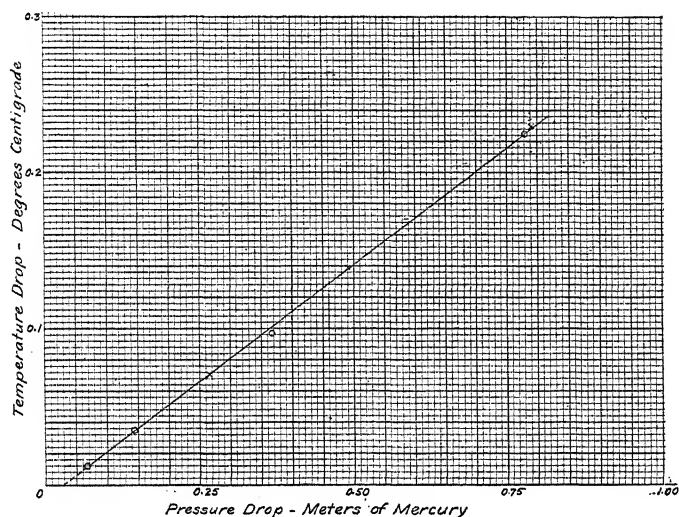


Fig. 6.

Preliminary Experiments, Effect of Radiation Shields.

Circles: No radiation shields

Crosses: 3 " "

these results also are indistinguishable. Radiation, then, is not to be looked for to explain the depression of these lines nor as a serious source of error.

*Heat Leakage by Convection.*—A turbulent motion of the gas surrounding the plug could possibly convey heat across from the walls of the casing without permanent absorption. If so, an increase in the inside diameter of the casing should reduce this error and shift the lines nearer to the origin. Now in the experiments of Fig. 6 (no shield), the annular space between the walls and the plug was about 6 mm. thick or about twice as thick as for the experiments of Fig. 4. The curve of Fig. 6 passes much nearer to the origin. Consequently convection must play a part.

It was not thought worth while to push these experiments further, the practical limit being apparently reached in the experiments of Fig. 6.

*Influence of Density.*—Where the mean density of the air is increased by operating at increased mean pressures, one should expect a minimizing effect upon the errors in the temperature readings, owing to the increased volumetric thermal capacity of the gas and the fact that the heat leakage

should not increase in the same ratio, if at all. This was verified by experimental runs, whose results are shown in Fig. 7, where it will be seen that curves corresponding to the higher pressures pass nearer the origin. Further confirmation in the final results were found and will be noted when these come up.

A second effect is observable here. The slopes of the lines fall off with increasing pressure. This again more properly comes up later, where the diminishing effect of increased mean pressure upon the Joule-Thomson coefficient is discussed.

*Derivation of the Joule-Thomson Coefficient from the Observations.*—We are now in a position to discuss the relation of  $\mu$  to these preliminary data. It is seen from what has gone before, that the continual reduction

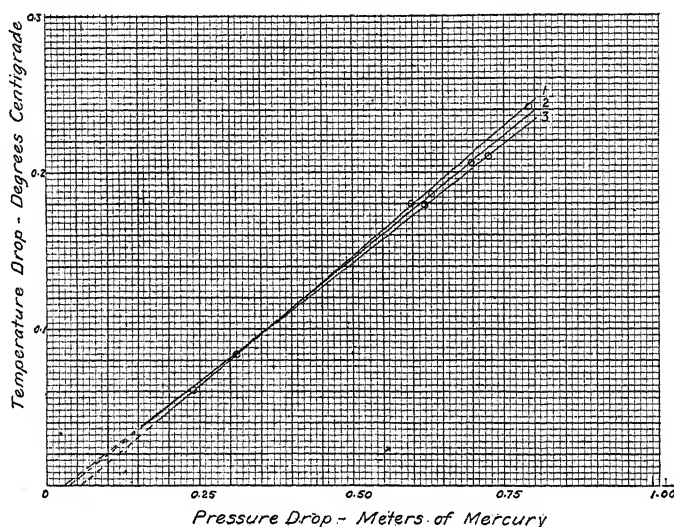


Fig. 7.

Preliminary Experiments, Varying Densities.

Curve 1: Mean pressure 2.5 meters mercury  
 " 2: " " 4.5 " "  
 " 3: " " 6.4 " "

of error by first reducing the heat-leak in obvious ways and secondly by increasing the mass of gas affected, has led in the main to the shifting of the curves of the type shown in Figs. 4, 6 and 7 so that their rectilinear portions (produced) are made to pass nearer to the origin without, however, changing their directions sensibly (exclusive, of course, of the pressure effect, which is entirely independent). The limiting ideal case, therefore, would be represented by a straight line passing through the origin, whose slope would be the true value of  $\mu$ . The obvious step then

to take is to make the actual runs under conditions as nearly ideal as feasible; and for the values of  $\mu$  adopt the values of the slopes of these straight lines without reference to their position. These values are taken to be as near the truth as the data are capable of yielding it.

The question may now be asked: Can anything be said as to the constancy of  $u + pv$  or  $H$  along these plots? It will be remembered that after the gas has passed through the plug the increment in the function  $H$  is equal to the gain of heat per gram of gas during its passage. Now it is conceivable that the heat-leak (measured in calories per second) might be of such magnitude as to make the slope of the plots too steep for a locus of constant  $H$ , or again, such as to make it too flat.

Start, for example, with one of the smaller pressure drops and suppose there is a small heat leak. The resulting temperature drop will be too small. Let this observation be represented by a point on the  $(p, t)$  diagram, say, near the origin on a plot like that of Fig. 6. The value of  $H$  for this point will be greater than its value,  $H_0$ , for the origin, which represents the state of the gas on the high pressure side of the plug. A straight line, then, drawn from the origin to a point  $A$ , say, when there is a heat leak, will pass from regions of lower to regions of higher  $H$  and will be too flat. Now suppose that the experiment be carried to a higher pressure drop and a second point  $B$  is determined. As a different and an extreme case of heat-leak, let us suppose, for the moment, that from the point  $A$  onward it remains constant. Then, since the rate of flow has been increased, due to the increased pressure-drop after we leave  $A$ , the gain per gram falls off. The line  $AB$ , therefore, passes from higher to lower values of  $H$  and is too steep. Now let us try another hypothetical law for the heat-leak, namely, that from the point  $A$  onward, it is proportional to the rate of flow of gas. Then the gain in heat per gram remains fixed and the two points  $A$  and  $B$  are characterized by the same value of  $H$ . A line plotted under this condition would be, therefore, a line of constant  $H$ , although this value would be greater than  $H_0$ . In general, then, starting from the point  $A$ : if the heat-leak varies more rapidly than the first power of the rate of flow of gas, the plot is too flat; if, less rapidly, the plot is too steep. In all cases the plot will lie under the line  $H_0 = \text{const.}$

The hypothesis that the heat-leak is proportional to the rate of flow of the gas seems a reasonable one to make for actual Joule-Thomson experiments, at least as a first approximation. For, as has been seen, the rate of flow is approximately proportional to the pressure-drop, which in turn is very closely proportional to the temperature-drop, and the latter again in turn would be expected to be proportional to the heat-leak,

certainly as far as conduction and radiation are concerned, and possibly convection too. In the case of convection, however, there seems to be a possibility that the leak would be speeded up in the case of turbulent motion and cut down in the case of stream-line motion, thus producing deviations from the proportionality relation on one side or the other. At present there is no way of deciding this point but the deviation in any case should be small.

If, then, the total heat-leak be small and if its deviation from being proportional to the rate of flow be likewise small, the deviations of our plots from true curves  $H = \text{const.}$  should be small to quantities of the second order of errors. It is hoped that this may be reasonably claimed for the final runs described in this paper.

In the light of what has just been developed, the actual plots are now susceptible of a further interpretation. The curve of Fig. 6, if completed by connecting up with the origin by the experimental determination of the requisite further points would of course curve upward in this region. This was actually done in some instances although some of the points were uncertain. This curvilinear portion of the plot, then, would represent a heat-leak varying more rapidly than a direct proportion of the flow-rate, while the remainder would represent a heat-leak proportional to this rate. Both the excess heat-leak and the uncertainty of the points near the origin are to be expected on account of the very slow, almost stagnant, flow of the air in experiments with such small pressure-drops.

As a matter of fact it is physically impossible for anything but an excess heat-leak to occur for this region. Otherwise we should have two curves,  $H$  and  $H_0$  say, for which  $H - H_0$  is finite, approaching intersection as a limit at an arbitrary point, namely, the origin of the diagram. We have, then, in other words, the surprising inference that it is impossible, in a throttling process, for a heat-leak to be continuously proportional to the rate of flow of gas below a finite limit of the flow and, likewise, of the pressure drop, although this would seem quite possible otherwise.

Referring now to Fig. 4, where the rate of flow of the air has been varied, other things being unchanged, by altering the porosity of the plug, the plots are explicable on the ground that the heat-leak is proportional to the rate of flow of the air in each case but that the constant of proportionality is smaller the greater the porosity of the plug.

*Zeros of the Differential Thermometers.*—In the determination of the slope of the lines just discussed, it is obviously unnecessary to know the precise value of the bridge setting when the pressure-drop is zero. This



knowledge was useful in the preliminary experiments in affording an index of the magnitude of the heat-leak. In the final run, therefore, the determinations of the zeros were discontinued. For they required time and at best were uncertain. The conditions for accuracy in a body of stagnant air in the pocket-like interior of the plug in the position shown in Fig. 3 are obviously unfavorable. The only way to put this accuracy on a par with the other readings would be to provide a way of opening the plug and sending a current of air through.

A matter well worth noting here is that, when this method of observation is employed, one is relieved of the necessity of a close adjustment to equality of the differential thermometers. The high side thermometer serves in reality merely as a sort of compensator for error due to fluctuations in the bath temperature, and an accurate knowledge of it even is unnecessary. For example the fundamental intervals of the two thermometers  $T_1$  and  $T_2$  differed by 0.3 per cent. Now suppose the temperature of the bath during a run changed by as much as 0.01 degree. Then the differential reading, after the steady state had been reached, would be changed by 0.00003 degree.

*Values given for the Temperatures and Absolute Pressures.*—In what follows  $\mu$  will be discussed as a function of temperatures and pressures. The pressures will be the means of the pressures above and below the plug. On the diagram they will be represented by the mid-points of the limited straight lines there plotted. As to temperatures, the variations are too small to make it worth while to correct the bath temperatures.

*Error from Moisture.*—Tests made at a fixed temperature and pressure, in which the air from the room was passed through long-used calcium chloride first; then the drying material was omitted; finally fresh calcium chloride was put in. The values obtained for  $\mu$  were 0.35, 0.34, and 0.33 degrees per meter respectively, showing a small increase due to moisture (and the poor quality of old calcium chloride). One should expect, then, that a final drying with phosphorus pentoxide would suffice. In the final runs this was used, with the exception of those at 30° C., and these, in spite of that, seem to fit well into the empirical formula derived from all the observations, as will appear later. The phosphorus pentoxide could be continually observed from the outside during the runs, and it was only at the ends of the longer ones that it had a gummy appearance.

*Conduction of Heat along the Thermometer Leads.*—After the final runs had been made, it was suspected that there might be an error arising from this cause. A new set of runs with the thermometer caps removed, exposing the thermometer wire to the stream of gas would test this point.

A number of observations were, therefore, made covering the same range of temperature and pressure. The galvanometer, of course, was quite unsteady under these conditions, but the agreement between the new and the old data was satisfactory. Thus the mean of ten runs with the caps off gave for  $\mu$  a value 0.265 while the mean of ten runs at the same temperatures and pressures with the caps on gave 0.267 degrees C. per meter of mercury pressure-drop. This was well within the accidental error of the runs with caps off.

*Regeneration Effect.*—After the experiments described in this paper had been completed, the paper by Trueblood, referred to in the beginning, appeared, in which is described a form of heat-leak arising from the peculiar geometry of the radial flow apparatus. The nature of this regeneration effect can best be described by quoting from Trueblood's paper: "At its open end, the plug is necessarily brought, for purposes of mechanical support, into good thermal contact with the metal walls of the case. As a result, fluid which has passed the low side thermometer is provided with a thermally short path of communication with fluid which has not yet passed through the plug. This results in a depression of the temperature of the latter, which, joining the low side fluid at a temperature therefore lower than then exists there, contributes to a still further depression of the temperature of the fluid which has not yet passed. This cumulative action, similar in principle to that taking place in certain types of liquid air machines, is limited by the ability of the bath to supply heat to the low side fluid. . . . The effect of the action just described on the observed temperature drop is indirect, since the fluid directly affected does not pass the low side thermometer. But the temperature depression produced in the neighborhood of the open end produces indirectly, by conduction through the walls of the plug, the walls of the case, the inside lagging and, doubtless, to some extent through the fluid itself, a depression of the temperature of fluid which does pass the low side thermometer; that is, there is an outward leak of heat from fluid between thermometers to fluid which has passed the low side thermometer with the result that the observed temperature drop is too high."

In the author's apparatus the regeneration effect, he believes, was negligible, although, as a matter of fact, its possibility was not foreseen. For, under the heading *heat leakage by conduction*, it was shown that a shift in position of the guard ring did not lead to a change in reading. Now since the temperature depression of the low side gas is due to the "regeneration" process should be manifest in greater and greater degree at points nearer and nearer to the mechanical support, it should have been shown up by the test in question, which, as has just been seen, was not



the case. This result may have been due to the unusually long and narrow proportions of the plug used. A natural remedy for the regeneration effect or any effect arising from conduction would seem to be the employment of a long plug.

## FINAL RESULTS.

Between August, 1915, and January, 1916, fourteen determinations of the Joule-Thomson coefficient ( $\mu$ ) were made at five temperatures between 0° C. and 100° C. and at mean pressures of 2.5, 4.5 and 6.4 meters of mercury.

In Table I. are collected the data summarized from each run. The first four columns need no comment. The "bridge readings" of column 5 correspond to the temperature-drop, without being reduced to degrees.

TABLE I.

1. Date.	2. Bath Temp.	3. Pressure (Meters).	4. Pressure Drop (Meters).	5. Bridge Readings.	6. Slope (Bridge Units per Meter).	Residuals.		9. Wt.	10. Intercepts (Bridge Units).
						7. Bridge Units.	8. Degrees.		
1916									
Jan. 10	15° 9	2.45	.763	14.26	0.698	—	—	0.2	
" "		6.50	.260 .694 .263	10.75 13.53 10.85	0.622	—	—	0.3	
1915									
Aug. 20	30° 0	2.49	.792 .599 .311 .241	4.67 3.48 1.63 1.18	0.635	-0.01 +0.02 0.00 -0.01	-0.0005 +0.001 .0000 -0.0005	1.0	-0.34
" 23	"	4.48	.697 .631 .298 .199	3.98 3.59 1.60 0.94	0.607	-0.01 0.00 +0.03 -0.03	-0.0005 0.0000 +0.0015 -0.0015	1.0	-0.24
" 26	"	6.40	.726 .620 .310 .255	4.07 3.46 1.63 1.32	0.588	-0.01 -0.02 0.00 0.01	-0.0005 -0.001 0.0000 +0.0005	1.0	-0.19
Nov. 24	50° 0	2.50	.747 .653 .343 .264	4.12 3.54 1.84 1.39	0.560	+0.02 -0.03 +0.01 0.00	+0.001 -0.0015 +0.0005 0.0000	1.0	-0.08
Nov. 29	"	4.50	.820 .689 .309 .193	4.37 3.77 1.71 1.03	0.535	-0.05 +0.05 +0.03 -0.03	-0.0025 +0.0025 +0.0015 -0.0015	0.5	+0.03
Dec. 1	"	6.33	.790 .711 .307 .193	4.28 3.90 1.74 1.14	0.529	-0.02 +0.02 0.00 +0.01	-0.001 +0.001 0.0000 +0.0005	1.0	+0.12

TABLE I. (Continued).

1.	2.	3.	4.	5.	6.	Residuals.		9.	10.
Date.	Bath Temp.	Pressure (Meters).	Pressure Drop (Meters).	Bridge Readings.	Slope (Bridge Units per Meter).	7. Bridge Units.	8. Degrees.	Wt.	Intercepts (Bridge Units).
1915									
Dec. 6	70°.0	2.53	.769 .500 .259	3.93 2.54 1.41	0.496	+0.02 -0.04 +0.02	+0.001 -0.002 +0.001	1.0	+0.1
Dec. 8	"	4.36	.747 .497 .247	3.84 2.66 1.44	0.480	-0.01 +0.01 -0.01	-0.0005 +0.0005 -0.0005	1.0	+0.26
Dec. 13	"	6.40	.718 .498 .255	3.72 2.70 1.55	0.468	0.00 +0.01 0.00	0.0000 +0.0005 0.0000	1.0	+0.36
Dec. 16	90°.0	2.44	.746 .509 .257	4.87 3.83 2.71	0.441	0.00 0.00 0.00	0.0000 0.0000 0.0000	1.0	+1.57
"	"	"	4.52 .747 .495 .248	4.91 3.88 2.81	0.421	-0.01 +0.02 -0.01	-0.0005 +0.001 -0.0005	1.0	+1.78
Dec. 15	"	6.35	.743 .506 .253	3.51 2.60 1.52	0.406	-0.02 +0.04 -0.02	-0.001 +0.002 -0.001	1.0	+0.51

This reduction is more simply done later. Column 6 gives the slope of the straight line that would be formed by plotting after the manner of Figs. 6 and 7, but actually they have been calculated by the method of least squares. The residuals in temperature, shown in column 8, give no evidence of a systematic deviation from the straight line and, moreover, the order of their magnitude is seldom over  $0^{\circ}.001$  C. The weights (column 9) assigned to the several runs are the same for all except three, where experimental difficulties of a temporary sort turned up during the respective runs. In the two runs of January 10 it was impractical to get more than two points for each straight line, so there was no least square reduction. That carried out on November 29 suffered from a derangement of the pressure controlling apparatus. The later runs included one less point each but the gain in observing experience about balanced the omission of one point, and so the weights for these were kept at unity. The "intercepts" (column 10) on the temperature axis, are known to within additive constants only, except for the runs at  $30^{\circ}$  C., where zero readings of the differential thermometers were taken. The only purpose these data serve here is to furnish further evidence for the conclusions stated under the head "*influence of density*." It will be seen that with ascending pressures, hence increasing densities, the intercepts progressively rise along the temperature axis, with one exception.

*The Joule-Thomson Coefficient Expressed as a Function of Temperature and Pressure.*—The values of the slopes (column 6) reduced to the proper units are taken as the values of  $\mu$  for reasons already given. This reduction is now made by the use of the multiplying factor 0.5178 together with formula (8).  $\mu$  is then expressed in degrees C., hydrogen scale, temperature-drop per meter of mercury pressure-drop.

The fourteen values of  $\mu$  thus obtained are classified in Table II.

TABLE II.

Approximate Pressure. (Meters.)	15° C.	30° C.	50° C.	70° C.	90° C.
	.358	.327	.290	.258	.232
2.50	(+.008)	(+.002)	(-.002)	(-.002)	(+.002)
	(+.002)	(+.001)	(.000)	(-.001)	(+.001)
		.312	.277	.250	.220
4.50		(-.001)	(.000)	(-.001)	(-.001)
		(-.002)	(-.002)	(.000)	(-.002)
	.319	.302	.274	.244	.213
6.40	(-.006)	(.000)	(+.002)	(+.002)	(-.001)
	(-.010)	(.000)	(+.004)	(+.003)	(-.002)

Upper line of residuals—five-constant formula.

Lower line of residuals—three-constant formula.

according to temperature and pressure. To these values were fitted, by least square computation, two empirical formulas, one a five-constant formula in  $t$ , the other a three-constant formula in  $t + 273$  or  $\tau$ . In the same table are included in parentheses, under each observed value of  $\mu$ , the residuals (obs.-calc.) obtained with each of these formulas, the upper one applying to the former, the lower, to the latter formula.

The five-constant formula was tried following upon an initial inspection of the observations, which showed that the variations of  $\mu$  were linear in  $p$  but not linear in  $t$ . Consequently a second degree term in  $t$  was included. The formula is

$$\mu = +0.3935 - 0.00691p - 0.001835t + 0.00000134t^2 + 0.0000325pt, \quad (9)$$

the units being as above specified.

Five constants make an unwieldy formula and seemed quite too many for the limited range of temperatures and pressures here employed. Resort was then had to the three-constant formula (4) suggested by the use of the equation of van der Waals. The least square reduction for this gave

$$\mu = -0.2599 + 181.96 \frac{1}{\tau} - 552.4 \frac{p}{\tau^2}, \quad (10)$$

where  $\tau = 273 + t$ . This formula is plotted in Fig. 8, which shows likewise the observed points.

The five-constant formula agrees a trifle more closely with the observations than does the other, as should be expected, having two more constants, but there is little to choose between them in this respect. All things considered, the three-constant formula is to be preferred; it is more rational and is simpler.

The computations of these formulæ were subjected to the standard

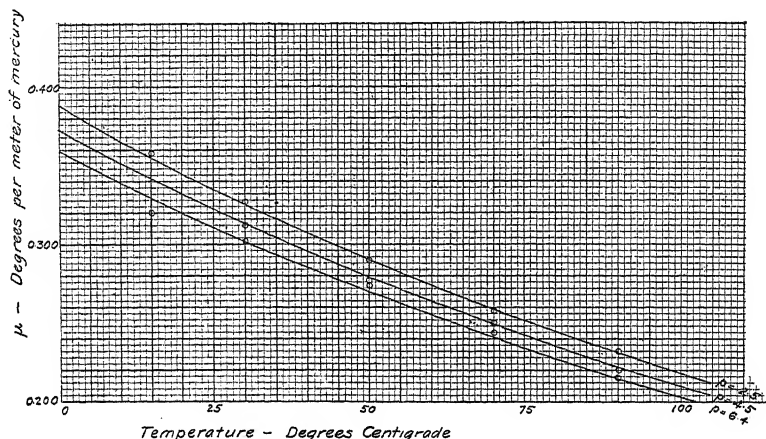


Fig. 8.

Joule-Thomson Coefficient, as a Function of Temperature and Pressure.

Curves are graphs of the formula:

$$\mu = -0.2599 + 181.96 \frac{1}{\tau} - 552.42 \frac{p}{\tau^2}$$

Circles are observed values.

numerical checks throughout. The smallness of the residuals, being on the whole less than one per cent. would indicate, at least for the present observations, that a better formula is not worth looking for.

*Comparison with Other Results.*—Joule and Thomson gave as their formula for air

$$\mu = 0.92 \left( \frac{273.7}{T} \right)^2 \text{ degrees C. per 100 inches of mercury,}$$

which, by reduction, is

$$\mu = 0.362 \left( \frac{273.7}{T} \right)^2 \text{ degree C. per meter of mercury.}$$

Their data have been embodied by Rose-Innes (*loc. cit.*) in the formula

$$\mu = -0.275 + 178 \frac{1}{T} \text{ reduced to degs. C. per meter of mercury,}$$

which resembles (10) omitting the pressure term. The observations of Joule and Thomson, it will be remembered, did not indicate an effect of pressure.

Buckingham, by making use of the results of Kester on carbon dioxide and of Joule and Thomson on oxygen, air, nitrogen, and hydrogen, applied the law of corresponding states and found two formulas of similar type, one of which is

$$\mu = \frac{a_1}{\tau - b_1} - d_1, \quad \tau = 273 + t.$$

A few numerical values calculated by the aid of this formula and the values given for the constants, will be found further on in Table III.

The only experiments published showing a pressure effect for air are those of Vogel and of Noell. The latter, being performed subsequently in the same laboratory may be considered as superseding the former. However, it is well to state Vogel's formula, namely,

$$\mu = 0.268 - 0.00086p \quad (\text{pressures in kg. per cm}^2),$$

which holds for the fixed temperature only.

The lengthy formula of Noell is quoted near the beginning of this paper. The pressure coefficient at 10° C. is practically the same in both cases. At 0° C. Noell's pressure coefficient is - 0.0015 while that found by the author is - 0.0074 the units in both figures being degrees C. and meters of mercury. The agreement here leaves something to be desired but it is well to note that the greater portion of the data to which Noell's formula was adjusted lay in a range of much higher temperatures and pressures than the temperatures and pressures of the present paper. It is interesting to note that Noell's formula is also linear in  $p$  and that both coefficients are negative.

Further corroborative evidence as to the negative sign may be found in the calculations of Dalton (*loc. cit.*) for hydrogen based on the empirical equations of state of Kamerlingh-Onnes. He plots numerous curves of the integrated Joule-Thomson effect, or in other words curves  $H = \text{const.}$  over a wide range in temperature and pressure, *i. e.*, from 1 to 100 atm. and - 215° C. to + 17° C. These curves in the ( $p, t$ ) plane were all concave downward, showing a negative pressure coefficient of  $\mu$  for hydrogen, over the whole field, which covered some inversion points and both positive and negative values of  $\mu$ . One would expect that an application of the law of corresponding states might lead to the same thing for all gases, although a general statement of this sort could not be made. In fact, accepting van der Waals' equation for the moment, a

glance at equation (3) shows that the sign of the expression

$$\left(6b - \frac{4a}{RT}\right)$$

determines the sign of the pressure coefficient. In regard to carbon dioxide, Natanson found a positive coefficient, and Kester found no dependence upon pressure except at 26.4 atm. and here an increase in  $\mu$  over its value at lower pressures was indicated. The matter on the whole for carbon dioxide seems undecided.

In Table III. are set forth for comparison a few numerical values for  $\mu$  calculated from the formulæ of other observers above quoted (omitting those of Vogel and Rose-Innes). In general the values of the present paper exceed the other values. The former, at 4.5 meters pressure are in closest agreement with those of Buckingham. It is of interest to note, however, that, choosing a temperature of 0° C. for example, the integrated cooling effect over a drop of about 7 atmospheres calculated by the author's formula agrees with that set down for "corresponding states." This was roughly the pressure-drop in the experiments of Joule and Thomson.

TABLE III.

t° C.	Joule and Thomson.	Corresponding States. (Buckingham.)	Noell $p = 1m.$	Present Paper.		
				$p = 1m.$	$p = 4.5m.$	$p = 6.4m.$
0	.362	.374	.365	.399	.373	.359
50	.259	.270	.244	.298	.278	.270
100	.194	.212	.165	.224	.210	.203

*Calculation of the Temperature of the Ice Point on the Thermodynamic Scale.*—The general relations are

$$\frac{v_{100}}{T_{100}} - \frac{v}{T_0} = \int_{T_0}^{T_{100}} \frac{\mu C_p}{T^2} dT \quad (\text{pressure} = \text{const.}), \quad (II)$$

$$v_{100} = v_0(1 + 100\alpha),$$

$$T_{100} - T_0 = 100.$$

The first of these is one of the two ways of integrating equation (2); the second equation defines, in terms of experiment, the mean temperature coefficient at constant pressure of the gas used in the experiment; while the last is a mathematical definition. The limits of integration are the two temperatures of the ice point and the steam point.

If we factor out a mean value of  $c_p$  (which varies but slightly within these limits) calling it  $\bar{C}_p$  and call the value of the remaining definite integral  $(I_{100} - I_0)$ , noting that  $1/v_0 = \rho_0$  the density of the gas at the

given constant pressure and at the ice point, then the combination of these three relations results in

$$T_0 = \frac{I}{\alpha} + \frac{T_0(T_0 + 100)}{100\alpha} \rho_0 \bar{C}_p (I_{100} - I_0)$$

without approximation.

The first term on the right is the temperature of the ice point on the scale of the gas in question. The second term is the quantity that must be added thereto in order to derive the same temperature on the thermodynamic scale. It is small and may be called a "correction." Its relative magnitude will be seen to come out only about 0.0037 times the first term.

Now the accuracy of this term is limited by that of the determinations of  $\mu$ , namely an order somewhat under one per cent. Expressed in degrees this would come to about 0°.01 C. The following two approximations therefore are permissible: First,  $T_0$  may be replaced by 273; secondly, the evaluation of the integral

$$\int_{T_0}^{T_{100}} \frac{\mu}{T^2} dT,$$

in which formula (9) or formula (10) must be used,  $t$  may be treated as if it were  $T - 273$  in formula (9) and in formula (10)  $\tau$  may be treated as if it were  $T$ . Further, or successive approximations are unnecessary. The above formula, with these approximations, then becomes

$$T_0 = \frac{I}{\alpha} + \frac{273 \times 373}{100\alpha} \rho_0 \bar{C}_p (I_{100} - I_0). \quad (12)$$

The value of  $(I_{100} - I_0)$  has been computed and found to be  
+ 0.000306 by (10), the three-constant formula for  $\mu$ ,  
and

+ 0.000308 by (9), the five-constant formula for  $\mu$ ,

the unit of pressure still being one meter of mercury at 0° C. in this locality.

The expression of this result in terms of absolute units and the calculation of the remainder of the formula were carried out by means of the following data:

density of mercury = 13.595 gm. per cm<sup>3</sup>,

$g = 980$  cm. per sec<sup>2</sup>,

$\bar{C}_p = 241$  cal per gm. per deg.,

1 cal. =  $4.187 \times 10^7$  ergs.

$\rho_0 = 0.001293 \frac{100}{76}$  gm. per cm<sup>3</sup>,

$\alpha = 0.00367282$ .

The value for  $g$  is rounded off from 979.937, the figure given as determined in this locality by the U. S. C. and G.S.; that for  $\bar{C}_p$  is a mean taken from the papers of Swann and Scheel and Heuse already mentioned. The value for  $\alpha$  was determined with great care and precision by Chapuis<sup>37</sup> at the standard thermometric pressure of one meter of mercury. The reciprocal of this is 272.270.

The correction term in (12) obtained by use of these data are

1.093 by the three-constant formula

and

1.100 by the five-constant formula,

whence, for the thermodynamic temperature of melting ice, we get

$T_0 = 273^\circ.36$  by the three-constant formula

and

$T_0 = 273^\circ.37$  by the five-constant formula.

The other most recent calculations are by Buckingham (1908) using the Joule-Thomson coefficient for various gases adjusted into a single reduced formula by the "law of corresponding states," and by Berthelot (1907) using the characteristic equation. Previously in 1903, he had computed values on the basis of the Joule-Thomson effect. These results are collected in Table IV.

TABLE IV.

Author.	$T_0$ .	Gas.	Method.	Reference Used Here.
Buckingham (1908)	273.273	Air	J-T effect	Buckingham <sup>31</sup> (1908)
	273.286	N <sub>2</sub>	"	"
	273.049	H <sub>2</sub>	"	"
	273.267	CO <sub>2</sub>	"	"
Berthelot (1903)	273.13	H <sub>2</sub>	"	Buckingham <sup>31</sup> (1907)
		N <sub>2</sub>		
		CO <sub>2</sub>		
		Air		
Berthelot (1907)	273.04	H <sub>2</sub> in Pt-Ir (1)	Characteristic equation	Berthelot <sup>32</sup> (1907)
	273.07	" " " (2)		
	273.10	" in <i>verre dur</i>		
	273.09	N <sub>2</sub>		

The value arrived at in this paper is higher than any of these; and, further, it will be noted that the method by the Joule-Thomson effect gives higher values than does the method using the characteristic equation of the gas. This fact has been noted by Buckingham and others but has not been considered to be satisfactorily explained. Rose-Innes and Berthelot (the latter while discussing his values obtained from hydrogen)



have suggested the possibility of a surface action at the walls of the containing bulb, such as the formation of a layer of gas whose density is different from that to be found in the interior. This, of course, affects the main term  $1/\alpha$ .

*Pressure Coefficient of  $\alpha$ .*—A partial comparison may be made here by taking into consideration the pressure coefficient of  $\alpha$  found by Chappuis<sup>37</sup> for nitrogen and that deducible for air from the results of this paper. Reference to equation (12) will show that the correction term is proportional to the density  $\rho_0$  (neglecting the pressure effect in  $\mu$ , an influence of the second order) and hence proportional to the pressure, which, in the present units, has the value 1.  $T_0$ , being a constant, this term is, therefore, the pressure coefficient of the reciprocal of  $\alpha$ . The pressure coefficient for  $\alpha$ , then, is computed to be 0.0000147 for air, while Chappuis finds the value 0.0000119 for nitrogen, or about four fifths as much, an agreement which, at least, is qualitative.

*Correction to the Constant-pressure Air Thermometer between the Ice and Steam Points.*—Here, since by definition, the centigrade gas and centigrade thermodynamic scales must agree at both end-points, a closer agreement among the results from various sources is to be expected.

We have, as before, the general relations

$$\frac{v}{T} - \frac{v_0}{T_0} = \int_{T_0}^T \frac{\mu c_p}{T^2} dT \equiv \bar{c}_p(I - I_0)$$

and

$$v = v_0(1 + \alpha t),$$

defining  $t$  on the gas scale in question.

Now let

$$T - T_0 = t_a$$

defining  $t_a$  on the thermodynamic scale and

$$T_0 = \frac{1}{\alpha} + \eta$$

where  $\eta$  stands for the correction term in (12).

Therefore, since  $v_0 = 1/\rho_0$ ,

$$T_0(1 + \alpha t) - T = TT_0\rho_0\bar{c}_p(I - I_0).$$

The left-hand member becomes

$$-t_a + T_0\alpha t = -t_a + t + \alpha\eta t.$$

Therefore the formula at once reduces to

$$t_a - t = \alpha\eta t - TT_0\rho_0\bar{c}_p(I - I_0),$$

in which  $t_a - t$  is the correction sought.

Replacing  $\eta$  now by its expression in (12) and noting that, for the same reasons as before it is permissible, in the right-hand member, to identify  $t$  with  $t_a$ ,  $c_p$  with  $C_p$ , and to replace  $T_0$  and  $1/\alpha$  by 273, the equation finally becomes

$$t_a - t = 273\rho_0\bar{c}_p\{3.73(\dot{I}_{100} - I_0)t - (I - I_0)(t + 273)\}, \quad (13)$$

which, inspection shows, vanishes at both the ice and steam points, as, of course, it should. The values of  $\bar{C}_p$  and  $\rho_0$  are the same as those given for the computation of (12).  $\bar{C}_p$  and  $\bar{c}_p$ , strictly speaking, are not identical, owing to the differing limits of their respective integrals, but it is obvious that their difference is not sufficient to affect the computations.

Corrections ( $t_a - t$ ) calculated by this formula for every 10° C. are to be found in the last column of Table V. The other columns are copied from Buckingham's article in the Phil. Mag., 1908.<sup>31</sup> The corrections are negative, that is, the numbers in the table are to be subtracted from the constant-pressure readings. The new values and those of Berthelot in this instance are seen to be practically identical. Chappius (*loc. cit.*), moreover, states that the air and nitrogen scales are sensibly the same in this interval; so that we have here a real basis for comparison. This is agreement within the accuracy of gas thermometry.

TABLE V.

*Thermodynamic Corrections to the Constant Pressure Nitrogen Scale Obtained by Various Writers, Compared with the Corrections to the Constant Pressure Air Scale Deduced from the Joule-Thomson Effect. 1,000 mm. = pressure.*

°C.	Rose-Innes, 1901.	Callendar, 1903.	Berthelot, 1903.	Buckingham, 1907.	Mean.	This Paper (Air), 1916.
10	0.0120	0.0109	0.010	0.0078	0.010	0.0100
20	0.0205	0.0188	0.017	0.0137	0.017	0.0170
30	0.0261	0.0236	0.022	0.0179	0.022	0.0212
40	0.0288	0.0260	0.024	0.0203	0.025	0.0235
50	0.0289	0.0260	0.024	0.0209	0.025	0.0238
60	0.0269	0.0240	0.022	0.0198	0.023	0.0218
70	0.0228	0.0204	0.019	0.0172	0.020	0.0189
80	0.0168	0.0151	0.014	0.0129	0.015	0.0141
90	0.0092	0.0081	0.007	0.0071	0.008	0.0068

*Variation of  $c_p$  with Pressure.*—Recent work of Holborn and Jacob<sup>38</sup> on  $c_p$  for air at 60° C. and at pressures from 1 to 200 atmospheres affords a further comparison of the present work with that of other observers in that the Joule-Thomson effect can be utilized to evaluate the pressure coefficient of  $c_p$ .

Writing equation (2) in the form

$$\mu c_p = T \left( \frac{\partial v}{\partial T} \right)_p - v$$

and differentiating with respect to  $T$  at constant  $p$ , we get

$$\mu \left( \frac{\partial c_p}{\partial T} \right)_p + c_p \left( \frac{\partial \mu}{\partial T} \right)_p = T \left( \frac{\partial^2 v}{\partial T^2} \right)_p = - \left( \frac{\partial c_p}{\partial p} \right)_T, \quad (14)$$

the last member following directly from a well-known equation in thermodynamics.

Now the left-hand member can be evaluated by using either formula (9) or (10) for  $\mu$  and (say) Swann's values (*loc. cit.*) of  $c_p$  and  $(\partial c_p / \partial T)_p$ . It should be noted that, in regard to units,  $c_p$  may be expressed in calories per gm. per deg., since the equation is homogeneous in that physical quantity. Also since  $\mu$  is here expressed in degrees per meter of mercury and since the data of Holborn and Jacob for  $(\partial c_p / \partial p)_T$  are expressed in atmospheres for the pressure unit, the left hand member must be multiplied by 0.76 to make the two sets of results comparable. Finally  $T$  must be put equal to  $273 + 60 = 333$ , the temperature of Holborn and Jacobs' experiments.

Now Swann gives at one atmosphere,

$$c_p = 0.241 \quad \text{and} \quad \left( \frac{\partial c_p}{\partial T} \right)_p = 0.000016,$$

then by the 3-constant formula (10) also at 1 atmosphere

$$\mu \left( \frac{\partial c_p}{\partial T} \right)_p = 0.0000047,$$

$$\text{and} \quad c_p \left( \frac{\partial \mu}{\partial T} \right)_p = -0.0003901.$$

The sum of these two (of which the first is almost negligible) is  $-0.0003854$ . Multiplying by 0.76 we get

$$\left( \frac{\partial c_p}{\partial p} \right)_T = 0.000293 \text{ cal. per gm. per deg. per atm.}$$

The value of this quantity from Holborn and Jacobs' empirical interpolation formula, namely,

$$10^4 c_p = 2413 + 2.86p + 0.0005p^2 - 0.00001p^3,$$

is

$$0.000286 \text{ cal. per gm. per deg. per atm.}$$

and from their observations at their lowest range of pressures alone (namely 1 to 25 atmospheres) it is

$$0.000312 \text{ cal. per gm. per deg. per atm.}$$

The agreement here is about as good as would be expected. Moreover it is corroborative as regards a comparison between two widely different forms of continuous flow experiments.

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#### ERRATA.

Vol. XIII., April, 1919, in the article by A. F. Kovarik, entitled "The Automatic Registration of  $\alpha$ -particles,  $\beta$ -particles and  $\gamma$ -ray and X-ray Pulses," page 277, second line from bottom, the first  $P_2$  should read  $P_1$ ; page 278, line 28,  $4.77 \times 10^{-11}$  should read  $4.77 \times 10^{-10}$ ; page 280, line 32, first word "or" should read "for."

Vol. XIII., April, 1919, in the abstract by Irwin G. Priest, entitled "A New Formula for the Spectral Distribution of Energy from a Complete Radiator," in equations (2) and (3), pages 314 and 316,  $A - \frac{1}{3}$  should read  $A^{-1/3}$ ; line 6, page 317, "are" should read "an"; line 10, page 317,  $D^3$  should read  $D_2$ .

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